polarization can be expressed in terms of circular, this should strike you as odd—there's a subtlety here, called *topological phase*, that makes it all come out right in the end.

6.2.4 An Often-Ignored Effect: The Pancharatnam-Berry Topological Phase

When light traverses a non-planar path, *e.g.* in two-axis scanning, articulated periscopes, or just piping beams around your optical system, its polarization will shift. For reflection off mirrors, this isn't too hard to see: since **E** is perpendicular to **k**, a mirror whose surface normal has a component along **E** will change **E**. Make sure that you follow your polarization along through your optical system, or you may wind up with a nasty surprise.

A much more subtle fact is that the same is true for any system where light travels in a non-planar path, *e.g.* a fiber helix. Left and right circular polarizations have different phase shifts through such a path, giving rise to exactly the same polarization shift we get from following the mirrors; this effect is known as Pancharatnam's topological phase†, and is what accounts for the puzzling difference in the polarization behavior of linear and circularly polarized light upon reflection that we alluded to earlier (the corresponding effect in quantum mechanics is Berry's phase, discovered nearly 30 years after Pancharatnam's almost-unnoticed work in electromagnetics). This sounds like some weird quantum field effect, but you can measure it by using left and right hand circular polarized light going opposite ways in a fiber interferometer‡. These polarization shifts are especially important in moving-mirror scanning systems, where the resulting large polarization shift may be obnoxious. Thus it's often best to use circular polarization in scanning systems, as in the ISICL sensor of Example 1.12.

It sounds very mysterious and everything, but really it's just a consequence of spherical trigonometry; the **k** vector is normal to a sphere, and **E** is tangent to the sphere throughout the motion; depending on how you rotate **k** around on the surface, **E** may wind up pointing anywhere. Equivalently, 2×2 rotation matrices commute, but 3×3 ones don't.

If you follow your **k** vector around a closed loop enclosing a solid angle Ω, the relative phase of the right and left circular polarizations gets shifted by

$$\Delta\phi = 2\Omega. \tag{6.1}$$

A linearly-polarized beam will have its axis rotated by $\Delta\phi$. This is important in cases such as corner cube reflectors: a hollow retroreflector makes the **k** vector describe an eighth of a sphere ($\pi/2$ sr), so a full circuit rotates a linear polarization by 180°. However, a retroreflector performs only half of a full rotation, so the polarization shift is half that, *i.e.* 90°. This is pretty useful in interferometers based on polarizing cubes, because most of the light reflected on the first pass through the cube gets transmitted on the second pass.

Gotcha: Angle-Dependent Polarization Shifts In Metal Mirrors

Besides the geometric phase, real mirrors generally introduce a bit of ellipticity, especially at shorter wavelengths or with poorer coatings such as protected aluminum.

† S. Pancharatnam, 'Generalized theory of interference and its applications. Part 1. Coherent pencils', *Proc. Indian Acad. Sci* **44**, 2247–2262 (1956).

‡ Erna M. Frins and Wolfgang Dultz, 'Direct observation of Berry's topological phase by using an optical fiber ring interferometer', *Opt. Commun.* **136**, 354–356 (1997)

It can be a surprisingly large effect—up to a half wave of aberration difference between polarizations—so it's worth keeping track of it. Karlton Crabtree has a an excellent tutorial on this.†

6.2.5 Orthogonal Polarizations

We often describe two polarization states as *orthogonal*. For linear polarizations, it just means perpendicular, but what about circular or elliptical ones? The idea of orthogonal polarizations is that their interference term is 0, *i.e.*

$$\mathbf{E}_1 \cdot \mathbf{E}_2^* = 0 \tag{6.2}$$

Two elliptical polarizations are thus orthogonal when their helicities are opposite, their eccentricities equal, and their major axes perpendicular (*i.e.* opposite sense of rotation, same shape, axes crossed). It's an important point, because as we'll see when we get to the Jones calculus in Section 6.10.2, lossless polarization devices do not mix together orthogonal states—the states will change along the way, but will remain orthogonal throughout. One example is a quarter wave plate, which turns orthogonal circular polarizations into orthogonal linear polarizations, but it remains true even for much less well behaved systems such as single-mode optical fibers.

6.3 Interaction of Polarization with Materials

6.3.1 Polarizers

A polarizer allows light of one polarization to pass through it more or less unattenuated, while absorbing or separating out the orthogonal polarization. Any effect that tends to separate light of different polarization can be used: anisotropic conductivity, Fresnel reflection, double refraction, walkoff, and the different critical angles for *o*- and *e*-rays (related to double refraction, of course).

Polarizers are never perfectly selective, nor are they lossless; their two basic figures of merit at a given wavelength are the loss in the allowed polarization and the *open/shut ratio* of two identical polarizers (aligned versus crossed) measured with an unpolarized source, which gives the polarization purity. The best ones achieve losses of 5% or less and open/shut ratios of 10^5 or even more.

6.3.2 Birefringence

The dielectric constant ε connects the electric field $\mathbf{E}$ with the electric displacement $\mathbf{D}$. For a linear material, the most general form is a tensor relation, $\underline{\varepsilon}$

$$\mathbf{D} = \underline{\varepsilon}\mathbf{E}. \tag{6.3}$$

In isotropic materials the tensor is trivial, just ε times the identity matrix.‡ (See also Section 4.6.1.) Tensors can be reduced to diagonal form by choosing the right coordinate axes;

†Crabtree, Karlton, Polarization-Critical Optical Systems: Important Effects and Design Techniques', https://wp.optics.arizona.edu/optomech/wp-content/uploads/sites/53/2016/10/Tutorial_Paper-Crabtree.pdf.

‡Landau and Lifshitz, *The Electrodynamics of Continuous Media*, has a lucid treatment of wave propagation in anisotropic media, which the following discussion draws from.