MODERN RADIO TECHNIQUE

General Editor: J. A. RATCLIFFE

SUPER-REGENERATIVE RECEIVERS

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BY

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CAMBRIDGE

AT THE UNIVERSITY PRESS

1950

PUBLISHED BY
THE SYNDICS OF THE CAMBRIDGE UNIVERSITY PRESS

London Office: Bentley House, N.W. 1
American Branch: New York

Agents for Canada, India, and Pakistan: Macmillan

Printed in Great Britain at the University Press, Cambridge
(Brooke Crutchley, University Printer)

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PREFACE

The super-regenerative circuit was widely used, in wartime, in pulse responders for radar identification of ships and aircraft (I.F.F.). These units were produced in very large numbers and, for that reason, the initial design had to be particularly stable and reproducible. The achievement of these properties represented a great advance in technique and understanding, because the super-regenerative receiver previously had a reputation for unreliability. This book describes the operation of super-regenerative circuits in the light of these recent advances.

In view of the comparative shortage of information on the subject, however, the book is written to introduce the whole subject of super-regenerative theory and design and not only those parts relevant to wartime advances. Although addressed primarily to the specialist, it should be useful to the student and to the radio amateur. It begins with an outline of the super-regenerative principle which describes the linear and logarithmic modes of operation and introduces automatic gain-stabilization. Chapter 2 deals with the build-up of oscillations in a valve oscillator. No apology is made for the inclusion of this elementary treatment, for the subject is surprisingly absent from most text-books.

The following two chapters present an approximate theory of the linear mode, based upon material prepared jointly with Dr G. G. Macfarlane for a T.R.E.* Report and, later, for papers in Part III of the *Journal of the Institution of Electrical Engineers* (March–May 1946 and May 1948). A division of operation into the states of slope-control and step-control is made according to whether the circuit is brought slowly or quickly into the oscillatory state. The creation of these artificial and extreme states of operation is necessary to permit the derivation of simple formulae from the theory. Their relation to practical conditions is discussed in detail.

* Telecommunications Research Establishment.

A theory of sinusoidal quench applied to a triode oscillator leads to design data which are presented in an Appendix. These relate the properties of a super-regenerative receiver to the circuit parameters and valve electrode voltages. They are intended to be used as a guide to design and, as such, their usefulness is not strictly confined to sinusoidal quench.

The greater part of the book is thus concerned with the linear mode, which is the key to understanding all types of super-regenerative circuit. With this knowledge in mind, the treatment of the logarithmic mode and self-quenching in Chapter 6 is relatively simple.

The treatment of automatic gain-stabilization in Chapter 7 is similar to that used in the part of a monograph on I.F.F. written by the author as a contribution to the Scientific War Records of the Ministry of Supply (Air).

The final chapter describes a large number of super-regenerative circuits. Each is chosen to illustrate a type of receiver, and the whole chapter is intended to emphasize the range of applications of super-regeneration rather than to give precise instructions for the design of any particular circuit.

Attention is called to the list of symbols and to the short bibliography located at the end of the main text.

I believe this is the first book ever to be devoted entirely to super-regenerative receivers. It is unlikely to say the last word on the subject, but it will serve its purpose if it stimulates the reader into considering the super-regenerative receiver on the same basis as other types for U.H.F. applications.

I welcome this opportunity of expressing my appreciation of the assistance I have received from colleagues in Cambridge and Malvern in the course of preparing this book. In particular, I wish to thank Mr J. A. Ratcliffe for encouragement and helpful criticism; Mr K. E. Machin for reading manuscript and proofs; Dr G. G. Macfarlane, in association with whom the fundamental theory of Chapters 3–5 was developed for an earlier paper, and Mr R. W. Horne for preparing circuit diagrams. I am also indebted to

Professor F. C. Williams and Mr H. Wood of Manchester, and to Mr H. A. Wheeler, of New York, for discussions, during the war years, which helped to stimulate my interest in the subject.

I also wish to thank the following for permission to publish figures or extracts from published papers: The Institution of Electrical Engineers for the extract from Mr F. R. W. Strafford's paper in §6·3·2; The Institute of Radio Engineers for figs. 8·18 and 8·19; The *Wireless Engineer* for fig. 8·9; *Electronics* for fig. 8·17; and Messrs Raytheon for fig. 8·20. Finally, I thank the Chief Scientist, Ministry of Supply, for permission to publish this book.

J. R. W.

July
1949

CONTENTS

Chapter 1

INTRODUCTION TO THE SUPER-REGENERATIVE RECEIVER

1·1. The super-regenerative principle

A super-regenerative receiver is characterized by the repeated build-up and decay of self-oscillations in a valve oscillator, known as the *super-regenerative oscillator*, operating on, or near, the signal frequency. The circuit is made alternately oscillatory and non-oscillatory by the application of a periodic voltage to one of the electrodes of the oscillator valve. The source of this periodic voltage is usually a separate *quench oscillator*, although self-quenching may be arranged by a suitable choice of the grid leak and grid condenser of the super-regenerative oscillator. In either case the quench frequency is necessarily much lower than the natural frequency of the super-regenerative oscillator but must be higher than that of the signal modulation. Typical circuits of separately and self-quenched super-regenerative receivers are given in fig. 1·1.

In either separately or self-quenched systems the quench causes oscillations to build up in the super-regenerative oscillator and then permits them to decay. The oscillations build up from a signal voltage developed across the circuit by means of an inductive or capacitative coupling from the aerial. When there is no signal the oscillations start from the level of fluctuation noise existing in the circuit. During the period of build-up the oscillations may become as much as a million times greater than the signal. In this resides the gain of the super-regenerative receiver.

It is essential that the super-regenerative oscillations from one cycle of quench should die away before the next cycle commences. Otherwise the next cycle will build up from the dying oscillations instead of from the signal. For this, and other reasons, the choice of quench frequency, amplitude and wave-form must be made carefully. The actual choice is a complicated matter which is dealt with fully in later chapters. However, for sinusoidal quench, which is commonly used, the ratio of signal frequency to quench frequency usually lies between 100 and 1000. One of the reasons why the super-regenerative receiver is unsuitable for broadcast sound reception on the long and medium waves is now apparent. To receive a signal on 300 m. (1 Mc./sec.) we should require a quench frequency lying between 1000 and 10,000 c./sec., which is well within the band of audible frequencies.

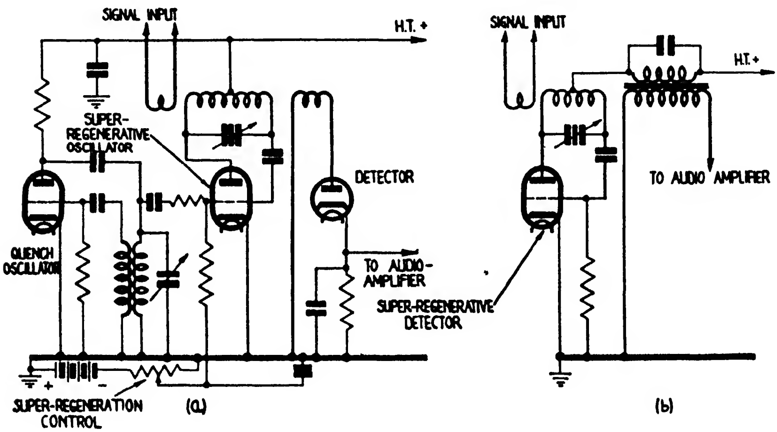


Fig. 1·1. Super-regenerative receivers. (*a*) Separate quench and detection. (*b*) Self-quench and self-detection.

1·2. Separate quench: linear and logarithmic modes

There are two clearly defined modes of operation with separate quench, according to whether or not the oscillations are allowed to build up to an equilibrium value, as in a normal valve oscillator, before they are quenched. If the oscillations which build up during a single quench cycle are quenched before they reach the limiting equilibrium amplitude determined by the valve characteristic, their peak amplitude is proportional to the signal (or noise) voltage from which the oscillations grew. The receiver is then said to operate in the *linear mode*. If, however, the build-up period is made long enough, the oscillations reach a steady value before they are quenched. In this, the *logarithmic mode*, the effect of an applied signal is to advance the time at which the oscillations reach a given amplitude. This is clearly shown in fig. 1·2, which compares the build-up and decay of oscillations in a single quench cycle in (*a*) the linear and (*b*) the logarithmic mode. It is shown later that the useful received output in the logarithmic mode increases in the ratio

$$\log_e (V_2/V_1),$$

when the incoming signal voltage is increased from V_1 to V_2, and it is this fact which gives rise to the name logarithmic mode. A separately quenched receiver can operate in either mode, depending upon the setting of the gain-control and upon the amplitude of the applied signal. A receiver is, however, termed linear or logarithmic according to the mode in which it is commonly used.

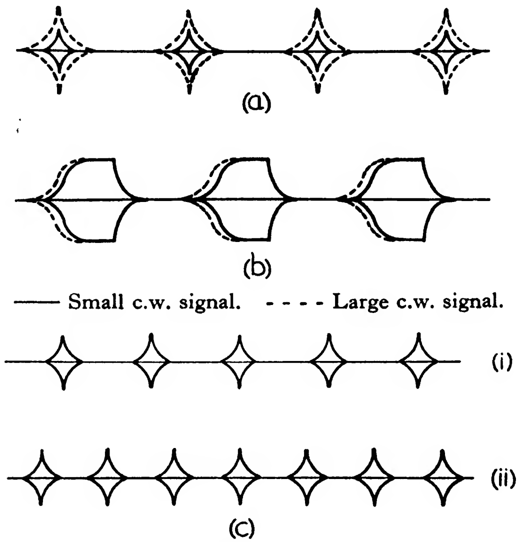


Fig. 1·2. Envelopes of super-regenerative oscillations. (*a*) Linear mode. (*b*) Logarithmic mode. (*c*) Self-quenching: (i) small c.w. signal; (ii) large c.w. signal.

In the linear mode the peak of the oscillation envelope occurs, in each quench cycle, at the same point which is independent of the signal amplitude. The amplitude of the peak is linearly dependent upon the signal amplitude. In the logarithmic mode the amplitude of the oscillation envelope is always the same, but a signal advances the apparent starting time of the oscillations, thus increasing the area under the envelope. Thus the area changes with the amplitude of the incoming signal. A suitable detector circuit converts this once more into a change of amplitude.

1·3. Self-quenching

The self-quenching receiver operates somewhat differently. The values of the components in the grid circuit of the super-regenerative oscillator are chosen so that the oscillations are periodically interrupted in a manner often known as 'squegging'. All the successive bursts of oscillation build up to the same critical amplitude before they are damped out. This amplitude is determined by the parameters of the squegging oscillator. When a signal is present, this amplitude is reached at an earlier time, and less time is also taken for the oscillations to decay back to the signal amplitude. Thus the effect of a c.w. signal is to decrease the interval between successive bursts of oscillation, i.e. to increase the quench frequency, without altering the form of each individual pulse. This is illustrated in fig. 1·2(*c*). Actual oscillograms showing the relationship between the oscillation pulse and the grid voltage in the various types of receiver are shown in fig. 1·3. The increase in repetition rate due to the presence of a c.w. signal causes an increase in the mean anode current of the super-regenerative oscillator. The self-quenching circuit is therefore self-detecting and is normally used in that way. Fig. 1·1 (*b*) is an example of such a circuit.

1·4. Reception of amplitude-modulated signals

The appearance of the output oscillation envelopes of the three types of super-regenerative receiver, when receiving a tone-modulated signal, is shown in fig. 1·4. In the linear mode the envelope of the output peaks follows the modulation wave-form, provided the latter is slow compared with the quench frequency. In the logarithmic mode the width of the 'pulses' increases and decreases according to the instantaneous amplitude of the signal, determined by the modulation wave. In the self-quenching receiver the output pulses cluster together at the modulation peaks and separate at the troughs in a manner characteristic of repetition-rate modulation.

The output from a linear detector following a super-regenerative receiver operating in the *linear* mode is proportional to the amplitude of the incoming signal. This mode of operation is therefore capable of reproducing accurately the signal modulation with a minimum amount of distortion. On the other hand, operation in the logarithmic mode and self-quenching both produce, after detection, an output proportional to the *logarithm* of the applied signal. This introduces considerable distortion at normal depths of modulation. From the point of view of fidelity, therefore, the linear mode is to

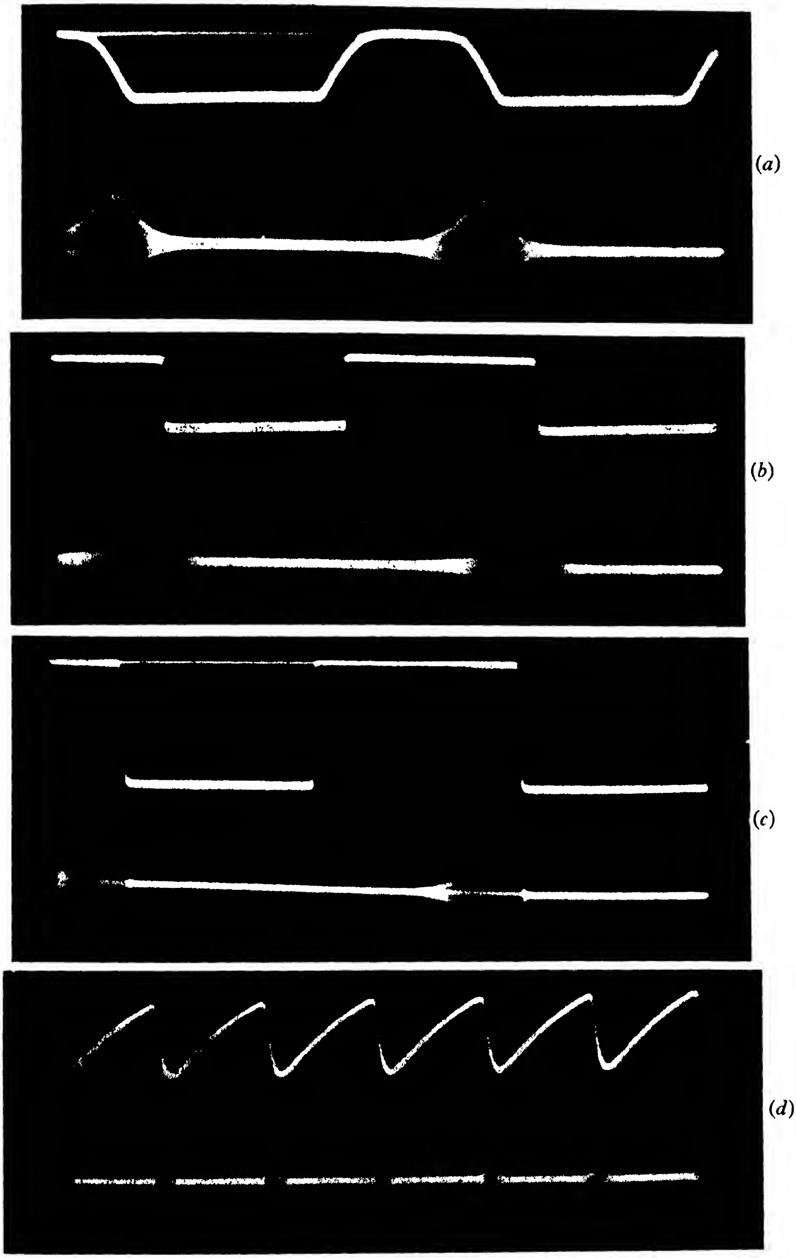


Fig. 1·3. Oscillograms of grid voltage and pulse of oscillations. (*a*) Linear mode: trapezoidal quench. (*b*) Linear mode: square-wave quench. (*c*) Logarithmic mode. (*d*) Self-quenching.

be preferred. Logarithmic operation has, however, certain advantages where fidelity is not important. The logarithmic relationship makes an interfering transient signal of very large amplitude

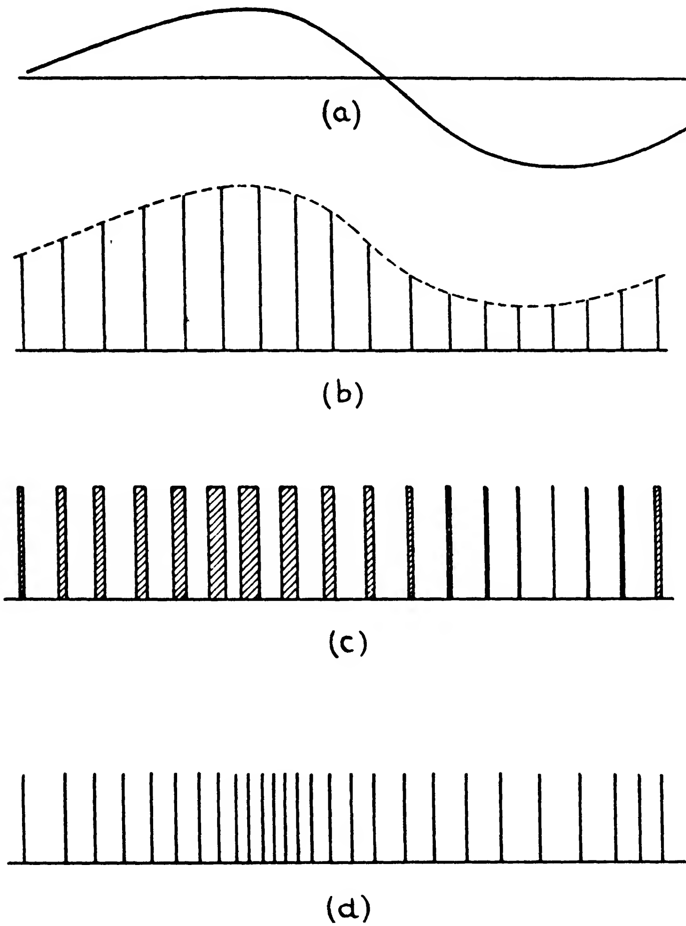


Fig. 1·4. Appearance of receiver output due to modulated signal. (*a*) Modulation wave-form. (*b*) Linear mode (even spacing, variable amplitude). (*c*) Logarithmic mode (even spacing, variable duration). (*d*) Self-quenching (variable spacing).

produce much less disturbance in the receiver output. This is one of the reasons for the much-vaunted 'interference-suppressing' properties of the super-regenerative receiver. In the same way wide variations (e.g. fading) of the signal amplitude can occur before there is any noticeable change in the receiver output level, so giving the effect of automatic volume control.

1·5. Automatic gain-stabilization

When the receiver is required to operate in the linear mode it is desirable to include an auxiliary circuit to maintain the output relatively constant, in spite of changes in signal amplitude, in the manner of automatic volume control. Because of the enormous

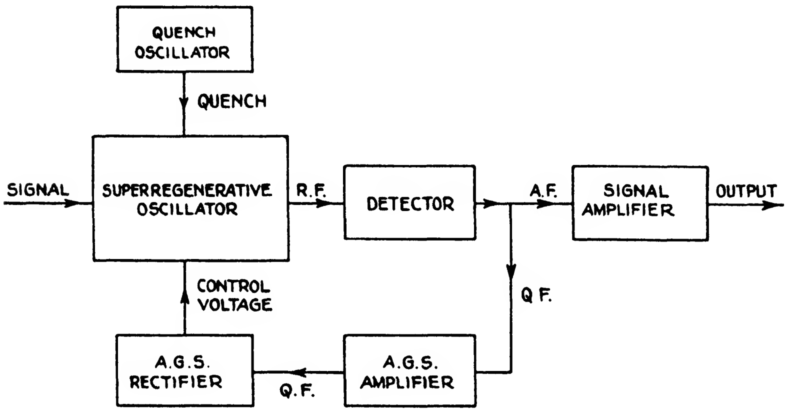


Fig. 1·5. Block diagram of a.g.s. system.

range of control possible in a super-regenerative receiver, the same circuit may be used to maintain the level of output *noise* constant in the absence of a signal. In this way the sensitivity of a search receiver may be held constant whilst tuning over a band of frequencies and in spite of changes in the other circuit parameters (e.g. aerial loading). The name 'automatic gain-stabilization' (a.g.s.) has been given to circuits which perform this function. Such circuits were first developed for use in the I.F.F. Mark III airborne responder* and were originally suggested by F. C. Williams.

The precise regularity of the output pulses from a receiver operating in the linear mode has already been emphasized. This means that the output carries a very strong quench-frequency component. In order readily to obtain a high gain in the a.g.s. amplifier it is usual to tune it to quench frequency. This is not essential, and is, in fact, in some cases a disadvantage, but these

* Chapter 8.

cases are rare. For the present the description is confined to quench-tuned a.g.s.

A block diagram of the a.g.s. system is given in fig. 1·5. The output from the detector of the super-regenerative receiver is applied to the input of a quench-tuned a.g.s. amplifier, as well as to the audio or video output circuits. The output from the a.g.s. amplifier is rectified and provides a d.c. control voltage. This voltage is used to control the super-regenerative gain, in the appropriate direction, so that any tendency for the receiver output to change is strongly opposed. The control voltage may be applied to the oscillator system in several alternative ways. A usual way, and probably the simplest, is to use it to provide bias to the grid of the super-regenerative oscillator, thus altering the receiver gain. Another method is to use it to control the quench amplitude by arranging the a.g.s. bias voltage to control the gain of a quench amplifier, from the output of which quench is supplied to the receiver. These methods are discussed in Chapter 7, which is devoted to a.g.s.

Chapter 2

THE SUPER-REGENERATIVE OSCILLATOR

2·1. Introduction

A fundamental process in the super-regenerative receiver is the build-up and decay of self-oscillations in the super-regenerative oscillator. In order to analyse the operation of the receiver it is necessary to develop the simplest possible equivalent circuit which will, however, represent it to an adequate degree of approximation. The linear mode is the easiest to analyse because the oscillator in this mode may be replaced, in the equivalent circuit, entirely by linear circuit elements. It is on this mode, then, that we shall concentrate for the time being. We shall find later that an analysis of the linear mode also goes far towards providing quantitative information regarding the logarithmic mode.

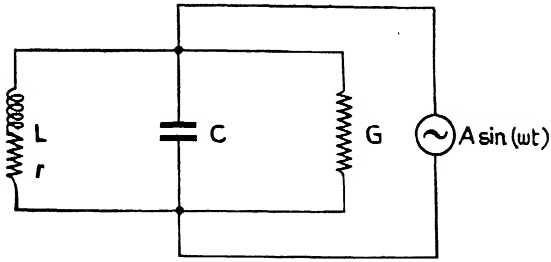
The valve in any oscillator circuit behaves approximately like a shunt negative conductance whose value is proportional to the mutual conductance at the operating electrode potentials. If the electrode potentials are varied, as they are by the quench in a super-regenerative receiver, the effective value of the shunt conductance undergoes a corresponding change.

Before proceeding with the analysis of a circuit having a time-varying shunt conductance, let us summarize the steady state and transient properties of a circuit having a fixed, positive value of the shunt conductance. A parallel *conductance*, rather than *resistance*, is used throughout because it is more convenient when considering the transition from positive, through zero, to negative values.

2·2. Parallel-tuned circuit: steady state

The properties of a parallel-tuned circuit, having a series resistance r and a shunt conductance G, are summarized in table 1. In later analysis, for simplicity, we put $r = 0$ and effectively increase the value of G in order to maintain the same circuit Q-factor. It may be shown from the formulae that the modification of the frequency-response characteristic by this approximation is slight for frequencies close to resonance, provided the Q-factor is reasonably large.

Table 1. *Properties of the parallel-tuned circuit*



Admittance:

$$Y=\frac{1}{r+Lj\omega}+G+Cj\omega.$$

Impedance:

$$Z=\left(\frac{L}{Cr+LG}\right)\left\{\frac{1-jr/\omega L}{1+[j/\omega(Cr+LG)]\,[\omega^2LC-(1+rG)]}\right\}$$

$$\doteqdot L\omega_0 Q_0\{1/[1+j(2Q_0\delta)]\}\quad (Q_0\gg 1,\ \delta\ll 1).$$

Magnitude:

$$|Z|\doteqdot\left(\frac{L}{Cr+LG}\right)\left\{\frac{(r/2L)+(G/2C)}{\sqrt{[(r/2L)+(G/2C)]^2+(\omega-\omega_0)^2]}}\right\}$$

$$\doteqdot L\omega_0 Q_0\{1/\sqrt{[1+(2Q_0\delta)^2]}\}\quad (Q_0\gg 1,\ \delta\ll 1).$$

Phase angle:

$$\phi=\tan^{-1}\frac{\omega-\omega_0}{(r/2L)+(G_0/2C)}$$

$$\doteqdot \tan^{-1}(2Q_0\delta)\quad (Q_0\gg 1,\ \delta\ll 1).$$

Symbols: $\omega_0=1/\sqrt{(LC)},\quad Q_0=\omega_0\Big/\left(\frac{r}{L}+\frac{G}{C}\right),\quad \delta=(\omega/\omega_0)-1.$

2·3. Transient oscillations in the parallel-tuned circuit

In the super-regenerative receiver we are concerned not with the steady state but with the continual starting and stopping of oscillations in a tuned circuit having an a.c. signal applied to it. Let us examine next the transient behaviour of the parallel circuit of fig. 2·1, in which all the damping is due to the shunt conductance G. The signal is represented by an equivalent current generator of amplitude A and radian frequency ω.

The differential equation for the voltage V across the circuit is

$$C\frac{dV}{dt}+GV+\frac{1}{L}\int V\,dt=A\sin(\omega t), \tag{1}$$

or

$$C\frac{d^2V}{dt^2}+G\frac{dV}{dt}+\frac{V}{L}=A\omega\cos(\omega t). \tag{2}$$

The complete solution of this equation for the oscillatory case in which we are interested may be written

$$V = k_1 e^{(-\alpha+j\omega_d)t} + k_2 e^{(-\alpha-j\omega_d)t} + \frac{A\sin(\omega t+\phi)}{\sqrt{[G^2+(\omega C - 1/\omega L)^2]}}, \tag{3}$$

where

$$\left.\begin{aligned} \alpha &= G/2C, \\ \omega_d &= \sqrt{\left[\frac{1}{LC} - \left(\frac{G}{2C}\right)^2\right]} = \sqrt{(\omega_0^2-\alpha^2)}. \end{aligned}\right\} \tag{4}$$

Note that

ω is the frequency of the applied signal,

ω_0 is the natural frequency of the circuit,

ω_d is the frequency of the damped oscillation, of which

α is the damping factor.

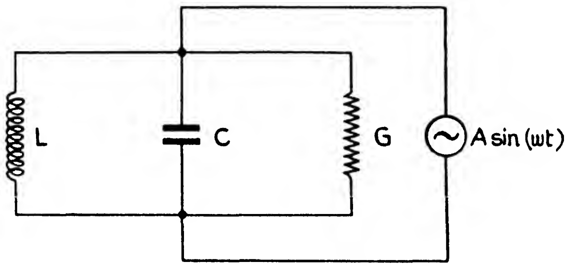


Fig. 2·1. Parallel-tuned circuit with applied signal.

The values of k_1 and k_2 may easily be found from the initial conditions, at $t = 0$, which are

$$V = 0 \quad \text{and} \quad A\sin(\omega t) = 0. \tag{5}$$

The resulting values of k_1 and k_2 are

$$k_1 = \frac{A[G\omega + (1/\omega C - \omega L)(-\alpha - j\omega_d)]}{2j\omega_d[G^2+(1/\omega C-\omega L)^2]},$$

$$k_2 = -\frac{A[G\omega + (1/\omega C - \omega L)(-\alpha + j\omega_d)]}{2j\omega_d[G^2+(1/\omega C-\omega L)^2]}. \tag{6}$$

If conditions are such that $\omega = \omega_0 = 1/\sqrt{(LC)}$ the signal is tuned to the resonant frequency of the circuit and

$$k_1 = \frac{A\omega_0}{2j\omega_d G}, \quad k_2 = -\frac{A\omega_0}{2j\omega_d G}. \tag{7}$$

The complete solution for the voltage across the circuit in this case is

$$V = \frac{A\omega_0}{2j\omega_d G} e^{(-\alpha+j\omega_d)t} - \frac{A\omega_0}{2j\omega_d G} e^{(-\alpha-j\omega_d)t} + \frac{A\sin(\omega_0 t)}{G}$$

$$= \frac{A}{G}\frac{\omega_0}{\omega_d} e^{-Gt/2C} \sin(\omega_d t) + \frac{A}{G}\sin(\omega_0 t). \quad (8)$$

The first term is the interesting transient term, and the second represents the steady state current.

This solution shows that initiation of the current in a parallel-tuned circuit causes the appearance of a transient voltage as well as the voltage normally expected in the steady state. If, however, G is made negative in equation (8) then the transient voltage builds up from the signal level and soon becomes large compared with the steady-state voltage. This clearly involves a supply of energy to the circuit from an external source, a state of affairs which holds in a valve oscillator. The next section shows that an oscillator valve behaves approximately like a negative conductance across the associated tuned circuit. It also appears that the value of this negative contribution to the parallel conductance is generally a simple function of the mutual conductance of the valve.

2·4. Transient oscillations in the tuned-anode oscillator

We shall now analyse a valve oscillator of the feed-back type and compare the results with those obtained above. Instead of allowing the oscillations to build up from circuit noise we shall again give them a definite level from which to start by applying a signal current $A\sin(\omega t)$ to the oscillatory circuit. Fig. 2·2(*a*) shows the circuit of a tuned-anode oscillator with a signal applied to the tuned circuit. The total losses in the circuit components are represented by the shunt conductance G_0. In the equivalent circuit of fig. 2·2(*b*) the valve is replaced by an equivalent current generator of magnitude $V_g g_m$ in parallel with anode conductance G_a. This is a useful alternative, for parallel circuits, to the more usual representation of the valve as an equivalent voltage generator μV_g in series with the anode resistance $R_a = 1/G_a$. It assumes, of course, 'ideal' valve characteristics, i.e. that $\mu = g_m/G_a$ for all values of g_m.

Now it can be assumed with a fair degree of accuracy for a negative-grid triode that no current flows in the grid circuit. If this is so, then the voltage V_g appearing across the grid-coupling inductance L_g is equal to VM/L, where M is the mutual inductance between the coils and V is the voltage across the parallel-tuned circuit.

The differential equation representing the equivalent circuit is

$$-g_m V_g + C\frac{dV}{dt} + (G_0 + G_a)V + \frac{1}{L}\int V dt = A\sin(\omega t). \qquad (9)$$

Substituting for V_g in terms of V, as above,

$$C\frac{dV}{dt} + \left(G_0 + G_a - \frac{M}{L}g_m\right)V + \frac{1}{L}\int V dt = A\sin(\omega t),$$

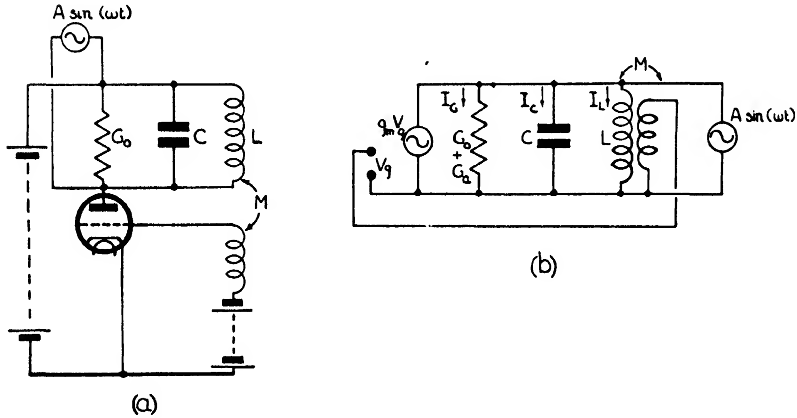


Fig. 2·2. Tuned-anode oscillator with applied signal.
(*a*) Oscillator circuit. (*b*) Equivalent circuit.

or, writing $G_a = g_m/\mu$, where g_m is the mutual conductance and μ the amplification factor of the valve,

$$C\frac{dV}{dt} + \left[G_0 + g_m\left(\frac{1}{\mu} - \frac{M}{L}\right)\right]V + \frac{1}{L}\int V dt = A\sin(\omega t). \qquad (10)$$

This is identical with equation (1) if we put G in that equation equal to

$$G = G_0 + g_m\left(\frac{1}{\mu} - \frac{M}{L}\right). \qquad (11)$$

If the coupling is such that $M/L > 1/\mu$ there is some value of g_m for which $g_m(M/L - 1/\mu)$ is greater than G_0, i.e. for which G is negative. When this is the case the coefficient of the transient term in equation (8) becomes

$$\frac{A}{G}\frac{\omega_0}{\omega_d}\exp\left\{\frac{1}{2C}\left[g_m\left(\frac{M}{L} - \frac{1}{\mu}\right) - G_0\right]t\right\}, \qquad (12)$$

and increases with time, quickly becoming large compared with the steady-state term $(A/G)\sin(\omega_0 t)$. The amplitude of these oscillations is seen to be proportional to that of the applied signal. The frequency of oscillations, ω_d, is slightly dependent upon the mutual conductance of the valve, as well as on the other circuit parameters, for

$$\omega_d = \sqrt{(\omega_0^2 - \alpha^2)},$$

where α is now given by

$$\alpha = \frac{1}{2C}\left[g_m\left(\frac{M}{L} - \frac{1}{\mu}\right) - G_0\right]. \qquad (13)$$

It appears, then, that the tuned anode oscillator may be represented approximately by the simple circuit of fig. 2·1 in which the value of G is

$$G = G_0 + G_v, \qquad (14)$$

where G_v is the contribution made to the parallel-circuit conductance by the valve and feed-back coupling, and is given by

$$G_v = -g_m\left(\frac{M}{L} - \frac{1}{\mu}\right). \qquad (15)$$

G_v is negative, zero or positive according to whether M/L is greater than, equal to or less than $1/\mu$. For a given arrangement of the oscillator circuit and a given choice of valve, the factor $(M/L - 1/\mu)$ is, to a first approximation, constant, in so far as μ is independent of g_m. We may therefore write

$$G_v = Kg_m \quad \text{and} \quad G = G_0 - Kg_m, \qquad (16)$$

where $K = (M/L - 1/\mu)$ is constant for a particular circuit arrangement.

2·5. Method employing the operator j

The above example is sufficient to illustrate the principles behind the representation of the valve oscillator as a negative resistance. For practical purposes, however, this result for the tuned anode oscillator may be obtained more readily from a steady-state analysis of the circuit. The latter method is particularly useful for dealing with more elaborate oscillator circuits for which the solution of the differential equations is not so easy.

First let us consider the steady-state conditions for maintaining the amplitude of the oscillations constant in a tuned-anode oscillator with no applied signal. From fig. 2·2(*b*) putting $A = 0$, we see that

$$-g_m V_g + I_G + I_C + I_L = 0.$$

But, as before, $V_g = (M/L)\,V.$

Substituting the values for I_G, I_C, I_L, we obtain

$$-g_m(M/L)+G_0+G_a+j\omega C+1/j\omega L = 0.$$

The imaginary terms give the frequency of oscillation

$$\omega^2 = 1/LC. \tag{17}$$

The real terms give the conditions for oscillation

$$G_0-g_m(M/L-1/\mu) = 0. \tag{18}$$

This corresponds to equation (11) with $G = 0$, and we again obtain the result

$$G_v = -g_m(M/L-1/\mu). \tag{19}$$

The values of G_v for the tuned-grid, Hartley, Colpitts and other oscillators may be obtained in this way.

2·6. Representation of a super-regenerative oscillator

In a super-regenerative oscillator the value of G in the simple equivalent circuit of fig. 2·1 is made to alternate from positive to negative values. For any valve oscillator G may be represented, to a close approximation, by the equation

$$G = G_0-Kg_m, \tag{20}$$

in which G_0 is the effective parallel conductance representing the losses in the quiescent circuit when the valve is not conducting, g_m is the mutual conductance of the valve and K is a constant for any given circuit arrangement.

Now we can cause G to alternate in two ways. The obvious method, and the one commonly used, is to vary one of the electrode voltages of the valve and thus cause a corresponding fluctuation of g_m. An alternative way, however, is to maintain g_m at a value exceeding G_0/K and to vary the value of G_0 by periodically applying additional damping across the circuit. This effect may be achieved in many ways, for example, by connecting a diode with an alternating anode voltage across the circuit, so that it only conducts periodically. The latter method is seldom used, but we shall have occasion to mention it again later.

In either case the equivalent circuit of fig. 2·1 is adequate to represent the super-regenerative oscillator, but the precise nature of the function $G(t)$ depends upon the method used for quenching the oscillations.

The argument above deals exclusively with the early part of the build-up period when the oscillations have not had time to build up to a large amplitude. If the oscillations are quenched before they approach a limiting amplitude determined by the valve characteristics, then the whole process of build-up and decay may be described on the basis of the equations relating to the circuit of fig. 2·1. This is, in fact, what is done in the analysis of the super-regenerative receiver in the linear mode in later chapters. If, however, the oscillations are allowed to build up to their equilibrium amplitude before they are quenched, then we need to know something of the factors causing the limitation of amplitude.

2·7. The limitation of oscillation amplitude

2·7·1. *Tuned-anode oscillator*

Let us again take the tuned-anode oscillator of fig. 2·2 as our example. Suppose that the supply voltages are arranged to make the quiescent mutual conductance (i.e. the mutual conductance in the absence of superimposed oscillatory voltages on the electrodes) high enough for oscillations to build up, once they are initiated, i.e.

$$g_m > G_0/(M/L - 1/\mu). \tag{21}$$

Immediately after switching on, the amplitude of the oscillations increases exponentially according to the law

$$V = V_0 \exp\left\{\frac{1}{2C}[(M/L - 1/\mu)g_m - G_0]\,t\right\}. \tag{22}$$

This law holds during the initial part of the build-up when the amplitude of the oscillations is small, and the operation is restricted to the linear portion of the valve characteristic. As the amplitude increases, however, the mutual conductance can no longer be regarded as constant and equal to its quiescent value, but the amplification factor, μ, may reasonably be assumed to remain constant for the purpose of a qualitative analysis. When μ is constant the anode-current–grid-voltage curves are displaced from each other along the V_g axis by amounts proportional to the difference in anode voltage between them. In this case, therefore, they may be represented by a single curve

$$i_a = f(V_g - V_a/\mu). \tag{23}$$

This is shown in fig. 2·3. The slope of this curve at any point gives the corresponding value of the mutual conductance. Let us suppose that P is the operating point on the characteristic determined by the supply voltages to the valve. The slope of the curve at this point is the mutual conductance g_m.

During the early part of the build-up period, the oscillation amplitude is small and the electrode voltages do not deviate appreciably from their steady values.

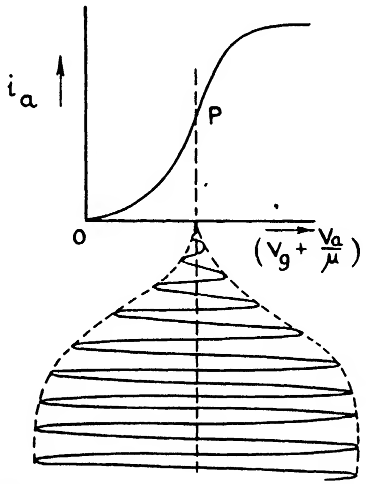


Fig. 2·3. Limitation of oscillation amplitude in negative-grid oscillator.

As the amplitude increases, the grid potential moves along an appreciable part of the characteristic curve. The value of the mutual conductance now changes during a cycle of oscillation, being decreased for large numerical values of the grid potential. Consequently the effective conductance of the circuit also varies. This reduces the rate of increase of the oscillatory voltage until, finally, a stable amplitude is reached. In the state of equilibrium the supply of energy by the valve is equal to the losses in the tuned circuit.

In order to determine the shape of the oscillation envelope during build-up, and the value of the limiting amplitude, it is necessary to solve a non-linear differential equation containing parameters describing the curvature of the valve characteristic. Appleton and van der Pol* and others have obtained approximate solutions representing the equilibrium value of the oscillatory voltages.

* See Bibliography.

Rocard* shows that the approximate form of the build-up envelope is

$$A(t) = \frac{e^{a(t-t_0)}}{\sqrt{[1+e^{2a(t-t_0)}]}}, \tag{24}$$

where a depends upon the circuit parameters.

2·7·2. *The effect of a grid leak and condenser*

The argument above refers to an oscillator circuit with fixed grid bias and a corresponding value of the initial mutual conductance g_m. If a grid leak and condenser are included, the mechanism by which the oscillation amplitude is limited is somewhat different.

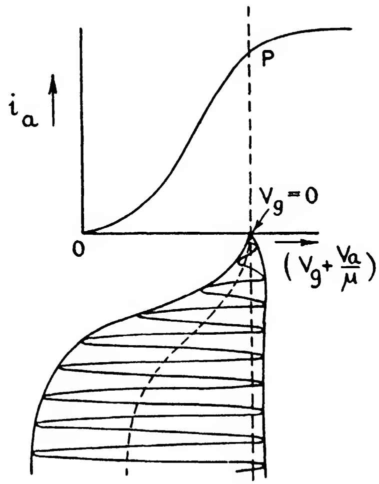


Fig. 2·4. Limitation of oscillation amplitude due to grid-leak condenser.

In the absence of any fixed bias the operating point determined by the steady electrode voltages is the point P (fig. 2·4) corresponding to $V_g = 0$. As the oscillations build up, grid current begins to flow during that part of the cycle in which the grid potential is positive. The pulses of grid current charge the grid condenser. Consequently the mean grid potential becomes increasingly negative as the oscillation amplitude grows. The anode current is cut off during a portion of each cycle, leaving only pulses of current at the oscillation frequency. During the build-up the energy supplied by these pulses

* See Bibliography.

more than overcomes the circuit losses, and the oscillation amplitude is thus enabled to grow. As the grid bias increases, the pulses become shorter in duration and supply less energy to the circuit. Eventually an equilibrium amplitude is reached at which the current pulses are just sufficient to maintain oscillation.

2·7·3. *The squegging oscillator*

This argument assumes that the time constant in the grid circuit is short enough to permit the mean grid potential to follow the oscillation amplitude. If that is not so, interrupted oscillations may

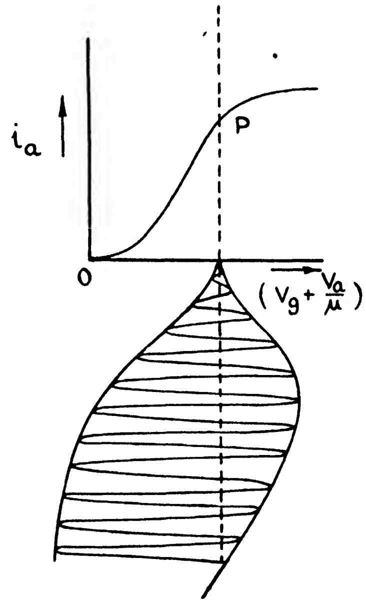


Fig. 2·5. Initiation of 'squegging'. Build-up of oscillations in squegging oscillator.

be produced in the manner often known as 'squegging'. If, for instance, the grid condenser is large it cannot be charged very quickly and the grid-bias voltage is no longer able to follow the build-up of oscillation amplitude. Thus the oscillations reach their limiting amplitude whilst the mean grid voltage is still descending. The process is pictured in fig. 2·5. The axis of the oscillations descends until the pulses of anode current are not sufficient to overcome the circuit losses. The oscillations then begin to decrease in amplitude. Since the grid bias can only change very slowly, the reduced amplitude results in an even shorter duration of the conducting periods. Thus less power is supplied by the valve and the oscillations quickly die away. At this stage the grid condenser is still charged and the valve remains cut off until the grid condenser has discharged through the grid leak. When the grid voltage recovers to a certain value oscillations again start to build up and the cycle is repeated. The period between successive bursts of oscillation depends upon the time constant in the grid circuit.

The squegging oscillator is discussed in more detail in connexion with self-quenching super-regenerative receivers in §6·4.

2·8. Practical oscillators

We have shown that an oscillator valve may usually be replaced, in the equivalent circuit, by a negative conductance proportional to its mutual conductance, g_m. The formulae of §2·4 give the value of the negative conductance in terms of the valve and circuit parameters. In obtaining these formulae it was assumed that the valve could be represented by a current source of magnitude $V_g g_m$ in parallel with a conductance $G_a = g_m/\mu$. We shall now see to what extent this is true for a practical oscillator.

The following assumptions are made in representing the valve as a current source $V_g g_m$ with a shunt conductance G_a:

(i) μ is constant for all values of anode current.

(ii) The mutual conductance g_m is constant and equal to its static value over the whole oscillatory cycle.

(iii) The electrode capacitances and lead inductances of the valve may be neglected, or, at any rate, assumed to be constant in value.

(iv) The effect of transit time in the valve is negligible.

In a practical valve (i) is closely true except for very small values of anode current. It is apparent later that this causes little departure of the characteristics of an actual super-regenerative receiver from those calculated on the basis of a constant μ. The reason is that the important properties of a super-regenerative receiver are determined at a time in the quench cycle when the anode current is relatively high.

Condition (ii) is only true for a valve having a constant slope over the entire range of excursion of the oscillatory electrode voltages.

For an average valve, however, the departure from linearity is not serious so long as the oscillations are never allowed to build up to more than a small fraction of their limiting amplitude. In any case the fundamental properties of the super-regenerative receiver, such as sensitivity and frequency response, are determined at a time when the oscillation amplitude is comparable with the signal amplitude and is thus very small. Departure from linearity later in the build-up period alters the nature of the output pulse of oscillations and is dealt with in Chapter 5 under the logarithmic mode.

The effect of the transit time of electrons in the valve is only important when oscillations are required near the highest frequency at which the valve can be made to oscillate. For an average oscillator triode this is 300 Mc./sec. (CV 6) and for a miniature valve about 600 Mc./sec. (6C4). There are, however, triode valves, such as the CV 90 and the American Lighthouse, that will oscillate in a cavity resonator at 3000 Mc./sec. A super-regenerative receiver has been constructed and successfully operated at 3000 Mc./sec. using one of these valves.

It is apparent, then, that the super-regenerative oscillator is closely represented by the equivalent circuit of fig. 2·1, with a value of G proportional to the mutual conductance, provided that

(i) the oscillation amplitude is a small fraction of its limiting value;

(ii) the electrode capacitances and lead inductances of the valve form a negligible part of the oscillatory circuit;

(iii) the valve is operating well below its limiting frequency.

At higher frequencies the electrode capacitances and inductance become by no means negligible. It is usually possible to make allowance for them, provided they are independent of electrode voltages, by lumping them with the external circuit parameters in the design formulae. This process has been used successfully at frequencies as high as 500 Mc./sec. Often, however, a part of the grid-anode capacitance, say, varies with anode current. This causes the nominal resonant frequency of the circuit to vary during the quench cycle. It appears later that this can result in an asymmetric frequency-response characteristic in a super-regenerative receiver.

The effect of approaching the limiting frequency of the super-regenerative oscillator is to add additional damping to the circuit, thus reducing the effective value of g_m in equation (20). There is also a reactive term in the valve impedance, which causes a frequency shift as described above.

In designing a super-regenerative oscillator at high frequencies it is necessary to take the usual precautions with regard to short leads, lay-out and earthing, to be found in any good text-book on high frequencies. Some examples of current practice in the design of super-regenerative oscillators are provided in Chapter 8.

Chapter 3

GENERAL THEORY OF SUPER-REGENERATIVE RECEPTION OF A SIGNAL: LINEAR MODE

3·1. Introduction

The theory of the super-regenerative receiver is concerned with the repeated build-up and decay of oscillations in a valve oscillator. The oscillations build up from the level of the received signal, which is developed across the resonant circuit. When the characteristics of the receiver are such that the oscillations are damped out before they have had time to build up to their limiting amplitude, the mode of operation is termed linear. That is because the maximum amplitude to which the oscillations rise is proportional to the amplitude of the applied signal.

It is demonstrated below that many of the important properties of a super-regenerative receiver are determined at the start of build-up. These properties, then, are common to both the linear and the logarithmic modes which only differ at the end, not the beginning, of the build-up period. For this reason the analysis of the linear mode given below is, in many respects, relevant to the logarithmic mode which eases our task when we come to consider the logarithmic mode of operation, in a later chapter.

We saw, in Chapter 2, how a valve oscillator could be closely represented by a parallel resonant circuit with a negative conductance shunted across it representing the valve (fig. 3·1). The value of the shunt conductance G was shown to take the form

$$G = G_0 - Kg_m,$$

in which G_0 is the value of the positive conductance damping the circuit. This simple equivalent circuit can also be used to represent a super-regenerative oscillator, in which case the total parallel conductance is a function of time. The precise nature of this function depends upon the characteristics of the quench voltage and is discussed more fully in a later chapter. For the present we may represent the super-regenerative oscillator by a parallel-tuned circuit with a shunt conductance G which is a function of time of the form

$$G(t) = G_0[1 - F(t)].$$

It is possible, on the basis of this simple circuit, to develop general formulae which demonstrate the main properties of a super-regenerative receiver. That is done in the present and the succeeding chapter. Only in Chapter 5, after the development of a general theory of super-regeneration, do we substitute the conductance-time functions corresponding to *particular* quench wave-forms.

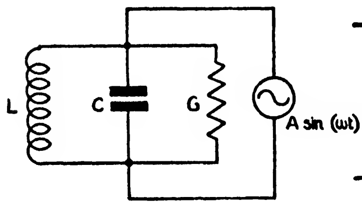


Fig. 3·1. Basic circuit.

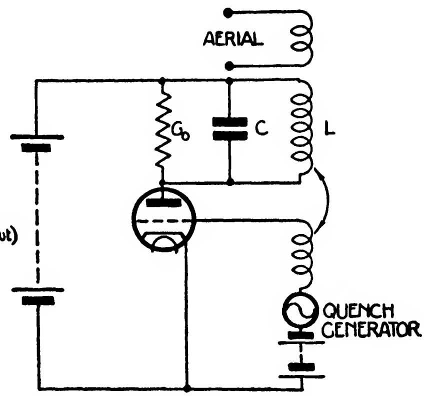


Fig. 3·2. Super-regenerative oscillator.

3·2. The quench cycle

The operation of a super-regenerative receiver is best understood by following the events in a single quench cycle. For this purpose we will take as our example a cycle of sinusoidal quench, remembering that our remarks are not confined to this, or to any other, particular quench variation. At this stage we are not concerned with the quench voltage itself but with the conductance variation it produces in the oscillatory circuit. It may, however, make the example clearer if it is related to a practical circuit. Let us therefore consider the operation of the super-regenerative oscillator of fig. 3·2. The triode oscillator valve is given a mean grid-bias voltage near the value for cut-off, on which the sinusoidal quench voltage, applied to the grid, is superimposed. The valve only conducts over a part of the quench cycle. If we represent this oscillator by the equivalent circuit of fig. 3·1, then it is apparent that the total circuit conductance

$$G = G_0 - Kg_m$$

varies with time in relation to the quench voltage in the way shown in fig. 3·3. Provided a value of g_m is reached such that $Kg_m > G_0$, the total conductance G alternates from positive to negative, and the conditions for super-regeneration are established. (The value of

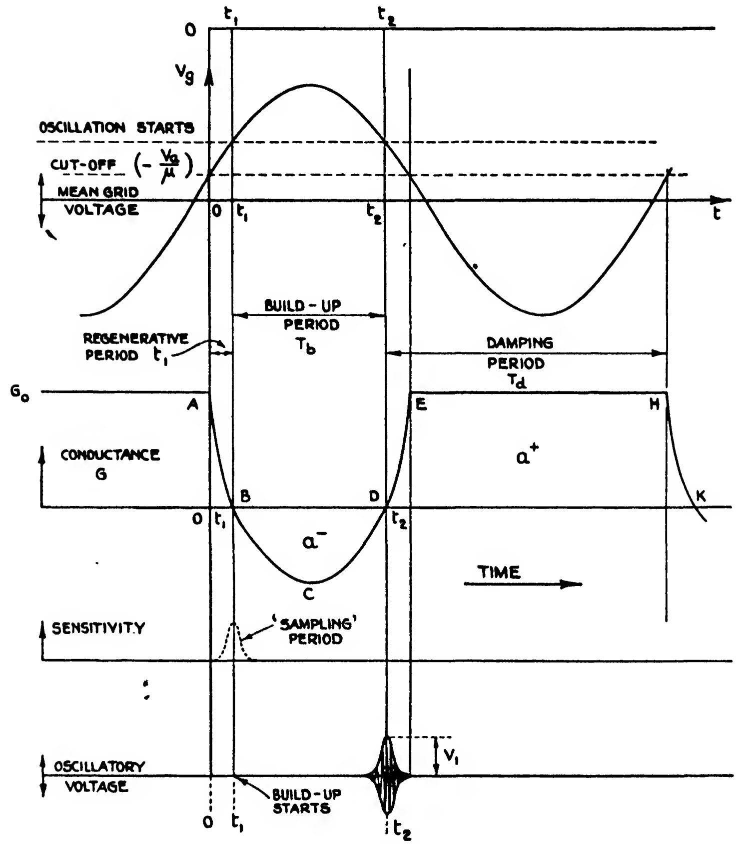


Fig. 3·3. Quench and conductance cycle: slope-controlled state.

K is G_0/g_{m0}, where g_{m0} is the value of the mutual conductance required just to produce oscillation.) During the quiescent part of the quench cycle, when the valve is not conducting, the total effective conductance is determined by the circuit losses, i.e.

$$G = G_0,$$

and the voltage across the circuit is that due to the applied c.w. signal current $A \sin(\omega t)$. When the valve starts to conduct it contributes an amount $-Kg_m$ to the circuit conductance G. The effective value of G is reduced and the voltage V across the circuit consequently increases. The initial period t_1 during which the value of G is reduced from G_0 to zero is known as the regenerative period. As soon as G becomes negative, self-oscillations begin to build up at the resonant frequency of the circuit formed by L and C. These oscillations build up from the incoming signal during the interval T_b and attain maximum amplitude at the instant t_2, when the conductance is once again zero. After that they are damped for the remainder of the cycle whilst the conductance is positive. In order that the oscillations in the next quench cycle shall build up from the signal $A \sin(\omega t)$, and not from the decaying oscillations, the damping period T_d must not be too short. The precise conditions for this appear later.

If the damping during the quench cycle is not sufficient to reduce the self-oscillations below noise level before the beginning of the next quench cycle, the receiver settles down in the 'coherent' state. In this condition, the oscillations in the circuit build up from the decaying oscillations from the preceding cycle of quench and the output pulses are, therefore, coherent in phase. Because of this the sensitivity is greatly reduced and the frequency response is altered. The coherent state produces a characteristic frequency response with multiple peaks separated by the quench frequency, as in fig. 3·4. These are caused by reinforcement of the signal by the decaying self-oscillation when the two are separated by an integral multiple of quench frequency. The relative heights of the peaks and troughs are determined by the relation between the amplitudes of signal and self-oscillation during the sensitive period.

The quench cycle of fig. 3·3 represents only one of two distinguishable states of operation. It is characterized by a period of regeneration (t_1). It is called *slope-controlled*, because the operation of the receiver is controlled by the slope of the conductance-time characteristic at the instant the conductance is zero. If, however, the transition from positive to negative conductance occurs very rapidly the operation is controlled by the values of the conductance just before and just after the instant of transition. In this case the receiver is said to be *step-controlled*, a state typified by rectangular-wave quench. A step-controlled receiver has very different properties from one which is slope-controlled. A super-regenerative receiver with sinusoidal quench usually operates in the slope-controlled state unless the amplitude of the quench greatly exceeds the grid-base of the valve. These two states of operation are clearly differentiated in the analysis that follows.

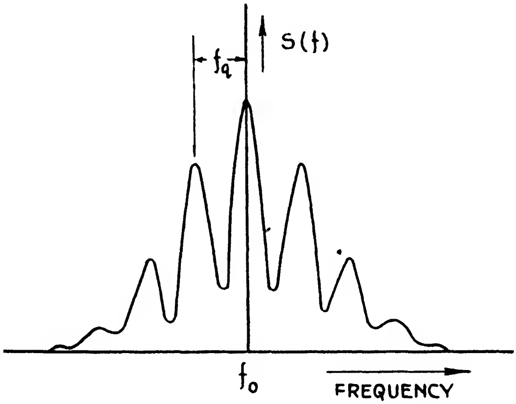


Fig. 3·4. Frequency response: coherent state.

3·3. Operation of the receiver

The super-regenerative receiver is only sensitive to the incoming signal for a small fraction of each quench period. The maximum sensitivity occurs exactly at the time when the total circuit conductance, G, is zero, at the end of the interval t_1. The signal voltage in the circuit at this instant plays the greatest part in the determination of the amplitude to which the self-oscillations build up, before they are quenched. An element of signal occurring before time $t = t_1$ has time to decay before build-up starts and, consequently, has less effect upon the peak amplitude than a similar signal occurring exactly at time $t = t_1$. An element of signal, arriving later than this instant, again has less effect upon the final amplitude, because part of the build-up period has expired before its arrival. The incoming signal is, therefore, sampled for a short period in each quench cycle. Over the greater part of the cycle the signal has a negligible effect upon the receiver. The way in which the sensitivity varies with time during a quench cycle depends upon the nature of the quench and upon valve and circuit parameters. The sampling process is pictured in fig. 3·3.

The sample of signal taken in the sensitive period on each quench cycle determines the amplitude to which the self-oscillations rise before they are quenched. The receiver output, therefore, cannot include more information about the signal than that obtained during these brief sensitive periods. That is why a super-regenerative receiver can only distinguish modulation frequencies which are low compared with the quench frequency. An alternation of carrier amplitude which occurred between two successive sensitive periods would go unnoticed.

It is in this sensitive period of the quench cycle that the frequency response of the receiver is determined. The effect of the signal upon the final amplitude of the self-oscillations depends upon the signal

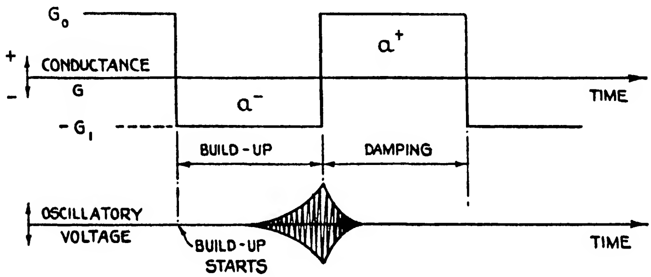


Fig. 3·5. Build-up and decay of oscillations during cycle of rectangular quench.

frequency as well as upon its amplitude. It is not easy to predict the precise nature of the frequency response by a physical argument, although we can say, even at this stage, that it is likely to depend upon the behaviour of the circuit conductance about the time $t = t_1$ at the peak of the sensitive period.

The self-oscillations which build up during the period of negative conductance, and are damped out for the remainder of the quench cycle, constitute the output of the super-regenerative oscillator. The receiver gain is the ratio of the peak amplitude of the oscillations at the instant $t = t_2$ to the amplitude of the signal applied during the corresponding sampling period. For the purpose of designing detector and output circuits for a super-regenerative receiver we are interested in the shape of the envelope of the build-up and decay of oscillations. In the conductance cycle of fig. 3·4, the value of G is continually changing throughout the cycle and the shape of the envelope is not easily predicted. Later analysis shows that it is usually, approximately, a Gaussian error-curve for the slope-controlled state. With rectangular quench, however, both build-up and decay are truly exponential, as shown in fig. 3·5. The pulse of oscillations in this case is only symmetrical if the negative value G_1 of G during the cycle is equal in magnitude to the positive value G_0.

The precise way in which the properties of the receiver, such as gain and frequency response, depend upon the circuit parameters, and upon the conductance variation, is determined by a mathematical analysis of the equivalent circuit of fig. 3·1. By making certain assumptions about the relation between the rate of conductance change and the radio-frequency it is possible to obtain a solution for the voltage at the peak of the pulse of oscillations.

3·4. Analysis of the linear mode

Many of the properties of a super-regenerative receiver operating in the linear mode may be demonstrated by an approximate mathematical analysis of the circuit of fig. 3·1. It is assumed, first of all, that a signal current $A\sin(\omega t)$ is applied to the circuit and there is no circuit noise. Only after developing a signal theory shall we consider the behaviour of the receiver due to the reception of both signal and noise.

Equating currents in the circuit of fig. 3·1 gives us the basic equation

$$C\frac{dV}{dt}+G(t)\,V+\frac{1}{L}\int V\,dt = A\sin(\omega t), \tag{1}$$

in which L and C form the resonant circuit, $G(t)$ is the effective value of the total circuit conductance, which is a function of time, V is the value of the voltage across the circuit, A is the amplitude of the signal current, and ω is the radian frequency of the signal.

Differentiating equation (1) gives a differential equation for the voltage V. Thus

$$\frac{d^2V}{dt^2}+\frac{G(t)}{C}\frac{dV}{dt}+\left[\omega_0^2+\frac{G'(t)}{C}\right]V = \frac{A\omega}{C}\cos(\omega t) = \frac{V_s G_0\omega}{C}\cos(\omega t), \tag{2}$$

where $V_s = A/G_0$ is the voltage across the tuned circuit at resonance when $G(t)$ holds its 'passive' value G_0, and $\omega_0^2 LC = 1$.

Let us eliminate the term dV/dt in equation (2) by changing the dependent variable V to y by the substitution

$$V = y\exp\left[-\frac{1}{2C}\int^t G(x)\,dx\right], \tag{3}$$

where x is a time variable.

The equation for y is then

$$\frac{d^2y}{dt^2}+[\omega_0^2+G'/2C-G^2/4C^2]y = \frac{A\omega}{C}\cos(\omega t)\exp\left[\frac{1}{2C}\int^t G(x)\,dx\right], \qquad (4)$$

where G' is the time derivative of conductance.

This equation can be solved for y provided we assume that

$$\omega_0^2 \gg \left|\frac{G'}{2C}-\frac{G^2}{4C^2}\right| \qquad (5)$$

for all values of G.

Let us make this assumption and examine its meaning later.

Equation (4) now becomes

$$\frac{d^2y}{dt^2}+\omega_0^2 y = f(t), \qquad (6)$$

where
$$f(t)=\frac{A\omega}{C}\cos(\omega t)\exp\left[\frac{1}{2C}\int^t G(x)\,dx\right]. \qquad (6a)$$

The general solution of this equation is

$$y = a\cos(\omega_0 t)+b\sin(\omega_0 t)+\frac{1}{\omega_0}\int_0^t f(x)\sin[\omega_0(t-x)]\,dx. \qquad (7)$$

From (3) and (7)

$$V=\exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right] \times\left[a\cos(\omega_0 t)+b\sin(\omega_0 t)+\frac{1}{\omega_0}\int_0^t f(x)\sin[\omega_0(t-x)]\,dx\right]. \qquad (8)$$

This formula (8) for the voltage across the circuit is applicable to any variation of conductance provided that condition (5) is satisfied. We shall now examine the condition further and see what it really means.

The expression in (5) is a relationship between the resonant frequency, the conductance variation and the time constant of the tuned circuit. We can see roughly what conditions are necessary to satisfy this expression by writing it in terms of the circuit Q-factor.

From table 1 ($r=0$) we know that the Q-factor of the resonant parallel circuit is

$$Q_0=\frac{\omega_0 C}{G}. \qquad (9)$$

If we assume that the maximum excursion of G is G_0, in either direction, we may replace G by G_0 in equation (5), giving

$$\omega_0^2 \gg \frac{G'}{2C} - \frac{G_0^2}{4C^2}. \qquad (10)$$

We know that G_0 is the maximum excursion in the positive direction. In normal arrangements the negative excursion rarely exceeds G_0 for reasons which will appear later.

Assuming that this substitution is justified and writing ω_0/Q_0 for G_0/C in (10), we obtain

$$\omega_0^2 \gg \frac{G'}{2C} - \frac{\omega_0^2}{4Q_0^2}.$$

The second term is negligible for values of Q_0 greater than about 5. The remaining condition is

$$\omega_0^2 \gg G'/2C,$$

i.e.

$$G' \ll 2C\omega_0^2.$$

But, from (9), $C\omega_0 = QG_0$;

thus

$$G' \ll 4\pi Q_0 G_0/T_0,$$

where $T_0 = 2\pi/\omega_0$ is the period of the radio-frequency self-oscillations. In all normal circuits Q is likely to exceed 10, so that $4\pi Q > 100$, and the condition is satisfied if the rate of change of conductance, G', does not exceed the value G_0/T_0. This means that our approximation is only justified when the conductance changes by less than G_0 in one cycle of the radio-frequency.

In the quiescent state, $t < 0$, the voltage V across the circuit is the steady-state voltage, provided that the oscillations from the previous quench cycle have been allowed to die away to a negligible amplitude. The quiescent voltage is given by

$$V = AZ$$

$$= \frac{A}{G}\{1/[1 + (j/\omega L G_0)(\omega^2 LC - 1)]\} \quad \text{(from table 1)}$$

$$= \frac{A\omega}{C}\,\frac{(\omega_0^2 - \omega^2)\cos(\omega_0 t) + (\omega G_0/C)\sin(\omega_0 t)}{(\omega_0^2 - \omega^2)^2 + (\omega G_0/C)^2}. \quad (t<0) \quad (11)$$

Knowledge of this is necessary to enable us to evaluate the constants in equation (8) by inserting the appropriate boundary conditions.

3·5. The slope-controlled state

The expression (8) for the voltage across the circuit is too general to be of any use to us as it stands. It is necessary to insert the boundary conditions and to replace G by a function representing the type of variation pictured in fig. 3·4. First, the boundary conditions. In the slope-controlled state G is continuous at $t = 0$ (fig. 3·4). Therefore, from (1), V and dV/dt must be continuous at $t = 0$. At $t = 0$, (8) gives

$$V(0) = a = \frac{A\omega}{C}\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}$$

and

$$\left(\frac{dV}{dt}\right)_0 = b\omega_0-(G_0/2C)\,a = \frac{A\omega}{C}\frac{\omega^2 G_0/C}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}. \tag{12}$$

Therefore

$$b\omega_0 = \frac{(A\omega/C)(G_0/2C)(\omega_0^2+\omega^2)}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}, \tag{13}$$

and, from (8), (12) and (13),

$$\begin{aligned} V = {} & \exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right]\frac{A\omega}{C\omega_0} \\ & \times\left[\frac{\omega_0(\omega_0^2-\omega^2)\cos(\omega_0 t)+(G_0/2C)(\omega_0^2+\omega^2)\sin(\omega_0 t)}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}\right] \\ & +\exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right]\frac{A\omega}{C\omega_0} \\ & \times\left\{\int_0^t \exp\left[\frac{1}{2C}\int^x G(x)\,dx\right]\cos(\omega x)\sin\omega_0(t-x)\,dx\right\} \\ = {} & V_0+V_1, \quad \text{say.} \end{aligned} \tag{14}$$

Thus the voltage across the circuit at any time $t > 0$ consists of two parts, V_0 and V_1. V_0 can be regarded as due to the lingering effects of the steady-state voltage. V_1 is the effect of the signal upon the circuit with time-varying conductance. It is apparent later that the relative importance of V_0 and V_1 depends upon the duration of the regenerative period t_1, before the conductance is negative and super-regeneration proper begins. If the period t_1 is sufficiently long to include a large number of cycles of the radio-frequency ω_0, then V_1 is by far the more important term. We shall now evaluate V_1 for a slightly less general conductance function, corresponding to that shown in fig. 3·3.

We may write, from equation (14), the value of V_1 in the form

$$V_1 = \frac{V_s G_0}{C}\frac{\omega}{\omega_0}\exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right]P_1,$$

where $$P_1 = \int_0^t \exp\left[\frac{1}{2C}\int^x G(x)\,dx\right]\cos(\omega x)\sin\omega_0(t-x)\,dx. \quad (15)$$

The conductance function of fig. 3·3 may be represented by

$$G = G_0[1-F(t)], \quad (16)$$

where $$F(0) = 0, \quad F(t_1) = 1 \quad \text{and} \quad F(t_2) = 1. \quad (17)$$

From this $$\int_0^t G(t)\,dt = G_0[t-\Phi(t)], \quad (18)$$

where $$\Phi(t) = \int_0^t F(t)\,dt. \quad (19)$$

We may now proceed to evaluate P_1, of equation (15), using equations (3) and (4).

Making the substitutions, this becomes

$$P_1 = \int_0^t \exp[(G_0/2C)\{x-\Phi(x)\}]\cos(\omega x)\sin\omega_0(t-x)\,dx.$$

Now the rough argument, earlier in this section, shows us that the sensitivity of the circuit is greatest at time $t = t_1$.

Let us change the time variable from x to $z = x - t_1$, thus changing the origin from $t = 0$ to $t = t_1$. Now P_1 becomes

$$P_1 = \int_{-t_1}^{t-t_1} \exp\{(G_0/2C)[(z+t_1)-\Phi(z+t_1)]\} \times \cos\omega(z+t_1)\sin\omega_0(t-t_1-z)\,dz. \quad (20)$$

The exponential under this integral is, in fact, a measure of the sensitivity. It is greatest at time $t = t_1$ and falls off on either side thereof. The particular form of equation (20) enables us to expand about the instant $t = t_1$ and, by neglecting unimportant terms, to reach a reasonable and informative approximation to the value of P_1. Thus

$$\Phi(z+t_1) \doteqdot \Phi(t_1) + zF(t_1) + (\tfrac{1}{2}z^2)F'(t_1),$$

and, because $F(t_1) = 1$, from equation (17)

$$\Phi(z+t_1) \doteqdot \Phi(t_1) + z + (\tfrac{1}{2}z^2)F'(t_1).$$

The approximate value of the exponential consequently becomes

$$\begin{aligned}\exp[(G_0/2C)\{(z+t_1)-\Phi(z+t_1)\}] \\ &= \exp[(G_0/2C)\{t_1-\Phi(t_1)\}]\exp\left[-\frac{G_0F'(t_1)}{4C}z^2\right] \\ &= \exp\left[\frac{1}{2C}\int_0^{t_1}G(x)\,dx\right]\exp\left[-\frac{G_0F'(t_1)}{4C}z^2\right], \qquad (21)\end{aligned}$$

using equation (18).

The factor $\exp(-G_0F'(t_1)z^2/4C)$ is the sensitivity factor. It has the form of a Gaussian-error curve with a maximum at

$$z = 0 \quad (\text{i.e. } t = t_1).$$

It determines how the instantaneous sensitivity varies during the sampling period in a single quench cycle. It is during this period on each cycle that the receiver obtains its only information about the incoming signal. The sampling process is fundamental to the super-regenerative receiver.

The fact that this term, and therefore the entire expression for P_1, is only important for values of t near $t = t_1$ helps us to evaluate the integral of equation (20). Thus we can expand

$$\cos\omega(z+t_1)\sin\omega_0(t-t_1-z)$$

into four terms with factors

$$\begin{matrix}\sin\\ \cos\end{matrix}(\omega_0\pm\omega)z.$$

Of these, the terms with $\omega+\omega_0$ oscillate so rapidly that successive cycles in the integrand nearly cancel. On the other hand, the term with $\sin(\omega-\omega_0)z$ is zero at $z = 0$, where the sensitivity term is maximum, and therefore contributes very little. The remaining term with $\cos(\omega-\omega_0)z$ is by far the most important, so that P_1 is closely represented by the formula

$$P_1 = \tfrac{1}{2}\exp\left[\frac{1}{2C}\int_0^{t_1}G(x)\,dx\right]\int_{-t_1}^{t-t_1}\exp\left[-\frac{G_0F'(t_1)}{4C}z^2\right] \times\cos(\omega-\omega_0)z\,dz\sin[\omega_0t+(\omega-\omega_0)t_1]. \qquad (22)$$

We may again use the fact that the sensitivity factor falls off rapidly on both sides of $z = 0$ to replace the limits of integration by $\pm\infty$, without incurring much error, provided that

$$t \geqslant 2t_1 \quad \text{and} \quad \frac{G'(t_1)}{4C}t_1^2 > 3. \qquad (23)$$

These are the conditions for the exponential under the integral in equation (22) to be down to approximately 5 % of its maximum value at the limits of integration ($e^{-3} = 0{\cdot}05$). If these conditions are satisfied, the error involved in changing the limits of integration is negligible.

There only remains the problem of evaluating the integral

$$\int_{-\infty}^{+\infty} \exp\left[-\frac{G_0 F'(t_1)}{4C} z^2\right] \cos(\omega - \omega_0) z \, dz. \qquad (24)$$

It is a standard integral whose value is

$$2\sqrt{\left[\frac{\pi C}{G_0 F'(t_1)}\right]} \exp\left[-\frac{C(\omega-\omega_0)^2}{G_0 F'(t_1)}\right], \qquad (25)$$

and the value of P_1 is, consequently,

$$P_1 = \exp\left[\frac{1}{2C}\int_0^{t_1} G(x)\,dx\right] \sqrt{\left[\frac{\pi C}{G_0 F'(t_1)}\right]}$$
$$\times \exp\left[-\frac{C(\omega-\omega_0)^2}{G_0 F'(t_1)}\right] \sin[\omega_0 t + (\omega - \omega_0) t_1]. \qquad (26)$$

Thus we have evaluated P_1 and we may now use equations (15) and (26) to write down a new expression for the voltage V_1:

$$V_1 = V_s G_0 \sqrt{\left[\frac{\pi}{CG'(t_1)}\right]} \left(\frac{\omega}{\omega_0}\right) \exp\left[-\frac{C(\omega-\omega_0)^2}{G'(t_1)}\right]$$
$$\times \exp\left[-\frac{1}{2C}\int_{t_1}^{t} G(x)\,dx\right] \sin[\omega_0 t + (\omega-\omega_0) t_1]. \qquad (27)$$

Equation (27) is the basic formula for the slope-controlled state. It describes how the major part of the receiver output voltage depends upon the amplitude and frequency of the incoming signal and upon the circuit parameters. Before proceeding to interpret this formula it remains to show under what conditions it is permissible to neglect V_0 compared with V_1.

Now, from equation (14), the remaining part of the output voltage is

$$V_0 = \frac{V_s G_0}{C} \frac{\omega}{\omega_0} \exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right]$$
$$\times \left[\frac{\omega_0(\omega_0^2-\omega^2)\cos(\omega_0 t) + (G_0/2C)(\omega_0^2+\omega^2)\sin(\omega_0 t)}{(\omega_0^2-\omega^2)^2 + (\omega G_0/C)^2}\right]. \qquad (28)$$

This may be written

$$V_0 = \frac{V_s G_0}{C}\frac{\omega}{\omega_0}\exp\left[-\frac{1}{2C}\int_0^{t_1} G(x)\,dx\right]\exp\left[-\frac{1}{2C}\int_{t_1}^{t} G(x)\,dx\right]$$
$$\times\left[\frac{\omega_0(\omega_0^2-\omega^2)\cos(\omega_0 t)+(G_0/2C)(\omega_0^2+\omega^2)\sin(\omega_0 t)}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}\right]. \quad (29)$$

But
$$\int_0^{t_1} G(x)\,dx \doteqdot \frac{G_0 t_1}{2}, \quad (30)$$

for it is the area beneath the G-t curve (fig. 3·3) from $t = 0$ to $t = t_1$.

Thus we may write the expression for V_0:

$$V_0 \doteqdot \frac{V_s G_0}{C}\frac{\omega}{\omega_0}\exp\left(-\frac{G_0 t_1}{4C}\right)\exp\left[-\frac{1}{2C}\int_{t_1}^{t} G(x)\,dx\right]$$
$$\times\left[\frac{\omega_0(\omega_0^2-\omega^2)\cos(\omega_0 t)+(G_0/2C)(\omega_0^2+\omega^2)\sin(\omega_0 t)}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}\right]. \quad (31)$$

We now have a complete expression for the oscillatory voltage at any time t after the beginning of the conductance cycle. Let us examine the relative importance of the two parts V_0 and V_1.

From equations (27) and (31) the ratio of V_0 and V_1, at resonance, is

$$\frac{V_1(\omega_0)}{V_0(\omega_0)} = G_0 \exp\left(\frac{G_0 t_1}{4C}\right)\sqrt{\left[\frac{\pi}{C\,|\,G'(t_1)\,|}\right]}. \quad (32)$$

But
$$G'(t_1) \doteqdot G_0/t_1.$$

Therefore
$$\frac{V_1}{V_0} \doteqdot \sqrt{\left[\frac{\pi G_0 t_1}{C}\right]}\exp\left(\frac{G_0 t_1}{4C}\right). \quad (33)$$

Thus the condition for V_1 to be much greater than V_0 is

$$t_1 \gg 4C/G_0. \quad (34)$$

Because of the exponential in equation (33) it is adequate to write

$$t_1 > 12C/G_0, \quad (35)$$

for which V_0 is less than 1 % of V_1 at resonance. If this condition is satisfied the expression in equation (27) represents the only important contribution to the output voltage. This is the basic formula of the slope-controlled state. All subsequent formulae relating to slope-control are based upon equation (27).

In order to arrive at this approximate expression for the voltage V_1 we have made approximations that are only justified if the conditions in (23) and (35) are satisfied. These conditions define the slope-controlled state. Before attempting to interpret the equation (27) for V_1, let us examine further the meaning of the conditions for slope-control.

3·5·1. *The conditions for slope-control*

Equations (23) and (35) state that the conditions for slope-control are

$$t \geqslant 2t_1, \quad \frac{G'(t_1)}{4C}t_1^2 > 3, \quad t_1 > 12C/G_0. \tag{36}$$

The first of these merely states that we must not expect equation (27) to describe the output voltage too soon after the build-up starts. The second and third place a lower limit upon the duration of the regenerative period t_1. It is easy to show that the second and third conditions amount to the same thing.

The condition

$$\frac{G'(t_1)}{4C}t_1^2 > 3$$

may be written approximately

$$\frac{G_0}{4Ct_1}t_1^2 > 3,$$

i.e.

$$t_1 > 12C/G_0. \tag{37}$$

This only assumes that

$$G'(t_1) \doteqdot G_0/t_1,$$

which is a close enough approximation for our purpose. Equations (23) and (35) are thus identical.

It is interesting to see how many cycles of oscillation must be included in the interval t_1 to ensure slope-control. This is obtained by writing the third expression in equation (36) in terms of the Q-factor

$$\frac{12C}{G_0} = \frac{12Q}{\omega_0} = \frac{12Q}{2\pi}T_0,$$

where T_0 is the period of one oscillation.

According to this, the regenerative period t_1 must include more than about $2Q$ cycles of the self-oscillation. We may therefore state the condition for slope-control as follows: *For slope-control the regenerative period t_1 must include a large number of cycles of the self-oscillation.*

The precise condition is obtained by substituting the values for any particular circuit arrangement into equation (36). It should be noted that the conditions for slope-control state that the greater part of the sensitive period of the quench cycle should lie within the period of conductance-slope, that is to say, within the interval $t = 0$ to $t = 2t_1$. This is clear from the derivation of equation (23) in the last section. We shall remember it when we compare the conditions for the slope-controlled state with those for step-control.

3·5·2. *The oscillation envelope*

Equation (27) provides a good deal of information about the properties of a super-regenerative receiver operating in the linear mode and in the slope-controlled state, defined by equation (36). The linearity is immediately evident, for the voltage at any time is proportional to the signal voltage $V_0 (= A/G_0)$ across the quiescent circuit. It is also evident that the oscillations occur at the resonant frequency $f_0 = \omega_0/2\pi$ and not at the frequency of the signal. This is made clear by the appearance of the oscillatory term

$$\sin[\omega_0 t + (\omega - \omega_0) t_1], \tag{38}$$

which we shall not need to mention further. We are, however, interested in the coefficients of this term, which describe the variation of the oscillation amplitude with time and also its dependence upon the signal frequency.

The envelope of the pulse of oscillations is described by the factor

$$\exp\left[-\frac{1}{2C}\int_{t_1}^{t} G(x)\,dx\right]. \tag{39}$$

It has a maximum value at the end of the build-up period, at the instant $t = t_2$. Its shape may be demonstrated by expanding the expression about the time $t = t_2$, as we did for the sensitivity factor in equation (20).

The lower limit may be replaced by zero for the purpose of the expansion, although the original limit must be replaced when we come to consider the gain of the receiver. Thus, from equation (16),

$$\exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right] = \exp\left[-\frac{G_0}{2C}\int_0^t \{1 - F(x)\}\,dx\right].$$

Integrating, we have

$$\exp\left[-\frac{G_0}{2C}\int_0^t \{1 - F(x)\}\,dx\right] = \exp\left\{-\frac{G_0}{2C}[t - \Phi(t)]\right\}.$$

Changing the variable from t to $z = t - t_2$ we obtain

$$\exp\left\{-\frac{G_0}{2C}[z+t_2-\Phi(z+t_2)]\right\}. \tag{40}$$

But, as before,

$$\Phi(z+t_2) = \Phi(t_2) + zF(t_2) + (\tfrac{1}{2}z^2)F'(t_2).$$

But $F(t_2) = 1$, from equation (17). Therefore

$$\Phi(z+t_2) = \Phi(t_2) + z + (\tfrac{1}{2}z^2)F'(t_2).$$

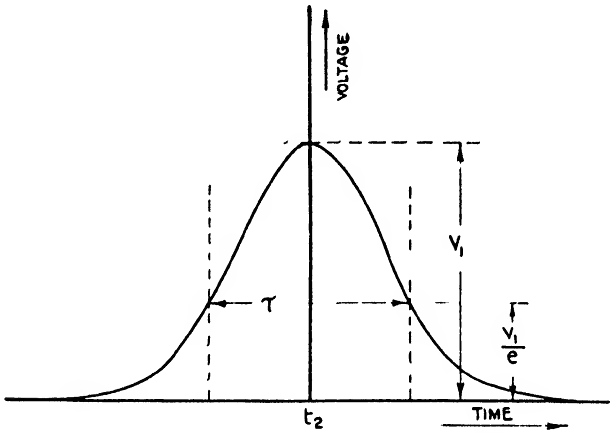


Fig. 3·6. Envelope of oscillations: slope-controlled state.

This makes the exponential factor

$$\exp\left\{-\frac{G_0}{2C}[t_2-\Phi(t_2)-(\tfrac{1}{2}z^2)F'(t_2)]\right\}$$

$$= \exp\left\{-\frac{G_0}{2C}[t_2-\Phi(t_2)]\right\}\exp\left[-\frac{G_0F'(t_2)}{4C}z^2\right]$$

$$= \exp\left\{-\frac{1}{2C}\int_{t_1}^{t_2}G(x)\,dx\right\}\exp\left[-\frac{G'(t_2)}{4C}z^2\right] \tag{41}$$

by replacing the lower limit.

The second factor in equation (41) describes the oscillation envelope of the output pulse. It is approximately a Gaussian error-curve, under the conditions of slope-control. The envelope of the oscillations in a single pulse is depicted in fig. 3·6. Its width, τ at $1/e$ of the peak amplitude, is

$$\tau = 4\sqrt{\left[\frac{C}{G'(t_2)}\right]}, \tag{42}$$

where $G'(t_2)$ is the slope of the conductance-time curve as it passes through zero at the end of the super-regenerative period.

3·5·3. *Super-regenerative and slope gain*

The factor $$\exp\left[-\frac{1}{2C}\int_{t_1}^{t_2} G(x)\,dx\right]$$

in equation (41) is the *super-regenerative gain*, measured at time t_2, at the peak of the output pulse. Expressed in nepers* it is

$$N_s = -\frac{1}{2C}\int_{t_1}^{t_2} G(x)\,dx = a^-/2C, \tag{43}$$

where a^- is the area under the negative portion of the conductance-time curve of fig. 3·3. The super-regenerative gain is due to the build-up of oscillations during the period of negative conductance, T_b $(= t_2 - t_1)$. It represents the ratio of the peak amplitude of the output oscillations to the value of the signal effective within the sampling period about the time $t = t_1$. There is, however, another gain factor which represents the net effect of the sampling process itself. From equation (27) we see that this term is

$$\mu_0 = G_0\sqrt{[\pi/C\,|\,G'(t_1)\,|]}. \tag{44}$$

We call this the *slope gain* because it arises from the period of conductance slope at the beginning of the conductance cycle when the conductance is less than G_0, but still positive. Wheeler has called it the regenerative gain. Expressed in nepers it is

$$N_0 = \tfrac{1}{2}\log_e\left[\frac{\pi G_0^2}{C\,|\,G'(t_1)\,|}\right] \text{ nepers}$$

$$= 4{\cdot}35\log_e\left[\frac{\pi G_0^2}{C\,|\,G'(t_1)\,|}\right] \text{ db.} \tag{45}$$

The total gain of the receiver is, therefore,

$$N_t = N_0 + N_s = \frac{a^-}{2C} + \tfrac{1}{2}\log_e\left[\frac{\pi G_0^2}{C\,|\,G'(t_1)\,|}\right] \text{ nepers.} \tag{46}$$

This is the gain, measured at the peak of the output pulse, for a signal on tune.

3·5·4. *The damping period*

After the oscillations have built up to their peak amplitude at the time $t = t_2$, they must decay so that they are smaller than the

* 1 neper ≑ 8·7 db.

incoming signal at the start of the next quench cycle. Otherwise the oscillations in the next cycle would build up from the decaying oscillations. Our analysis so far has assumed that one quench cycle can be treated as completely separate from the next. The condition for this is approximately

$$N_d > N_0 + N_s + 3. \tag{47}$$

This states that the oscillations are damped during the period T_d by an amount N_d nepers which is greater by 3 nepers (26 db.) than the amount by which they built up from the signal. Thus, at the beginning of the next quench cycle, the decaying oscillations are about twenty times less than the signal.

The value of N_d is

$$N_d = a^+/2C, \tag{48}$$

where a^+ is the area under the positive portion of the conductance-time curve of fig. 3·3.

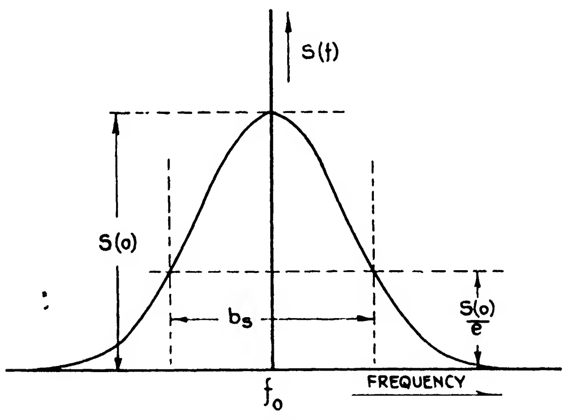


Fig. 3·7. Frequency response: slope-controlled state.

3·5·5. *Frequency response*

The receiver gain is reduced for a signal frequency differing from the resonant frequency. The frequency response of the receiver is described by the remaining factor of equation (27),

$$S(f) = \frac{f}{f_0} \exp\left[-\frac{4\pi^2 C(f-f_0)^2}{|G'(t_1)|}\right]. \tag{49}$$

It is plotted in fig. 3·7. The value of the expression is unity for $f = f_0$, and falls off for higher and lower frequencies. It is approximately a Gaussian error-curve for frequencies close to f_0. The band-width at $1/e$ of the peak is

$$b_s = \frac{1}{\pi}\sqrt{\left[\frac{G'(t_1)}{C}\right]} \quad \text{at } -1 \text{ neper,} \tag{50}$$

and depends only upon the conductance slope at time t_1, at the peak of the sensitive period, and upon the circuit capacitance.

We call b_s the *super-regenerative band-width.* It is the natural band-width of the slope-controlled linear super-regenerative receiver.

3·5·6. *Band-width and receiver gain*

The slope-gain factor

$$\mu_0 = G_0\sqrt{\left[\frac{\pi}{C\,|\,G'(t_1)\,|}\right]} \tag{51}$$

which arises in equation (27) comes from the period of conductance slope during which the frequency-response characteristics of the receiver are determined. A closer inspection of the origin of this factor provides a clearer physical picture of super-regenerative action and, ultimately, gives a more generally useful formula for receiver gain.

We have seen, in § 3·5·5, that the frequency response of the slope-controlled super-regenerative receiver is

$$S(f) = \frac{f}{f_0}\exp\left[-\frac{4\pi^2C(f-f_0)^2}{|\,G'(t_1)\,|}\right]. \tag{52}$$

It differs considerably from that of the quiescent circuit consisting of L, C and G_0, which is

$$A(f) = \frac{G_0/4\pi C}{\sqrt{[(f-f_0)^2+(G_0/4\pi C)^2]}}. \tag{53}$$

The slope gain (equation (51)) is merely the factor that takes account of the change in the frequency-response characteristic. It can be written as the ratio of the 'effective band-widths'* of the circuit in the two states defined, respectively, as

$$b_{es} = \frac{1}{|\,S(f_0)|}\int_{-\infty}^{+\infty}|\,S(f)\,|\,df \tag{54}$$

* It must be noted that these differ from the energy band-widths used in noise calculations and defined by

$$b_{ns} = \frac{1}{|\,S(f_0)\,|^2}\int_{-\infty}^{+\infty}|\,S(f)\,|^2 df, \quad b_n = \frac{1}{|\,A(f_0)\,|^2}\int_{-\infty}^{+\infty}|\,A(f)\,|^2 df.$$

b_e and b_{es} are the areas beneath the respective amplitude-frequency response and $$b_e = \frac{1}{|A(f_0)|}\int_{-\infty}^{+\infty} |A(f)|\,df. \tag{55}$$

Now, from (54) and (52),

$$b_{es} = \int_{-\infty}^{+\infty} (f/f_0)\exp\left[-\frac{4\pi^2C(f-f_0)^2}{|G'(t_1)|}\right]df$$
$$\doteqdot \frac{1}{2}\sqrt{\left[\frac{|G'(t_1)|}{\pi C}\right]}, \tag{56}$$

and, from (55) and (53),

$$b_e = \int_{-\infty}^{+\infty} \frac{G_0/4\pi C}{\sqrt{[(f-f_0)^2+(G_0/4\pi C)^2]}}df \tag{57}$$
$$\doteqdot G_0/2C.$$

Thus $$\frac{b_e}{b_{es}} = \frac{G_0}{2C}2\sqrt{\pi}\sqrt{\left[\frac{C}{|G'(t_1)|}\right]} = G_0\sqrt{\left[\frac{\pi}{C|G'(t_1)|}\right]}. \tag{58}$$

$G_0\sqrt{\left[\frac{\pi}{C|G'(t_1)|}\right]}$ is the expression, appearing in equation (27), which we call the slope gain of the super-regenerative receiver. If we substitute for this factor in the basic equation (27) for slope control, the expression for the peak-output voltage becomes

$$V = V_s\left(\frac{b_e}{b_{es}}\right)\exp\left(\frac{a^-}{2C}\right)S(f)\sin(\omega_0 t+\phi), \tag{59}$$

and the total voltage gain of the receiver is

$$\mu_t = \frac{b_e}{b_{es}}\exp\left(\frac{a^-}{2C}\right). \tag{60}$$

We see later that the expressions in equations (59) and (60) are independent of the type of conductance or quench cycle, but, nevertheless, they apply only to a receiver operating in the linear mode.

Equation (60) may also be written

$$\mu_t = 2\sqrt{\pi}\left(\frac{b}{b_s}\right)\exp\left(\frac{a^-}{2C}\right) = \sqrt{2}\left(\frac{b_n}{b_{ns}}\right)\exp\left(\frac{a^-}{2C}\right), \tag{61}$$

curves. b_n and b_{ns} are the areas beneath the corresponding energy-frequency response curves. Evaluation of the integrals for b_{ns} and b_n, with the values of $S(f)$ and $A(f)$ substituted from (52) and (53), shows that

$$b_n = G_0/4C, \quad b_{ns} = \frac{1}{2\sqrt{2}}\sqrt{\left[\frac{|G'(t_1)|}{\pi C}\right]}.$$

Consequently $$\frac{b_n}{b_{ns}} = \frac{G_0}{\sqrt{2}}\sqrt{\left[\frac{\pi}{C|G'(t_1)|}\right]} = \frac{1}{\sqrt{2}}\frac{b_e}{b_{es}}. \tag{58a}$$

where $b = G_0/2\pi C$ is the band-width of the quiescent circuit at -3 db. and b_n, b_{ns} are defined in the footnote. Equation (61) expresses the voltage gain in terms of the 'natural' band-widths, the circuit capacitance and the area under the negative portion of the conductance-time curve. It is a useful practical formula.

3·6. The step-controlled state

The whole of the analysis for slope-control depends upon the assumption that the period of conductance-slope, t_1, is finite and includes a very large number of cycles of the resonant frequency. We shall now discuss the other limiting case in which the conductance moves rapidly to a negative value at the commencement of the cycle and rapidly back to G_0 at some later time. The rectangular-wave quench cycle is a particular form of this, the *step-controlled state*, but we need not confine ourselves to the case in which the negative conductance remains constant throughout the super-regenerative period.

In order to analyse the performance of the receiver in the step-controlled state it is necessary to go back to the basic circuit equation (1) which is

$$\frac{dV}{dt} + \frac{G(t)}{C} V + \omega_0^2 \int V dt = A \sin(\omega t), \tag{62}$$

and to substitute a step function for the conductance $G(t)$.

The step-controlled quench cycle is illustrated in fig. 3·8. The conductance changes instantaneously from G_0 to $-G_1$ at time $t = 0$. The function $G(t)$ is therefore represented by

$$\left.\begin{aligned} &G(t) = G_0 \quad (t < 0) \\ \text{and} \qquad &G(t) = -G_1(1 + F(t)) \quad (t > 0), \end{aligned}\right\} \tag{63}$$

where $F(0) = 0$ and $F(t)$ represents the deviation of $G(t)$ from the value $-G_1$ during the super-regenerative period. For a rectangular-wave quench cycle, which is the most likely practical example of step-control, $F(t) = 0$ and $G(t) = G_1$ during the super-regenerative period.

If the change of conductance during the super-regenerative period is gradual so that $G'(t)$ is small enough to satisfy the condition

$$\omega_0^2 \gg \frac{G'}{2C} - \frac{G^2}{4C^2} \quad \text{(from equation (5))},$$

then we may solve the basic equation (1) as before (§3·4) arriving at equations (6*a*), (8) and (11) for the voltage V across the circuit at any time t after the oscillations have started to build up. According to these equations, V is given by

$$V = \exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right]\left\{a\cos(\omega_0 t) + b\sin(\omega_0 t)\right.$$
$$\left. + \frac{A\omega}{C\omega_0}\int_0^t \exp\left[\frac{1}{2C}\int_0^x G(x)\,dx\right]\cos(\omega x)\sin\omega_0(t-x)\,dx\right\} \qquad (64)$$

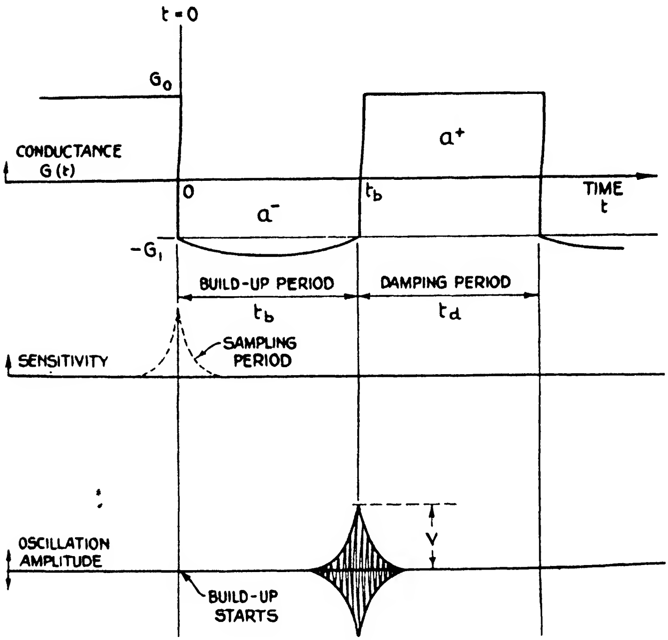


Fig. 3·8. General conductance cycle: step-controlled state.

for values of t greater than zero and by

$$V = \frac{A\omega}{C}\left[\frac{(\omega_0^2-\omega^2)\cos(\omega_0 t) + (\omega G_0/C)\sin(\omega_0 t)}{(\omega_0^2-\omega^2)^2 + (\omega G_0/C)^2}\right] \qquad (65)$$

for values of t less than zero on the chosen time scale.

The values of the coefficients a and b in equation (64) may be obtained from the boundary conditions at $t = 0$. The voltage V is continuous through $t = 0$, i.e.

$$V(-0) = V(+0). \qquad (66)$$

From the basic equation (1) it is apparent that

$$\left(\frac{dV}{dt}\right)_{+0} - \left(\frac{dV}{dt}\right)_{-0} = V(G_0+G_1)/C. \tag{67}$$

In other words, the instantaneous change of conductance causes a corresponding discontinuity, of amount $V(G_0+G_1)/C$, in the time rate of change of the voltage V.

From equation (64) $\quad V(+0) = a$

and from (65)

$$V(-0) = \frac{A\omega}{C}\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}.$$

Consequently, using equation (66),

$$a = \frac{A\omega}{C}\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}. \tag{68}$$

Similarly, from (64), (65) and (67),

$$\left(\frac{dV}{dt}\right)_{+0} - \left(\frac{dV}{dt}\right)_{-0} = b\omega_0 - (G_0/2C)\,a - \frac{A\omega}{C}\frac{\omega^2 G_0/C}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}$$

$$= [(G_0+G_1)/C]\,a.$$

When the value of a just obtained (68) is inserted and the expression rearranged we obtain the value of b, which is

$$b = \frac{A\omega}{C\omega_0}\frac{(G_0/2C)(\omega_0^2+\omega^2)+[(G_0+G_1)/C](\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}. \tag{69}$$

The complete expression for the voltage V now follows after the values of a and b are substituted from equations (68) and (69) into equation (64). In this way we obtain

$$V = \exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right](P_0+P_1), \tag{70}$$

where

$$P_0 = \frac{A}{C}\frac{\omega}{\omega_0}\left\{\left[\frac{\omega_0(\omega_0^2-\omega^2)\sin\omega_0 t}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}\right]\right.$$
$$\left.+\left[\frac{\{(G_0/2C)(\omega_0^2+\omega^2)+[(G_0+G_1)/C](\omega_0^2-\omega^2)\}\cos\omega_0 t}{(\omega_0^2-\omega^2)^2+(\omega G_0/C)^2}\right]\right\} \tag{71}$$

and $$P_1 = \frac{A}{C}\frac{\omega}{\omega_0}\int_0^t \exp\left[\int^x G(x)\,dx\right]\cos(\omega x)\sin\omega_0(t-x)\,dx. \tag{72}$$

The frequency-dependent factors P_0 and P_1 determine the response of the receiver in the step-controlled state. We shall proceed, first, to simplify these expressions.

From (72) and (63)

$$P_1 = \frac{A}{C}\frac{\omega}{\omega_0}\int_0^t \exp\left\{-\frac{G_1}{2C}\int_0^x [1+F(x)]\,dx\right\}\cos(\omega x)\sin\omega_0(t-x)\,dx$$

$$= \frac{A}{C}\frac{\omega}{\omega_0}\int_0^t \exp\left\{-\frac{G_1}{2C}[x+\Phi(x)]\right\}\cos(\omega x)\sin\omega_0(t-x)\,dx, \qquad (73)$$

where $$\Phi(x) = \int_0^t F(x)\,dx.$$

In the integrand, the exponential factor, which represents the sensitivity, has a strong maximum at $x = 0$. It is therefore a sufficient approximation to neglect the term $\Phi(x)$ which is zero at $x = 0$. Moreover, for $t \gg 2C/G_1$, we may replace the upper limit by infinity. Then

$$P_1 = \frac{A}{C}\frac{\omega}{\omega_0}\int_0^\infty \exp\left(-\frac{G_1}{2C}x\right)\cos(\omega x)\sin\omega_0(t-x)\,dx. \qquad (74)$$

The integral may be expanded as the sum of four standard integrals of the form

$$\int_0^\infty e^{-\alpha x}\,{\sin \atop \cos}(\beta x)\,dx.$$

The resulting approximate expression for P_1 is

$$P_1 \doteqdot \frac{A}{G_1}\frac{G_1/2C}{\sqrt{[(\omega-\omega_0)^2+(G_1/2C)^2]}}\sin(\omega_0 t-\psi_1), \qquad (75)$$

where $$\psi_1 = \tan^{-1}\left[\frac{2C(\omega-\omega_0)}{G_1}\right]$$

provided that $\omega_0 \gg G_1/2C$ and $(\omega-\omega_0)/\omega_0 \ll 1$.

Now, from (71), the approximate expression for P_0 is

$$P_0 \doteqdot \frac{A}{C}\frac{\omega}{\omega_0}\frac{\omega_0(\omega_0^2-\omega^2)\sin\omega_0 t+(G_0/2C)(\omega_0^2+\omega^2)\cos(\omega_0 t)}{(\omega^2-\omega_0^2)^2+(\omega G_0/C)^2}$$

$$\doteqdot \frac{A}{G_0}\frac{G_0/2C}{\sqrt{[(\omega^2-\omega_0^2)^2+(G_0/2C)^2]}}\sin(\omega t+\psi_0), \qquad (76)$$

where $$\psi_0 = \frac{2C(\omega-\omega_0)}{G_0}$$

provided that $$\omega_0 \gg G_0/2C$$

and $$\frac{\omega-\omega_0}{\omega_0} \ll 1.$$

If we substitute the values of P_0 and P_1 from (75) and (76) into equation (70) we get, to the degree of approximation stated above,

$$V = \frac{A}{2C}\exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right] \times \left\{\frac{\sin(\omega_0 t+\psi_0)}{\sqrt{[(\omega-\omega_0)^2+(G_0/2C)^2]}}+\frac{\sin(\omega_0 t-\psi_1)}{\sqrt{[(\omega-\omega_0)^2+(G_1/2C)^2]}}\right\},$$

in which $$\psi_0 = \tan^{-1}\{[2C(\omega-\omega_0)]/G_0\}$$

and $$\psi_1 = \tan^{-1}\{[2C(\omega-\omega_0)/G_1\}.$$

Simplified still further this becomes, to the same degree of approximation,

$$V = V_s[(G_0+G_1)/G_1]\exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right] \times \left\{\frac{G_0 G_1/4C^2}{\sqrt{[(\omega-\omega_0)^2+(G_0/2C)^2]}\sqrt{[(\omega-\omega_0)^2+(G_1/2C)^2]}}\right\}\sin(\omega_0 t+\psi_3), \tag{77}$$

where $V_s = A/G_0$ and ψ_3 is a phase angle dependent upon ω but of no further interest here.

Equation (77) for step-control corresponds to equation (27) for the slope-controlled state, and describes the dependence of the self-oscillation in the super-regenerative circuit upon the signal voltage and the circuit parameters. We shall derive from it formulae for the step-controlled state corresponding to those already derived for slope-control.

3·6·1. *The oscillation envelope*

The linearity of the receiver is again immediately evident, for the voltage at any time is proportional to the signal voltage $V_s = A/G_0$ across the quiescent circuit at resonance. The oscillations occur at the resonant frequency $f_0 = \omega_0/2\pi$ and not at the signal frequency. This is shown by the oscillatory term

$$\sin(\omega_0 t+\psi_3). \tag{78}$$

The envelope of the output pulse of self-oscillations is described by the factor

$$A(t) = \exp\left[-\frac{1}{2C}\int_0^t G(x)\,dx\right]. \tag{79}$$

It reaches its maximum value at the end of the build-up or super-regenerative period at time $t = t_b$ (fig. 3·8). Substituting for $G(x)$ from equation (63) it becomes

$$A(t) = \exp\left[\frac{G_1}{2C}\int_0^t \{1+F(x)\}\,dx\right]$$
$$= \exp\left[\frac{G_1}{2C}\{t+\Phi(t)\}\right].$$

But $\Phi(t) = 0$ at time $t = t_b$ when the oscillations have their maximum amplitude. Putting $\Phi(t) = 0$ and changing the variable from t to $z = t - t_b$ we obtain

$$A(t) = \exp\left[(G_1/2C)(z+t_b)\right]$$
$$= \exp(G_1 t_b/2C)\exp(G_1 z/2C) \quad (z<0). \tag{80}$$

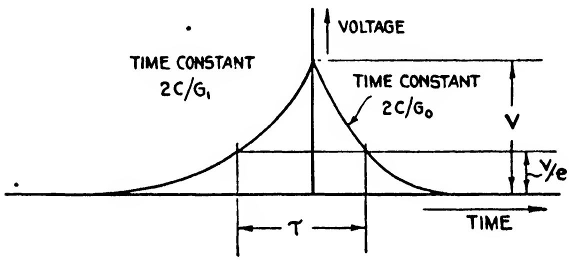


Fig. 3·9. Oscillation envelope: step-controlled state.

The second term represents the exponential build-up of the oscillations. The exponential decay ($z > 0$) is given by

$$A(t) = \exp(G_1 t_b/2C)\exp(-G_0 z/2C) \quad (z>0). \tag{81}$$

Thus the output pulse of oscillations builds up and decays exponentially, but it is important to note that the pulse is only symmetrical when the time constant of build-up is equal to that of decay, i.e. when $G_1 = G_0$. This is illustrated by fig. 3·9.

The duration or width of the pulse at $1/e$ of the peak is

$$\tau = 2C\left(\frac{1}{G_1}+\frac{1}{G_0}\right). \tag{82}$$

3·6·2. *Super-regenerative and step gain*

The super-regenerative gain in the step-controlled state is the exponential factor

$$\exp\left[-\frac{1}{2C}\int_0^{t_b} G(x)\,dx\right], \tag{83}$$

which represents the build-up ratio of oscillations during the super-regenerative period t_b. Expressed in nepers the super-regenerative gain is

$$N_s = -\frac{1}{2C}\int_0^{t_b} G(x)\,dx = a^-/2C, \tag{84}$$

where a^- is the area under the negative portion of the conductance-time curve of fig. 3·8.

The factor
$$(G_0+G_1)/G_1 \tag{85}$$

in equation (77) is the step-factor representing the net effect of the sampling process. It corresponds to the factor $G_0\sqrt{\left[\frac{\pi}{C|G'(t_1)|}\right]}$ for the slope-controlled state. The conditions for the oscillations from one quench cycle to decay so that they are small compared with the signal at the beginning of the next cycle must still be obeyed. They are precisely the same as those quoted for slope-control in §3·5·4 (equations (47) and (48)).

3·6·3. *Frequency response*

The frequency response of the linear receiver in the step-controlled state is approximately represented by the factor

$$S(f) = \frac{(G_0/4\pi C)(G_1/4\pi C)}{\sqrt{[(f-f_0)^2+(G_0/4\pi C)^2]}\sqrt{[(f-f_0)^2+(G_1/4\pi C)^2]}}. \tag{86}$$

It is similar to the response of a pair of loosely coupled tuned circuits, each of resonant frequency f_0, but having Q-factors $2\pi f_0 C/G_0$ and $2\pi f_0 C/G_1$ respectively. Thus it appears that the step-controlled receiver may be usefully represented by a pair of loosely coupled circuits followed by a linear amplifier. This is shown in fig. 3·10 which gives the value of the coefficient of coupling necessary for equivalence of gain as well as of frequency response.

There is no precise expression for the super-regenerative band-width, but it is possible to show that the following formula is a close enough approximation, in all practical cases:

$$b_s \doteqdot \frac{G_0 G_1}{\pi C(G_0+G_1)} \quad \text{at } -6\text{ db}. \tag{87}$$

The formula is accurate for $G_0 = G_1$, in which case the band-width is

$$b_s = G_0/2\pi C \quad \text{at } -6\text{ db}. \tag{88}$$

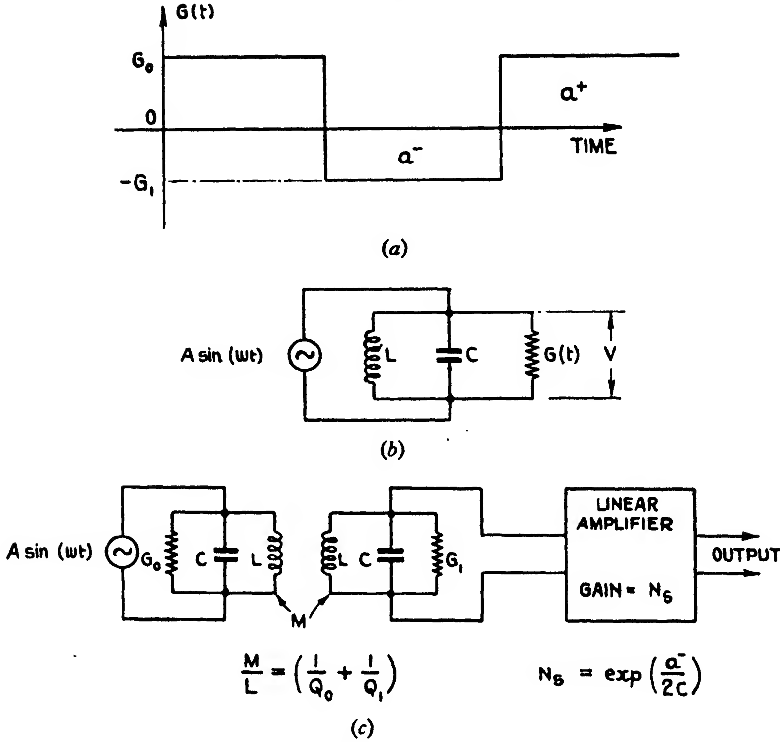


Fig. 3·10. Representation of a step-controlled super-regenerative receiver. (*a*) Conductance cycle. (*b*) Equivalent circuit used in analysis. (*c*) Circuit having similar gain and frequency response to step-controlled receiver.

3·6·4. *Band-width and step-gain*

The step-gain factor may be written as the ratio of two band-widths in the same way as the corresponding expression (51) for slope-control.

Thus
$$\mu_0 = (G_0+G_1)/G_1 = b_e/b_{es}, \tag{89}$$
in which
$$b_e = \frac{1}{|A(f_0)|}\int_{-\infty}^{+\infty} |A(f)|\, df, \tag{89a}$$

$$b_{es} = \frac{1}{|S(f_0)|}\int_{-\infty}^{+\infty} |S(f)|\, df, \tag{89b}$$

$$A(f) = \frac{G_0/4\pi C}{\sqrt{[(f-f_0)^2+(G_0/4\pi C)^2]}}, \tag{89c}$$

$$S(f) \doteqdot \frac{(G_0/4\pi C)(G_1/4\pi C)}{\sqrt{\{[(f-f_0)^2+(G_0/4\pi C)^2][(f-f_0)^2+(G_1/4\pi C)^2]\}}}. \quad (89d)^*$$

The total voltage gain of the step-controlled receiver is, therefore,

$$\mu_t = (b_e/b_{es}) \exp(a^-/2C). \quad (90)$$

This is identical with equation (60) relating to the slope-controlled state. It is a general expression for the voltage gain of the linear super-regenerative receiver and is true whatever the nature of the conductance function.

A more practical form of the step-gain factor is written in terms of the 'natural' band-widths b and b_s, given by

$$b = G_0/2\pi C \quad \text{at } -3\,\text{db.}, \quad (91)$$

$$b_s = G_0 G_1/[\pi C(G_0+G_1)] \quad \text{at } -6\,\text{db.} \quad (92)$$

Hence the step-gain factor is, in these terms,

$$\mu_0 = (G_0+G_1)/G_1 = \tfrac{1}{2}(b/b_s). \quad (93)$$

3·7. Interpretation of the theory

We have seen above that the properties of the super-regenerative receiver operated in the linear mode can be described in terms of simple formulae in two particular states of operation, which we call slope- and step-controlled, respectively. These are both idealized states in the sense that a practical receiver cannot operate precisely in either.

The formulae for the linear mode are summarized in table 2. They show that there is a general expression for the gain of the receiver, which is common to both states, but the characters of the frequency-response curve and of the output pulse of oscillations differ considerably from one state to the other. So does the dependence of band-width upon the circuit parameters. In the slope-controlled state the receiver band-width depends upon the conductance slope at the beginning of the super-regenerative period. In the step-controlled state, however, the band-width depends upon the extreme values of conductance, G_0 and G_1.

3·7·1. *Transition from slope-control to step-control*

The division of the operation of the receiver into two states is artificial. It has been made, necessarily, in order to obtain simple

* In order to prove equation (89) it is necessary to use a more precise expression for $S(f)$ than (89d). The approximations made in arriving at (89d) are not valid for values of f approaching infinity. The proof is not included here.

Table 2. *Summary of formulae for the linear super-regenerative receiver. Based upon the equivalent circuit of fig.* 3·1 *and the conductance cycles of fig.* 3·8 *(step-control) and fig.* 3·3 *(slope-control)*

	Quiescent circuit ($G=G_0$)	Step-controlled receiver	Slope-controlled receiver
Frequency response	$A(f)=\dfrac{G_0/4\pi C}{\sqrt{[(f-f_0)^2+(G_0/4\pi C)^2]}}$	$S(f)=\dfrac{(G_0/4\pi C)(G_1/4\pi C)}{\sqrt{\{[(f-f_0)^2+(G_0/4\pi C)^2][(f-f_0)^2+(G_1/4\pi C)^2]\}}}$	$S(f)=(f/f_0)\exp\left[-\dfrac{4\pi^2C(f-f_0)^2}{\lvert G'(t_1)\rvert}\right]$
Band-width	$b=G_0/2\pi C=f_0/Q$ at -3 db.	$b_s \doteqdot G_0G_1/[\pi C(G_0+G_1)]$ at -6 db.	$b_s=\dfrac{1}{\pi}\sqrt{\left[\dfrac{\lvert G'(t_1)\rvert}{C}\right]}$ at $-8{\cdot}7$ db.
Energy band-width	$b_n=G_0/4C=(\tfrac{1}{2}\pi)\,b$	$b_{ns}\doteqdot G_0G_1/[4C(G_0+G_1)]=(\tfrac{1}{4}\pi)\,b_s$	$b_{ns}=\dfrac{1}{2\sqrt{2}}\sqrt{\left[\dfrac{\lvert G'(t_1)\rvert}{\pi C}\right]}=\tfrac{1}{2}\sqrt{(\tfrac{1}{2}\pi)}\,b_s$
Super-regenerative gain	—	$\mu_s=\exp\left[-\dfrac{1}{2C}\displaystyle\int_0^{t_b}G(x)\,dx\right]=\exp\left(\dfrac{a^-}{2C}\right)$ $N_s=(a^-/2C)$ nepers $=8{\cdot}7(a^-/2C)$ db.	$\mu_s=\exp\left[-\dfrac{1}{2C}\displaystyle\int_{t_1}^{t_2}G(x)\,dx\right]=\exp\left(\dfrac{a^-}{2C}\right)$ $N_s=(a^-/2C)$ nepers $=8{\cdot}7(a^-/2C)$ db.
Slope- or step-gain factor	—	$\mu_0=(G_0+G_1)/G_1=b_n/b_{ns}=\tfrac{1}{2}(b/b_s)$	$\mu_0=G_0\sqrt{\left[\dfrac{\pi}{C\lvert G'(t_1)\rvert}\right]}=(b_e/b_{es})=2\sqrt{\pi}\left(\dfrac{b}{b_s}\right)$
Pulse shape	—	$A(f)=\exp(-Gz/2C)$ where $\begin{cases}G=G_0(z<0)\\G=-G_1(z>0)\end{cases}$	$A(f)=\exp\left[-\dfrac{\lvert G'(t_2)\rvert}{4C}z^2\right]$
Pulse width	—	$\tau=2C\left(\dfrac{1}{G_0}+\dfrac{1}{G_1}\right)$ at $-8{\cdot}7$ db.	$\tau=4\sqrt{\left[\dfrac{C}{\lvert G'(t_2)\rvert}\right]}$ at $-8{\cdot}7$ db.

formulae describing the behaviour of the receiver in the linear mode. The distinction between the two states is made in the rate of change of conductance at the *beginning* of the super-regenerative period, when the value of the conductance passes through zero (time $t = t_1$, fig. 3·3; time $t = 0$, fig. 3·8). We can see the reasons for the distinction, and also obtain much useful information regarding intermediate states, by considering a simplified conductance cycle, in which the transition from positive to negative values of G is linear.

The slope-controlled state is characterized by a sensitive period* bounded by an error curve and lying wholly within the period of conductance slope. As the rate of change of conductance is increased the skirts of the sensitivity-time curve overlap the period of constant conductance, and assume the exponential shape characteristic of step-control. Material deviation from the slope-controlled state only occurs when an appreciable area of the sensitivity curve falls outside the period of conductance slope. The condition for this is stated in equation (23), and the physical manifestation is the spreading of the skirts of the frequency-response curve.

It is impossible to give simple formulae for the properties of a receiver in the state between slope and step-control. By following the argument above it is, however, quite possible to estimate the extent and nature of the departure from slope-control in any given case.

3·7·2. *Hybrid conductance cycles*

So far we have only considered as examples symmetrical conductance cycles in which the rate of change of conductance at the

* The description of the operation of the receiver in terms of a sensitive period is very useful in giving a kind of physical picture of the process of reception. It is, however, a little misleading if taken too far, and a word of caution is, perhaps, necessary. The sensitivity-time curves sketched, for instance, in fig. 3·3, can be regarded as measuring the effect of a very narrow impulse upon the amplitude to which the oscillations rise in the later part of the cycle. If the impulse occurs before $G=0$, then the amplitude of the transient has diminished somewhat, due to damping, before the build-up starts. It has less effect, therefore, than a similar current impulse occurring exactly when $G=0$. An impulse occurring at a later time has to compete with the rising oscillations and thus has less effect, again. When discussing a c.w. signal, however, the sensitive period itself has little meaning. In this case the circuit is in a steady state long before the conductance is changed, a current $A \sin \omega t$ producing a steady voltage $V_s = A/G_0$ across the circuit at resonance. This state of affairs persists right up to the time the conductance begins to change and the picture of the rising sensitivity curve has less meaning. As soon as the signal is started or stopped in the vicinity of the sensitive period, however, the 'memory' of the circuit must be taken into consideration. It must be emphasized that this is a limitation only in the physical picture, not in the theory.

beginning of the super-regenerative period is the same as that at the end of the cycle (both measured when G crosses the axis at $G = 0$). There is, however, nothing in the theory to associate these two rates of change. It is feasible to have a conductance cycle in which the transition from positive to negative values is gradual, whilst the return, at the end of the super-regenerative period, is rapid, or vice versa. In such circumstances the state of operation (slope- or step-controlled) is determined by the transition at the beginning of the cycle, which determines the frequency response, even though the output pulse shape is that associated with the opposite state.

It is, therefore, possible to design a super-regenerative receiver to have a certain frequency-response characteristic and, independently, to have a certain shape of output pulse, within the general limits of the theory, by applying an asymmetric quench wave-form of suitable shape.

Chapter 4

RECEPTION AND DETECTION OF SIGNAL AND NOISE: LINEAR MODE

4·1. Introduction

The output of a super-regenerative receiver, when receiving a c.w. signal, is a regular succession of similar pulses of equal amplitudes. When there is no signal, however, the oscillations on each quench cycle build up from the sample of circuit noise occurring within the sensitive period at the beginning of the cycle. Under these circumstances the amplitudes differ from one pulse to the next. The fluctuation in amplitude of the pulses of oscillation constitutes noise in the receiver output. In spite of this fluctuation the time interval between the peaks of successive pulses remains constant. This is because the oscillations reach their maximum amplitude at a definite point on each quench cycle, when the value of the conductance passes through zero at the end of the super-regenerative period. It is important to note this regularity in the spacing, because it has practical importance in the design of systems for a.g.s., which are considered in a later chapter.

Because of the rather complicated sampling process that occurs in the super-regenerative receiver, it is by no means obvious how the fluctuation of the output pulses depends upon the noise in the input circuits. The noise changes its nature in the super-regenerative process and a new definition is necessary to describe the noise fluctuation in the receiver output.

4·2. Signal and noise before detection

A super-regenerative receiver, activated by noise alone, builds up pulses whose amplitudes vary from one pulse to another. The same is true when a steady c.w. signal is also present, but the fraction of pulses in a very large number which exceed in amplitude a certain value is different in the two cases. When the c.w. signal is much greater than noise the deviation of the amplitude from its mean is negligible.

Since the amplitudes are subject to random variations it is impossible to state precisely the amplitude which a pulse occurring at a specified time will have. What we can state is the probability that the amplitude shall lie between certain limiting values. In particular, if these limits are x and $x+dx$, the probability can be written

$$P(x, x+dx) = p(x)\,dx. \tag{1}$$

The function $p(x)$ is then the probability distribution.

The noise that is observed at the output arises from the random variation in amplitude of the pulses. When the amplitudes are examined after detection the probability distribution depends upon the law of the detector. But knowledge of the detector law is not the only prerequisite for finding that distribution. The statistical behaviour of the instantaneous voltage at the input to the detector must be known. It is necessary here to define clearly the voltage fluctuation that corresponds to noise in the receiver. For this purpose we are not interested in the voltage distribution in a single pulse. We want to know the distribution of instantaneous values of voltage measured at the peaks of successive pulses and due to the build-up of oscillations from noise alone.

In view of the linearity of amplification it is reasonable to expect that this distribution will be similar to that of the noise itself, namely, a normal Gaussian law with

$$p(x) = \{1/[\sigma\sqrt{(2\pi)}]\}\exp(-x^2/2\sigma^2). \tag{2}$$

The standard deviation σ is the r.m.s. value of the voltage at the end of the build-up period, i.e. at time t_2, averaged over a large number of quench periods. This is the r.m.s. noise voltage in the output of the super-regenerative receiver. The averaging process which is carried out here is not quite the usual kind. Usually the noise is continuous in time and the average is taken over a long interval. Here, however, the average is taken over discrete samples of noise, taken successively at equal intervals of time. In spite of this difference the r.m.s. noise voltage at the output, defined in the above way, is linearly dependent upon the r.m.s. noise voltage from which the oscillations grow, i.e.

$$\sigma^2 = \overline{V_{ns}^2} \times (\text{total receiver power gain}) \tag{3}$$

$$= 2\overline{V_{ns}^2}\left(\frac{b_n}{b_{ns}}\right)^2 \exp\left(\frac{a^-}{C}\right) \quad \text{(from equation (61), Chapter 3)},$$

where $\overline{V_{ns}^2}$ is the mean square noise voltage in the circuit, from all sources, at the time the oscillations start to build up. $\overline{V_{ns}^2}$ is the noise measured within the *super-regenerative* noise band-width b_{ns}.

In order to prove this relationship it is necessary to go back to the fundamental equation for the particular state and insert a function, representing the noise fluctuation, in place of the signal $A\sin(\omega t)$. Because of the obvious nature of the equation (3) the proof is not included here, but a proof referring to the slope-controlled state is provided in Appendix 1.

If, in addition to the noise, there is a steady c.w. signal $A\sin(\omega t)$ applied to the circuit, then the resultant voltage at the peak of the output pulse is that due to both signal and noise. According to the basic equation of the linear mode (equation (27), Chapter 3), the voltage to which the pulse builds up on signal alone is

$$V_{\text{sig.}} = V_1 \sin[\omega_0 t + (\omega - \omega_0) t_1], \tag{4}$$

where
$$V_1 = V_s \mu_t S(f).$$

The resultant voltage from signal plus noise at the peak of the pulse is therefore

$$\begin{aligned} V &= V_{\text{sig.}} + V_{\text{noise}} \\ &= [x + V_1 \cos(\omega - \omega_0) t_1] \sin(\omega_0 t) + [x + V_1 \sin(\omega - \omega_0) t_1] \cos(\omega_0 t), \end{aligned} \tag{5}$$

where x may have any value from minus to plus infinity with probability distribution of equation (2). The ratio of the signal power to the noise power at the end of the build-up period is

$$V_1^2/2\sigma^2.$$

Equations (4) and (5) define the voltage distribution due to signal and noise at the input to the detector. We are now in a position to discuss the effect of detection on the radio-frequency pulses.

4·3. Detection of high-frequency pulses from super-regenerative circuit

The action of the detector is to suppress the high-frequency f_0 and to give an output proportional to some function of the amplitude of the input pulse. The process of detection changes the probability distribution of the pulse amplitudes. We will derive an expression for the new distribution of amplitudes after detection.

From equation (5) the amplitude of a pulse before detection is

$$A_i = \sqrt{[\{x+V_1\cos(\omega-\omega_0)t_1\}^2+\{x+V_1\sin(\omega-\omega_0)t_1\}^2]}. \qquad (6)$$

To simplify the writing, let us put

$$a = V_1\cos(\omega-\omega_0)t_1, \quad b = V_1\sin(\omega-\omega_0)t_1. \qquad (7)$$

But the probability that A lies in the range A to $A+dA$ is the product of the probability that $x+a$ lies in the range y to $y+dy$ and $x+b$ in the range z to $z+dz$ integrated over all possible values of y and z that are consistent with equation (6), i.e. with

$$A_i^2 = y^2+z^2. \qquad (8)$$

Furthermore the probability that $x+a$ lies in the range y to $y+dy$ is, from equation (2),

$$p(y)\,dy = \frac{1}{\sigma\sqrt{(2\pi)}}\exp\left[-\frac{(y-a)^2}{2\sigma^2}\right]dy,$$

and the probability that $x+b$ lies in the range z to $z+dz$ is

$$p(z)\,dz = \frac{1}{\sigma\sqrt{(2\pi)}}\exp\left[-\frac{(z-b)^2}{2\sigma^2}\right]dz.$$

Then the probability that

$$A_i^2 < y^2+z^2 < (A_i+dA_i)^2$$

is

$$p(A_i)\,dA_i$$

$$= \frac{1}{2\pi\sigma^2}\int_{-A_i}^{+A_i} dy\int_{A_i}^{A_i+dA_i} dz\exp\left[-\frac{1}{2\sigma^2}\{(y-a)^2+(z-b)^2\}\right]$$

$$= \frac{A_i dA_i}{2\pi\sigma^2}\exp\left[-\frac{(A_i^2+V_1^2)}{2\sigma^2}\right]\int_{-A_i}^{+A_i}\frac{\exp\left\{\frac{1}{\sigma^2}[ay+b(A_i^2+y^2)]\right\}dy}{(A_i^2+y^2)}$$

$$= \frac{A_i dA_i}{2\pi\sigma^2}\exp\left[-\frac{(A_i^2+V_1^2)}{2\sigma^2}\right]\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\exp\left[\frac{A_i}{\sigma^2}(a\cos\theta+b\sin\theta)\right]d\theta$$

$$= \exp\left[-\frac{(A_i^2+V_1^2)}{2\sigma^2}\right] I_0\left(\frac{V_1 A_i}{\sigma^2}\right)\frac{A_i dA_i}{\sigma^2}, \qquad (9)$$

where I_0 is the modified Bessel function of zero order.

The probability distribution for the amplitude of the pulses at output from a detector whose law is

$$A = F(A_i) \qquad (10)$$

is obtained from equation (9) by changing the variable from A_i to A. This is most easily accomplished by inverting the detector law, equation (10), so that

$$A_i = \Phi_1(A). \tag{11}$$

Then

$$p(A)\,dA = \exp\left[-\frac{(\Phi_1^2+V_1^2)}{2\sigma^2}\right] I_0\left(\frac{V_1\Phi_1}{\sigma^2}\right)\left(\frac{\Phi_1}{\sigma^2}\right)\frac{d\Phi_1}{dA}\,dA. \tag{12}$$

We will now interpret equation (12) for a detector having (*a*) a linear law and (*b*) a square law.

4·3·1. *Linear-law detector*

In this case (putting scale-factor unity for simplicity)

$$\Phi_1(A) = A, \tag{13}$$

and so
$$p(A)\,dA = \exp\left[-\frac{(A^2+V_1^2)}{2\sigma^2}\right] I_0\left(\frac{V_1 A}{\sigma^2}\right)\left(\frac{A}{\sigma^2}\right)dA. \tag{14}$$

The ratio $V_1^2/2\sigma^2$ is the signal-to-noise power ratio at the input to the detector.

Let
$$r = V_1/\sigma. \tag{15}$$

Then the probability density $p(A)$, which is the relative frequency with which a pulse of amplitude between A and $A+dA$ occurs, is dependent upon r. It is plotted in fig. 4·1, in which the variable is A/σ for a range of values of r.

The mean value of the amplitude is

$$\begin{aligned}\bar{A} &= \int_0^\infty A.p(A)\,dA \\ &= \sigma\sqrt{(\tfrac{1}{2}\pi)}\exp(-\tfrac{1}{4}r^2)\left[(1+\tfrac{1}{2}r^2)\,I_0(\tfrac{1}{4}r^2)+(\tfrac{1}{2}r^2)\,I_1(\tfrac{1}{4}r^2)\right],\end{aligned} \tag{16}$$

and the mean-square deviation of the amplitude is

$$\overline{(A-\bar{A})^2} = 2\sigma^2(1+\tfrac{1}{2}r^2)-(\bar{A})^2. \tag{17}$$

The deviation of the amplitude from its mean value is random and constitutes noise in the detector output. As the ratio r increases, the mean value of the output becomes proportional to the signal amplitude at input. The deviation from linearity indicates distortion for an amplitude-modulated input signal. The degree of distortion is, however, slight for $r > 1$. Since the output pulses vary in amplitude about a mean value which is not zero even in the absence of signal, it is scarcely correct to call the ratio of the mean

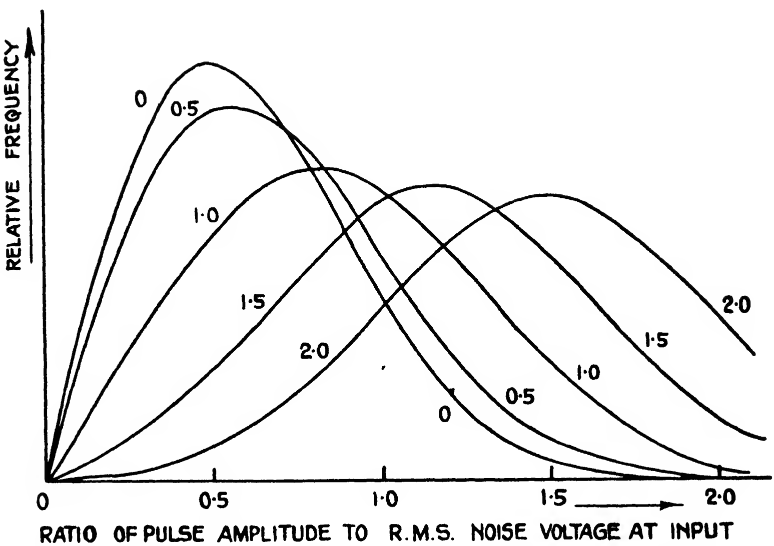


Fig. 4·1. Relative frequency with which a pulse of amplitude $A+dA$ occurs after a linear-law detector. (Parameter on curves is r.)

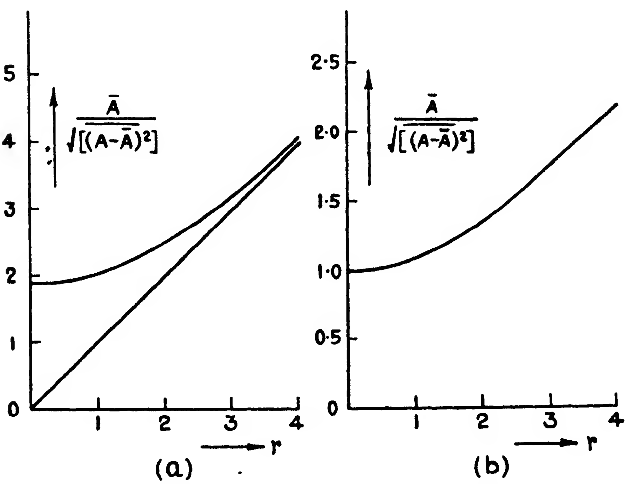


Fig. 4·2. Growth of apparent signal out of noise. (*a*) Linear-law detector. (*b*) Square-law detector.

amplitude to the r.m.s. deviation of the amplitude the signal-to-noise ratio. This difficulty arises whenever signal distortion is present. In spite of this we can exhibit both effects by plotting the apparent signal-to-noise voltage ratio as a function of the signal-to-noise voltage ratio at input as in fig. 4·2(*a*).

The detector output is a succession of d.c. pulses all of the same shape and spaced by equal intervals of time, but with unequal amplitudes. The mean value of the amplitudes taken over a large number of pulses is $\bar{A}$, equation (16), and the mean-square deviation of the amplitudes is $\overline{(A-\bar{A})^2}$, equation (17).

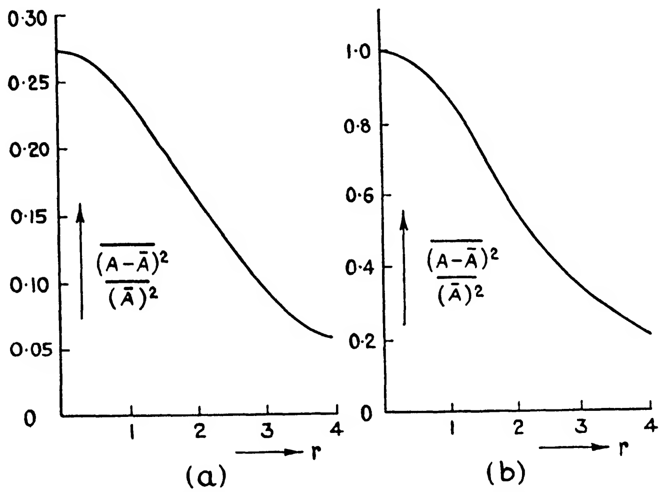


Fig. 4·3. Distribution of energy between line spectrum and continuous spectrum. (*a*) Linear-law detector. (*b*) Square-law detector.

The ratio of the total energy in the noise to that in the apparent signal is shown later (§4·4·6) to be

$$\{\overline{(A-\bar{A})^2}\}/(\bar{A})^2. \tag{18}$$

It is therefore a function of the maximum signal to r.m.s. noise-voltage ratio and is plotted in fig. 4·3(*a*). The value of the expression given in (18) falls off very like an error curve from a maximum value of 0·278 when r is zero and vanishes when r becomes large.

4·3·2. *Square-law detector*

In this case (putting scale factor unity)

$$\Phi_1(A) = \sqrt{A}, \tag{19}$$

and so $$p(A)\,dA = \exp\left[-\frac{(A+V_1^2)}{2\sigma^2}\right] I_0\left(\frac{V_1\sqrt{A}}{\sigma^2}\right)\frac{dA}{2\sigma^2}. \tag{20}$$

The probability density $p(A)$ shows a marked difference from the corresponding distribution for the linear detector.

The mean value of the amplitude is

$$\begin{aligned}\bar{A} &= \int_0^\infty A\,.\,p(A)\,dA \\ &= 2\sigma^2(1+V_1^2/2\sigma^2) \\ &= 2\sigma^2(1+\tfrac{1}{2}r^2),\end{aligned} \tag{21}$$

and the mean-square deviation of the amplitude is

$$\begin{aligned}\overline{(A-\bar{A})^2} &= 4\sigma^4(1+V_1^2/\sigma^2) \\ &= 4\sigma^4(1+r^2).\end{aligned} \tag{22}$$

In order to allow direct comparison of amounts of distortion with the linear and square-law detectors, fig. 4·2 (*b*) has been drawn. It gives the apparent signal-to-noise voltage ratio at output as a function of the signal-to-noise voltage ratio at input.

The ratio of the energy in the noise to that in the signal is

$$\frac{\overline{(A-\bar{A})^2}}{(\bar{A})^2} = \frac{1+r^2}{(1+\frac{1}{2}r^2)^2}. \tag{23}$$

The distribution of energy between the noise and the signal is shown in fig. 4·3 (*b*). The curve resembles the corresponding curve for the linear detector, but the vertical scale is different by a factor of almost four. A greater proportion of energy resides in the noise from a square-law detector than from a linear detector. The reason is evident from a comparison of the corresponding probability distributions of amplitude. The most probable amplitude for the linear detector is greater than zero, whereas it is more nearly zero for the square-law detector. The law of a practical detector can be approximately represented by a square law for small signals and a linear law for large signals. The distribution of energy between signal and noise will therefore lie somewhere between the limiting cases of figs. 4·3 (*a*) and (*b*).

The physical picture of the transition from noise to a large steady c.w. signal is made clearer by an examination of the frequency spectrum of the detector output, which is made in the next section.

4·4. Frequency spectrum of the rectified voltage at the detector output

Let us assume first of all that the circuit is free from all random disturbances such as noise and that the incoming signal is a pure c.w. oscillation. Then, if the output from the super-regenerative oscillator is fed directly into a perfect linear detector, the output signal is a succession of identical d.c. pulses, each having a shape determined by the particular type of quench or conductance waveform used. We shall determine first of all the frequency spectrum of a single pulse and then of a train of pulses of equal amplitude. We shall then be able to discuss the spectrum due to noise when the pulses have a random fluctuation in amplitude.

4·4·1. *The spectrum of a single rectified pulse: slope-controlled state*

Equation (41) in Chapter 3 shows that the envelope of the oscillations in a single pulse from a slope-controlled receiver is a Gaussian-error curve, described by

$$A(t) = \exp\left[-z^2/(\tfrac{1}{2}\tau)^2\right], \tag{24}$$

where
$$\tau = 4\sqrt{\left[\frac{C}{|G'(t_2)|}\right]}.$$

This equation also describes the voltage pulse from a perfect linear detector.

A Gaussian error-curve has the unique property that its spectrum or Fourier transform has the same shape. The transform of the error curve of equation (24) is

$$\begin{aligned} S(f) &= \sqrt{\pi}\,(\tfrac{1}{2}\tau)\exp\left[\tfrac{1}{4}(-\pi^2\tau^2 f^2)\right] \\ &= 2\sqrt{\left[\frac{\pi C}{|G'(t_2)|}\right]}\exp\left[-\frac{4\pi^2 f^2 C}{|G'(t_2)|}\right]. \end{aligned} \tag{25}$$

Thus both the frequency response and the spectrum of a pulse at the output of the slope-controlled receiver have the shape of a Gaussian-error curve. It is important to note that the frequency response depends upon the slope $G'(t_1)$ at the *beginning* of the build-up period, whilst the spectrum depends upon the slope $G'(t_2)$ at the *end*.

Equation (25) describes the spectrum of a pulse at the output from a linear detector. The band-width of the spectrum at $1/e$ of the peak is

$$B = \frac{1}{2\pi}\sqrt{\left[\frac{|G'(t_2)|}{C}\right]}. \tag{26}$$

The spectrum of a single pulse is drawn in fig. 4·4 and the meaning of B clearly shown.

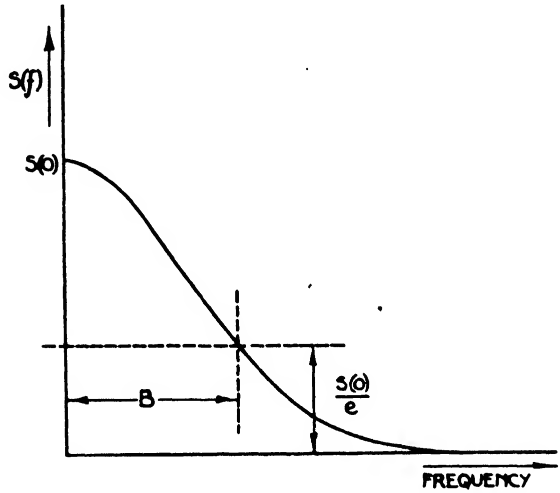


Fig. 4·4. Frequency spectrum of a single pulse (after detection).

4·4·2. *The spectrum of a succession of pulses: slope-controlled state*

In practice, when the incoming signal is a large steady c.w. signal, the detector output consists of a succession of pulses recurring at the quench frequency. The spectrum of the rectified pulses is then an array of discrete lines spaced by the quench frequency, f_q. The amplitude A_n of the line of frequency nf_q is obtained from equation (25) by replacing f by nf_q and multiplying the result by f_q. Thus, neglecting the phase factor,

$$A_n = 2f_q \sqrt{\left[\frac{\pi C}{|G'(t_2)|}\right]} \exp\left[-\frac{4\pi^2 n^2 f_q^2 C}{|G'(t_2)|}\right]. \qquad (27)$$

The spectrum of a succession of similar pulses, representing the detector output due to a c.w. signal, is shown in fig. 4·5. The envelope is the spectrum of a single pulse times a constant. Comparison of figs. 4·4 and 4·5 should make this clear.

4·4·3. *The spectrum of a linearly-rectified pulse: step-controlled state*

The pulse of oscillations in the step-controlled state rises and falls exponentially. The pulse envelope is described by equations (80) and (81), Chapter 3. The build-up is given by

$$A(z) = A_0 \exp(G_1 z/2C) = A_0 \exp(\alpha z),$$

where $\alpha = G_1/2C$ and $z < 0$, and the decay by

$$A(z) = A_0 \exp(-G_0 z/2C) = A_0 \exp(-\beta z), \tag{28}$$

where $\beta = G_0/2C$ and $z > 0$. A_0 is the amplitude at the peak of the pulse and z is time measured from that peak. The pulse is thus normally asymmetric, unless $G_0 = G_1$.

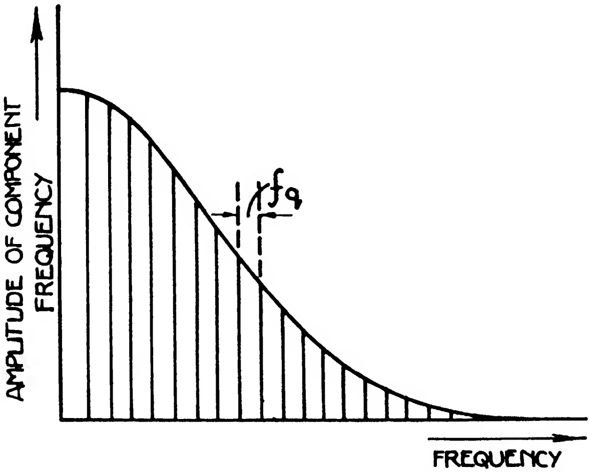


Fig. 4·5. Line spectrum of a succession of pulses due to a c.w. signal.

The spectrum of a single pulse of this description is

$$S(\omega) = S_1(\omega) + \bar{S}_1(-\omega),$$

where

$$S_1(\omega) = \int_{-\infty}^{\infty} e^{i\omega z} A(z)\, dz = \int_0^{\infty} e^{-i\omega z - \beta z}\, dz + \int_0^{\infty} e^{i\omega z - \alpha z}\, dz$$

$$= \frac{1}{\beta + i\omega} + \frac{1}{\alpha - i\omega} = \frac{\alpha + \beta}{(\alpha\beta + \omega^2) + i\omega(\alpha - \beta)}, \tag{29}$$

and $\bar{S}_1$ denotes the complex conjugate.

The real spectrum is thus

$$S(f) = \frac{\alpha + \beta}{(\alpha\beta + \omega^2) + i\omega(\alpha - \beta)} + \frac{\alpha + \beta}{(\alpha\beta + \omega^2) - i\omega(\alpha - \beta)}$$

$$= \frac{2(\alpha + \beta)(\alpha\beta + \omega^2)}{(\omega^2 + \alpha^2)(\omega^2 + \beta^2)}, \tag{30}$$

where $\omega = 2\pi f$.

This is the spectrum of a single pulse.

The band-width at half the maximum is given approximately by

$$\beta \doteqdot \frac{1}{2\pi}(\alpha\beta)^{\frac{1}{2}} \doteqdot \frac{\sqrt{(G_0 G_1)}}{4\pi C} \quad \left(\tfrac{1}{4} < \frac{G_0}{G_1} < 4\right). \tag{31}$$

4·4·4. *The spectrum of a succession of pulses: step-controlled state*

The spectrum of a succession of these pulses, all of equal amplitude, is again a line spectrum, bounded by a curve proportional to that of equation (30) above. The amplitude of the line of frequency nf_q is again obtained by replacing ω in equation (30) by $2\pi nf_q$ and multiplying by $2\pi f_q$. Thus

$$A_n = \frac{4\pi f_q(\alpha+\beta)(\alpha\beta+4\pi^2 n^2 f_q^2)}{(4\pi^2 n^2 f_q^2+\alpha^2)(4\pi^2 n^2 f_q^2+\beta^2)}$$

$$\doteqdot \frac{4\pi f_q(\alpha+\beta)}{\alpha\beta+4\pi^2 n^2 f_q^2} \quad \text{for } \alpha \doteqdot \beta. \tag{32}$$

4·4·5. *The effect of amplitude modulation upon the output spectrum*

When the incoming c.w. signal is amplitude-modulated, each pulse has an amplitude determined by the sample observed at the start of the corresponding cycle of quench. If the samples are taken sufficiently often, the envelope of the pulse peaks is a true replica of the modulation voltage.

The effect of the modulation upon the output spectrum is shown in fig. 4·6. Each line of the spectrum now has side-bands corresponding to the component frequencies of the modulation. To avoid interference between side-bands from adjacent lines, the highest modulation frequency must be less than half the quench frequency. That is another way of saying that the modulated signal must be sampled at least twice during each cycle of modulation, which is a minimum requirement. In practice the problems of demodulation are such as to require the quench frequency to be several times higher than the highest modulation component. This is discussed further in Chapter 5.

4·4·6. *The output spectrum due to signal and noise*

The spectra described in the preceding sections refer to the output of a receiver in which the signal is so large that noise may be neglected. We shall now consider the output spectrum when the signal is small, or absent altogether. Under these circumstances the detector output is a succession of d.c. pulses, all having the same shape and spaced by equal intervals of time, but with unequal amplitudes as shown in fig. 4·7 (*a*). The mean value of the amplitudes taken over a large number of pulses is $\bar{A}$. The mean square deviation of the amplitudes is $\overline{(A-\bar{A})^2}$. The expressions describing $\bar{A}$ and $\overline{(A-\bar{A})^2}$ are given in §§4·3·1 and 4·3·2 for the linear- and square-law detectors respectively.

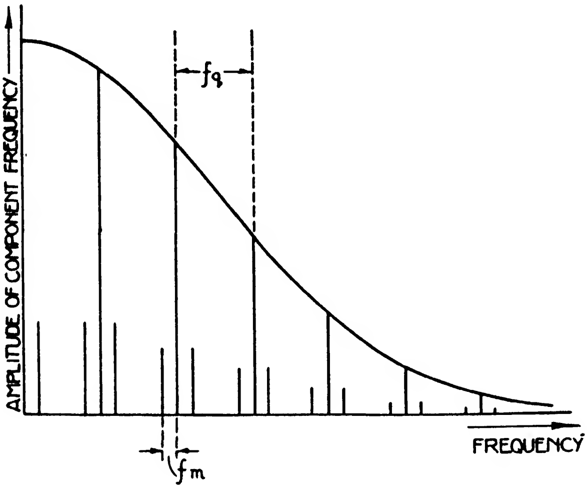


Fig. 4·6. Line spectrum of a succession of rectified pulses due to an amplitude-modulated c.w. signal.

The spectrum of a succession of pulses of unequal amplitudes has been analysed by G. G. Macfarlane.* The spectrum consists of two parts.

(*a*) A line spectrum with separation of lines in frequency by the quench frequency. This may be regarded as the signal spectrum, even though it still exists at zero signal due to the appreciable mean amplitude of the noise pulses after detection.

The power in the line of frequency mf_q is

$$2\pi f_q \frac{(\bar{A})^2}{T_q} \,|\, S(2m\pi f_q) \,|^2,$$

where $S(2m\pi f_q)$ is the value of the spectral function $S(\omega)$ at the frequency $\omega = 2m\pi f_q$. The formulae for $S(\omega)$ under slope-control and step-control conditions are given in §§4·4·1 and 4·4·3 respectively.

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The band-width at half the maximum is given approximately by

$$\beta \doteqdot \frac{1}{2\pi}(\alpha\beta)^{\frac{1}{2}} \doteqdot \frac{\sqrt{(G_0 G_1)}}{4\pi C} \quad \left(\tfrac{1}{4} < \frac{G_0}{G_1} < 4\right). \tag{31}$$

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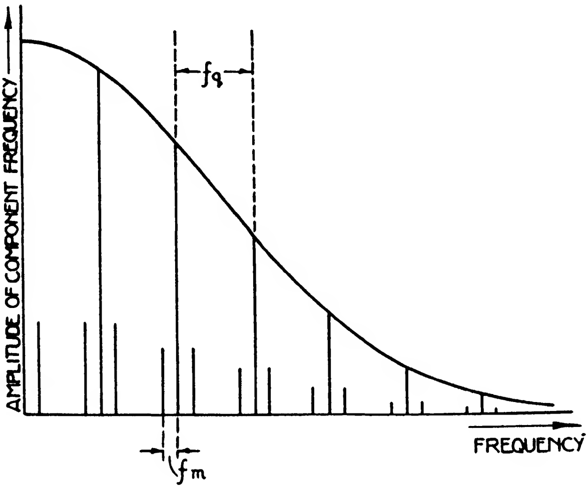

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* G. G. Macfarlane, 'On the energy-spectrum of an almost periodic succession of pulses', *Proc. Inst. Radio Engrs* (in the Press).

If the power in the lines is plotted against frequency, the power spectrum of fig. 4·5 is obtained. The envelope of the lines is proportional to the energy-density spectrum of a single pulse.

(*b*) A continuous-energy distribution of density

$$\overline{(A-\bar{A})^2}\frac{|S(\omega)|^2}{T_q}$$

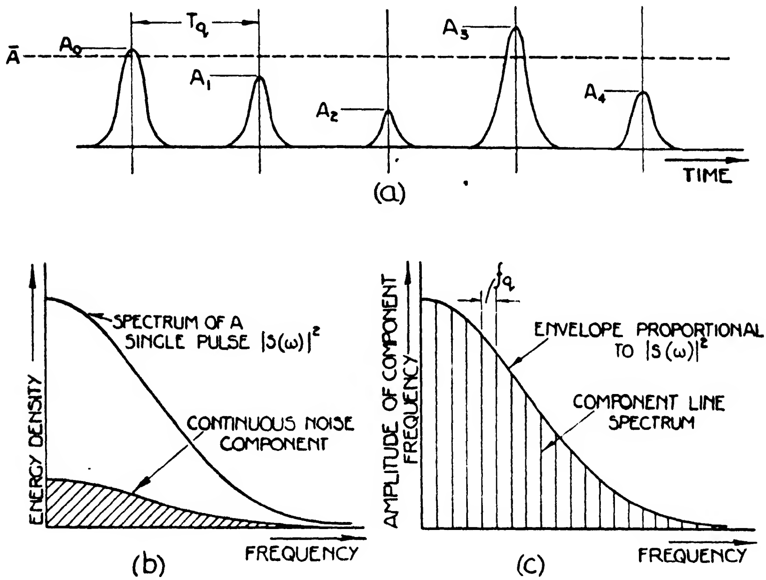


Fig. 4·7. Spectrum of a succession of rectified pulses due to noise. (*a*) Rectified output pulses. (*b*) Continuous energy spectrum (corresponding to deviation from mean). (*c*) Line spectrum (corresponding to mean amplitude).

with intensity proportional to the energy-density spectrum of a single pulse. This is the noise spectrum, because it is due to the fluctuation of the pulse amplitudes about the mean value. The formation of the spectrum due to signal and noise is shown in fig. 4·7.

We can now see what happens to the output spectrum as the signal grows out of noise. For zero signal in a linear detector, for example, the ratio of the total energy in the noise or continuous spectrum to that in the signal or line spectrum is

$$\frac{\overline{(A-\bar{A})^2}}{(\bar{A})^2},$$

as quoted in §4·3·1. As the signal is increased from zero, the proportion of the total energy in the noise spectrum is diminished. Finally, when the signal is much greater than noise (say > 10 × noise) a negligible proportion of the output energy goes into the noise spectrum and the output pulses are, to all intents and purposes, regular in amplitude. Fig. 4·3 shows the partition of energy between the signal and noise spectra for values of the signal to noise ratio up to 4.

4·5. Noise in practical receivers

The output from a linear detector following a super-regenerative oscillator consists of a series of pulses of different amplitudes (fig. 4·7). The ratio of the r.m.s. deviation to the mean amplitude of the pulses is the apparent signal to noise ratio at the detector output.

We have seen that the signal energy appears in discrete lines at quench frequency and its harmonics (including zero frequency), and that signal modulation appears as side-bands on these lines. The noise energy is distributed in proportion throughout the whole spectrum (fig. 4·7).

The highest modulation side-band that can be distinguished has a frequency equal to half the quench frequency, f_q. It is therefore necessary to restrict the post-detector band-width, by means of a suitable filter, so that only the side-bands of a single line are accepted. It is usual to use a low-pass filter, in which case the band-width must be less than $\frac{1}{2}f_q$ in order to avoid interference from the lower side-band of the line at quench frequency.

It is evident from the nature of the spectrum that restricting the post-detector band-width to precisely $\frac{1}{2}f_q$ by means of a low-pass filter reduces the signal and noise in the same ratio. That is to say, the signal to noise ratio within a band-width $\frac{1}{2}f_q$ after the detector is exactly the same as that measured over the entire spectrum. Noise collected at any frequency within the super-regenerative or radio-frequency band-width is effective in producing output in this narrow pass-band.

If the same reduction of post-detector band-width were applied to a conventional radio-frequency amplifier, having a similar frequency response and equivalent noise input, it would result in a reduction of noise relative to signal at the output. Ideally the reduction would be in the ratio of twice the post-detector band-width to the radio-frequency energy band-width, but it is not so great in practice because of intermodulation in a practical detector. At worst, therefore, the signal-to-noise ratio at the output of a super-regenerative receiver is less than that at the output of a tuned radio-frequency amplifier with similar characteristics in the ratio f_q/b_{ns}. The maximum numerical value of this ratio is about $\frac{1}{8}$ (−16 db.). The advantage of the average super-heterodyne receiver is not as great as this would suggest. Intermodulation in the diode detector probably reduces the figure to more nearly −6 db. There are, unfortunately, no reliable comparative measurements of the two types of receiver to confirm this figure. The super-regenerative receiver can never be *better* on this count because the nature of the output corresponds to complete intermodulation. That is the fundamental reason for the high noise-factor associated with super-regenerative receivers.

Reduction of the post-detector band-width below $\frac{1}{2}f_q$ reduces the noise in proportion but also reduces the frequency range available for modulation side-bands. The noise in a given post-detector band-width is constant and independent of quench frequency. There is consequently no point in increasing the quench frequency in the hope of improving the signal-to-noise ratio at the output. This is clear from an examination of the output spectrum. Changing the quench frequency alters the frequency scale of the spectrum without affecting the vertical scale. The relationship between signal and noise within a given post-detector band-width is thus unchanged.

The noise in any receiver arises from thermal fluctuations in the input circuits and shot-noise fluctuations of anode current in the first valve. Let us consider a super-regenerative receiver consisting of a tuned-anode oscillator with separate quench. The signal is injected into the tuned-anode circuit where it is compared directly with both thermal and shot-noise contributions. It undergoes no amplification by the valve. In a radio-frequency amplifier of conventional design, however, the signal is amplified before it is compared with the valve noise. At first sight, therefore, it appears that the shot noise is more important, in a super-regenerative receiver, by a factor of the same order as the amplification to be expected from a tuned radio-frequency amplifier. This factor is about ten, or less, at frequencies for which the super-regenerative receiver is suitable.

A super-regenerative oscillator is, however, only sensitive for a short period about the start of oscillation in each quench cycle. Thus the shot noise sampled during this period corresponds to the current required to start oscillation, which is considerably less than the normal operating current of the same valve used as a tuned radio-frequency amplifier. It is obviously an advantage to cause oscillation to occur at as low a current as possible. This can be done by making the oscillator feed-back coupling much closer than is usual for a valve oscillator.

Under these circumstances the shot noise need be no greater in relation to the signal than it is in a tuned radio-frequency amplifier. In any case it is good practice to precede the super-regenerative stage by a grounded-grid or other radio-frequency amplifier to prevent radiation of super-regenerative oscillations. Thus there need be no difference in the noise generated in the input circuits of superheterodyne and super-regenerative receivers. The only difference in noise properties between the two types of receiver is due to the nature of the output, described in detail above.

It must also be remembered when comparing the properties of super-regenerative and superheterodyne receivers that the frequency-response curve of the latter, in the slope-controlled state, has a more desirable shape. The ratio of energy band-width to the width of the amplitude response at, say, -6 db. is smaller for the super-regenerative receiver, thus providing less noise for a given nominal band-width.

There might appear to be some advantage in connecting the tuned circuit to the grid of the super-regenerative oscillator. This would result in a reduction of shot noise compared with signal in the ratio of the mutual inductance between the grid and anode coils to the inductance of the grid coil. Under the conditions of close coupling necessary to ensure low noise (see above) this ratio is of the order unity and there is little difference between the various types of oscillator.

Chapter 5

SINUSOIDAL AND OTHER PARTICULAR KINDS OF QUENCH

5·1. Introduction

The general theory of the linear super-regenerative receiver, developed in the preceding chapters, assumes very little about the kind of quench voltage applied to the super-regenerative oscillator. The resulting formulae are sufficient to describe the operation of any receiver operating within the restrictions imposed for either slope-control or step-control. They are also useful in estimating the properties of a receiver operating in the transitional state between slope- and step-control. It is possible, however, to derive more convenient formulae relating to certain particular kinds of quench. Of these, sinusoidal quench has the greatest practical importance because of the ease with which it can be generated and applied. It is the form of external quench most often used in the design of super-regenerative receivers.

It is shown below that a super-regenerative receiver using sinusoidal quench usually operates in or near the slope-controlled state. We shall use the formulae for the slope-controlled state throughout the analysis relating to sinusoidal quench. The extent to which this is justified is discussed in §5·4·5.

The theoretical analysis of a receiver using a specified quench-voltage variation is necessarily approximate. That is because the corresponding conductance function depends upon the relationship between the mutual conductance of the oscillator valve and the potential of the electrode to which the quench is applied. This relationship is non-linear and differs from one valve to another. It is possible to approximate to a portion of the valve characteristic by a simple expression, based on the $\frac{3}{2}$ power law of the ideal valve. The resulting formulae are extremely useful in showing how the gain and frequency response depend upon easily measurable circuit parameters, such as quench frequency, quench amplitude, Q-factor, etc.

The analytical method is only convenient when the quench voltage is a simple function of time and, even then, is necessarily approximate. When greater accuracy is desired or when the quench function cannot be simply expressed, graphical methods must be used.

We have chosen here to use the analytical method to provide design data for sinusoidal quench and, to a lesser extent, for rectangular quench. Used with circumspection the data are adequate for most design purposes. The graphical method of deriving the conductance function, and predicting from it the properties of a receiver, is described first as an introduction to the analytical method.

5·2. Conductance cycle due to sinusoidal quench: graphical method

In Chapter 3 formulae were derived which describe the properties of a super-regenerative receiver in terms of the circuit

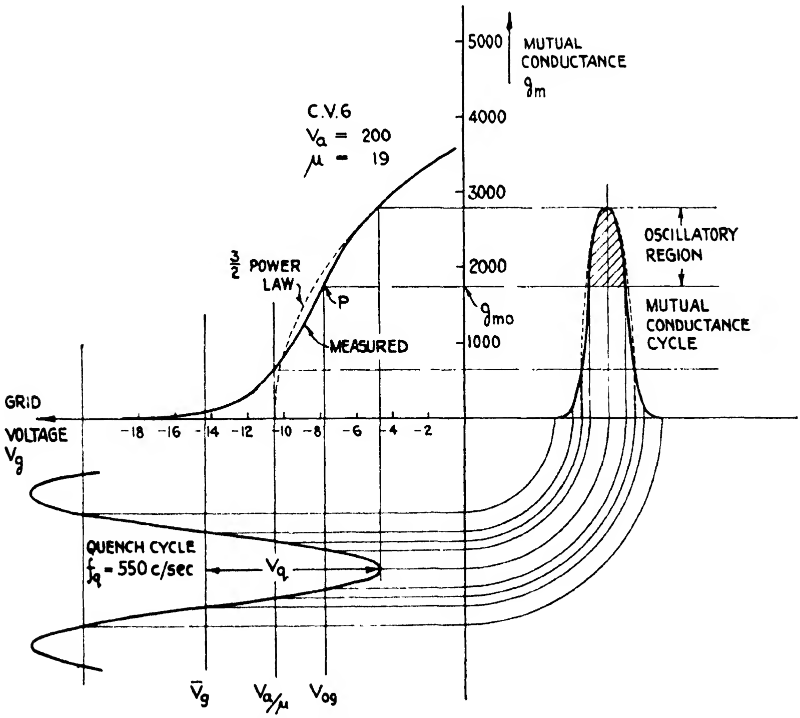


Fig. 5·1. Variation of mutual conductance with sinusoidal grid quench.

parameters and the conductance function $G(t)$ representing the shunt conductance in the equivalent circuit (fig. 3·1). We shall now examine the conductance function due to sinusoidal quench.

Fig. 5·1 shows the variation of mutual conductance during a single cycle of sinusoidal grid quench applied to a CV6 triode oscillator. The curves relate to an actual receiver, operating on a low frequency, from which the oscillograms in this and other chapters were obtained. The mutual conductance cycle has been computed by projection from the sinusoidal quench cycle below the V_g-axis, as shown. The experimental value of the voltage ($V_{0g} = -7·8$) at which oscillations will just begin, is also indicated. It provides the value of the constant k in the equation

$$G = G_0 - k g_m, \tag{1}$$

representing the total effective circuit conductance during the quench cycle. For, when oscillation is just possible, $G = 0$ and

$$k = G_0/g_{m0}, \tag{2}$$

where g_{m0} is the mutual conductance corresponding to the grid voltage V_{0g}. Using this value of k, the conductance cycle is plotted, according to equation (1), in fig. 5·2. We are now in a position to determine the properties of the receiver with this particular quench voltage and its resulting conductance variation.

The measured properties of the circuit and valve, in addition to those given in figs. 5·1 and 5·2, are:

Quiescent circuit

Band-width b	= 10·6 kc./sec. (at −3 db.)
Capacity C	= 873 $\mu\mu$F.
Natural frequency f_0	= 250 kc./sec.
Shunt conductance G_0	= $2\pi Cb$
	= 58·2 μmhos
Quench frequency f_q	= 550 c./sec.

Valve

Amplification factor μ	= 19
Anode voltage V_a	= 200 V.
Theoretical cut-off V_a/μ	= 10·5 V.

From fig. 5·2 we see that the value of the conductance slope at the beginning of the build-up period is

$$G' = 0·29\,\mu\text{mhos}/\mu\text{sec}.$$

Let us first see whether the condition for slope-control is satisfied. The duration of the regenerative period t_1 is, from the

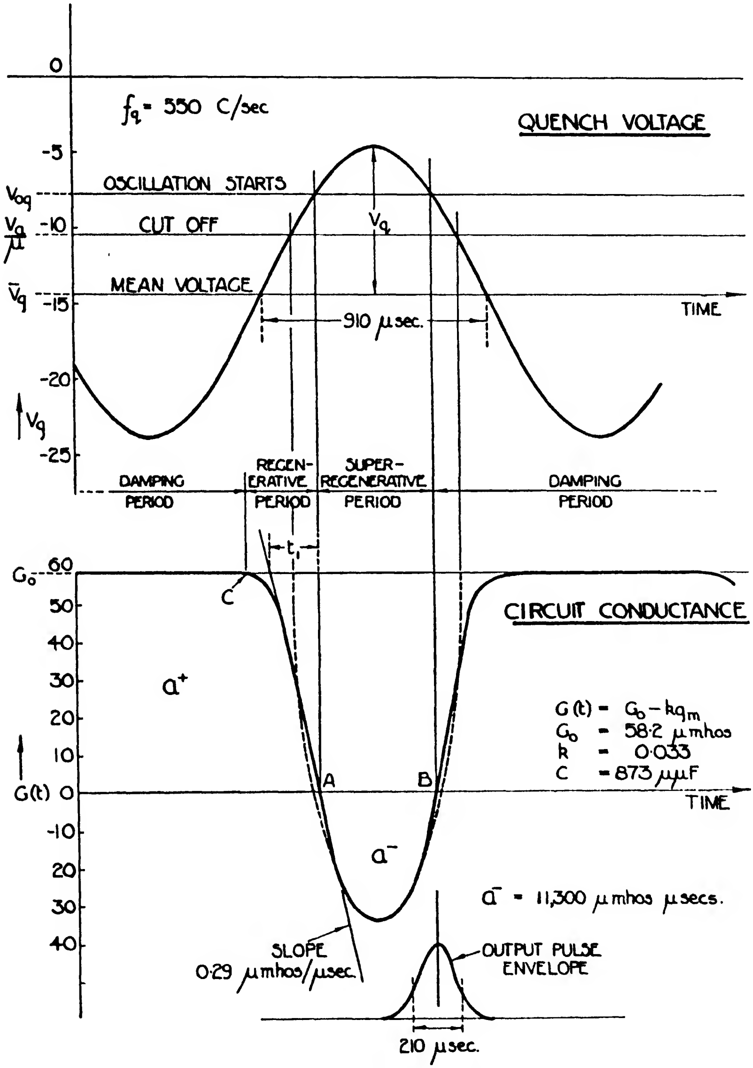


Fig. 5·2. Formation of conductance cycle: sinusoidal quench (corresponding to conditions of fig. 5·1).

diagram, about 200 μsec. This is obtained by projecting back the tangent to the curve at the point A, to cut the G_0 abscissa. The resulting value of t_1 is somewhat smaller than would be obtained by measuring from the point C at which the conductance starts to depart from the value G_0, but the former method conforms more nearly to that used in deriving the condition for slope control (§3·5·1).

The condition is

$$t_1 > 12C/G_0 \quad \text{(equation (37), Chapter 3).}$$

Now $$12C/G_0 = \frac{12 \times 873 \times 10^{-12}}{58{\cdot}2 \times 10^{-6}} \text{ sec.} = 180\,\mu\text{sec.}$$

Therefore, with $t_1 = 200\,\mu$sec., this condition is satisfied, and we may use the formulae for slope-control with a fair amount of confidence.

The band-width and slope-gain factor are determined by the conductance slope G' at the beginning of the super-regenerative period. The formula for the band-width is

$$b_s = \frac{1}{\pi}\sqrt{\left(\frac{|G'|}{C}\right)} \quad \text{(at } -1 \text{ neper)} \quad \text{(equation (50), Chapter 3).} \quad (3)$$

The slope of the conductance curve when $G = 0$ is 0·29 μmhos/μsec., obtained graphically from fig. 5·2. Using the value of C given, the band-width is

$$b_s = \frac{1}{\pi}\sqrt{\left(\frac{0{\cdot}29}{873 \times 10^{-12}}\right)} = 5{\cdot}8 \text{ kc./sec.}$$

The slope-gain factor is

$$\mu_0 = 2\sqrt{\pi}\,(b/b_s) \quad \text{(equation (3)),}$$

or $$N_0 = \log_e[2\sqrt{\pi}\,(b/b_s)] \text{ nepers} = 1{\cdot}87 \text{ nepers.}$$

The area a^- under the negative portion of the conductance-time curve determines the super-regenerative gain which is

$$N_s = a^-/2C \text{ nepers} \quad \text{(equation (43), Chapter 3).} \qquad (4)$$

The area, obtained graphically, is

$$a^- = 11{,}300\,\mu\text{mho.}/\mu\text{secs.,}$$

and the corresponding value of gain

$$N_s = \frac{11{,}300}{2 \times 873} = 6{\cdot}5 \text{ nepers.}$$

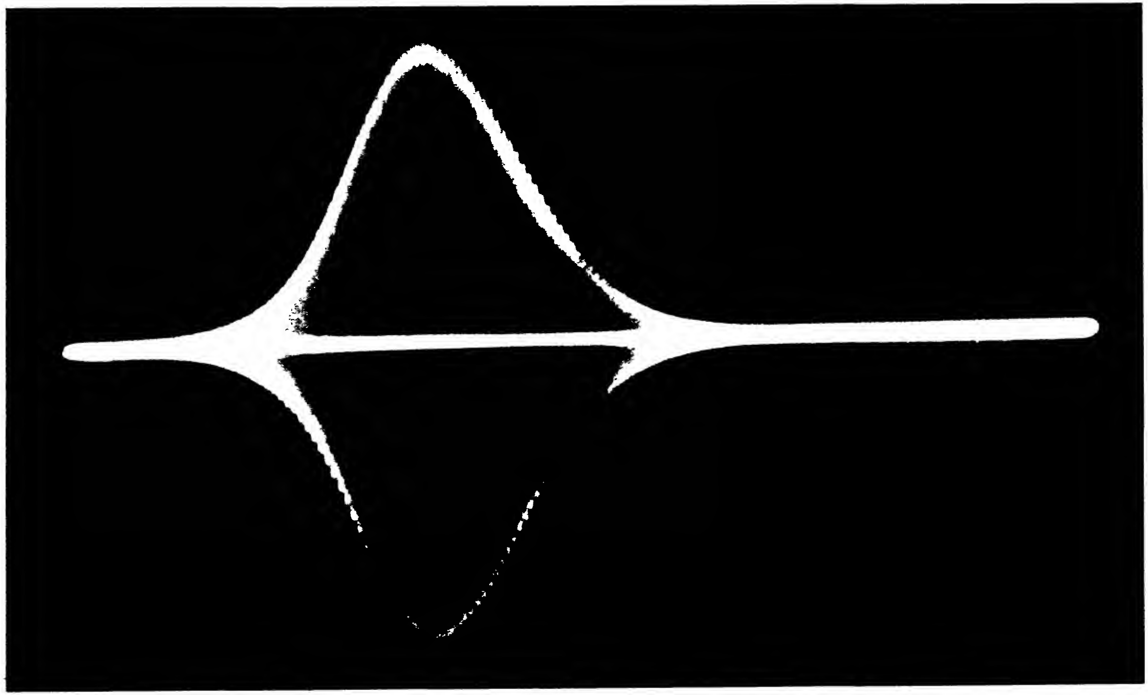


Fig. 5·3. Output pulse of oscillations: sinusoidal quench.

The total receiver gain is, consequently,

$$N_t = N_0 + N_s = 1{\cdot}87 + 6{\cdot}5 = 8{\cdot}42 \text{ nepers} = 72{\cdot}8 \text{ db.}$$

The peak of the output pulse of oscillations occurs at the end of the super-regenerative period at the instant when the conductance is again zero. Its width or duration at one neper below the peak is given by

$$\tau = 4\sqrt{\left[\frac{C}{|G'|}\right]} \quad \text{(equation (42), Chapter 3),} \qquad (5)$$

where G', the conductance slope, has the same value (but opposite sign) as that at the beginning of the super-regenerative period. In this case

$$\tau = 4\sqrt{\left(\frac{873 \times 10^{-12}}{0{\cdot}29}\right)} \text{ sec.} = 220\,\mu\text{sec.}$$

The oscillation envelope has the shape of a Gaussian-error curve. This is clearly shown in fig. 5·3 which is an oscillogram of the actual pulse of oscillations. It is convincing evidence that our assumption of slope-control is justified.

Table 3 compares the properties of the receiver, calculated by the graphical method from the known values of circuit parameters, with those actually measured on the receiver. A third column gives the values of the same quantities computed by the analytical method described below.

Table 3

Property	Symbol	Measured	Calculated	
			Graphical method	Analytical method (see Appendix 3)
Slope gain	N_0	16·3 db.	16·3 db.	—
Super-regenerative gain	N_s	—	56·5 db.	57·8 db.
Total gain	$N_t = (N_0 + N_s)$	69 db.	72·8 db.	74·0 db.
Band-width	b_s	5·8 kc./sec.	5·8 kc./sec.	6·08 kc./sec.
Pulse-width	τ	—	220 μsec.	210 μsec.

5·3. Approximate theory of sinusoidal quench

The direct effect of changing the anode or grid voltage of an oscillator valve, by the application of quench, is to alter the mutual conductance g_m. We have seen in Chapter 2 that this is generally related to the effective conductance shunting the tuned circuit by the formula

$$G = G_0 - kg_m, \tag{6}$$

where G is the effective conductance in the equivalent circuit of the oscillator, G_0 is the value of G when the valve is cut off, and k is a constant for a given circuit arrangement.

Now, in an ideal triode the anode current i_a is related to the anode voltage V_a by the $\frac{3}{2}$ power law

$$i_a = K\left(V_g + \frac{V_a}{\mu}\right)^{\frac{3}{2}}. \tag{7}$$

Current ceases in the valve when $V_g = -V_a/\mu$. This is known as the grid voltage for cut-off. The mutual conductance is

$$g_m = \left(\frac{\partial i_a}{\partial V_g}\right)_{V_a=\text{const.}} = \tfrac{3}{2}K\sqrt{\left(V_g + \frac{V_a}{\mu}\right)}. \tag{8}$$

Now, if the voltage producing the quenching action is sinusoidal in form we may put, for anode quench,

$$V_a = \bar{V}_a + V_q \sin(\omega_q t - \theta),$$

and for grid quench

$$V_g = \bar{V}_g + V_q \sin(\omega_q t - \theta), \tag{9}$$

where $\omega_q = 2\pi f_q$ is the quench radian frequency, $\bar{V}_a$ is the average anode voltage, $\bar{V}_g$ is the average grid voltage, and V_q is the peak voltage of quench oscillating about either $\bar{V}_a$ or $\bar{V}_g$.

From equations (6) and (8) the effective conductance G is given by

$$G = G_0 - \tfrac{3}{2}kK\sqrt{\left(V_g + \frac{V_a}{\mu}\right)} = G_0\left[1 - \sqrt{\left(\frac{V_g + V_a/\mu}{V_{0g} + V_a/\mu}\right)}\right], \tag{10}$$

where V_{0g} is the voltage at which oscillation commences (at time $t = t_1$ when $G = 0$).

Substituting from equation (9) for grid quench:

$$G(t) = G_0\left\{1 - \sqrt{\left[\frac{\bar{V}_g + (V_a/\mu) + V_q \sin(\omega_q t - \theta)}{\bar{V}_g + (V_a/\mu) + V_q \sin(\omega_q t_1 - \theta)}\right]}\right\}, \tag{11}$$

where t_1 is the time at which oscillation starts, defined by the equation

$$V_{0g} = \bar{V}_g + V_q \sin(\omega_q t_1 - \theta).$$

Rearranged, equation (11) becomes

$$G(t) = G_0\left\{1 - \sqrt{\left[\frac{\{\bar{V}_g + (V_a/\mu)\}/V_q + \sin(\omega_q t - \theta)}{\{\bar{V}_g + (V_a/\mu)\}/V_q + \sin(\omega_q t_1 - \theta)}\right]}\right\}. \tag{12}$$

But as $G = G_0$ when $t = 0$, the expression under the root sign is zero. Thus

$$\frac{\{\bar{V}_g+(V_a/\mu)\}/V_q-\sin\theta}{\{\bar{V}_g+(V_a/\mu)\}/V_q+\sin(\omega_q t_1-\theta)} = 0,$$

i.e.

$$\sin\theta = \{\bar{V}_g+(V_a/\mu)\}/V_q. \tag{13}$$

Also, by definition $V_g = V_{0g}$ when $t = t_1$, so that, from equation (9) for grid quench,

$$V_{0g} = \bar{V}_g + V_q \sin(\omega_q t_1 - \theta),$$

i.e.

$$\sin(\omega_q t_1-\theta) = \{V_{0g}-\bar{V}_g\}/V_q. \tag{14}$$

Thus the effective conductance is given by the formula

$$G = G_0\left[1-\sqrt{\left\{\frac{\sin(\omega_q t-\theta)+\sin\theta}{\sin(\omega_q t_1-\theta)+\sin\theta}\right\}}\right] \tag{15a}$$

in the interval $0 < \omega_q t < 2\theta$ during which the valve is conducting and

$$G = G_0 \tag{15b}$$

in the interval $(\pi+2\theta) < \omega_q t < 2\pi$ during which the current in the valve is cut off. Although derived for grid quench this formula also applies to anode quench but with a different interpretation of θ as below.

Table 4

Grid quench	Anode quench
$\sin\theta = \dfrac{\bar{V}_g+(V_a/\mu)}{V_q}$	$\sin\theta = \dfrac{\bar{V}_a+\mu V_g}{V_q}$
$\sin(\omega_q t_1-\theta) = \dfrac{V_{0g}-\bar{V}_g}{V_q}$	$\sin(\omega_q t_1-\theta) = \dfrac{V_{0a}-\bar{V}_a}{V_q}$

Fig. 5·4 shows the meaning of the symbols used above in relation to a cycle of (*a*) grid and (*b*) anode quench, and the formation of the corresponding cycle of effective conductance. It also shows, incidentally, that the same conductance variation may be produced by applying a suitable quench voltage either to the grid or to the anode of the super-regenerative oscillator. It will be noticed that all the parameters are directly measurable.

We have determined the value of the circuit conductance effective during a cycle of sinusoidal quench in terms of the quiescent conductance and the electrode voltages of the oscillator. We are now in a position to substitute this value in the general formulae of Chapter 3 and to obtain more practical formulae relating only to sinusoidal quench.

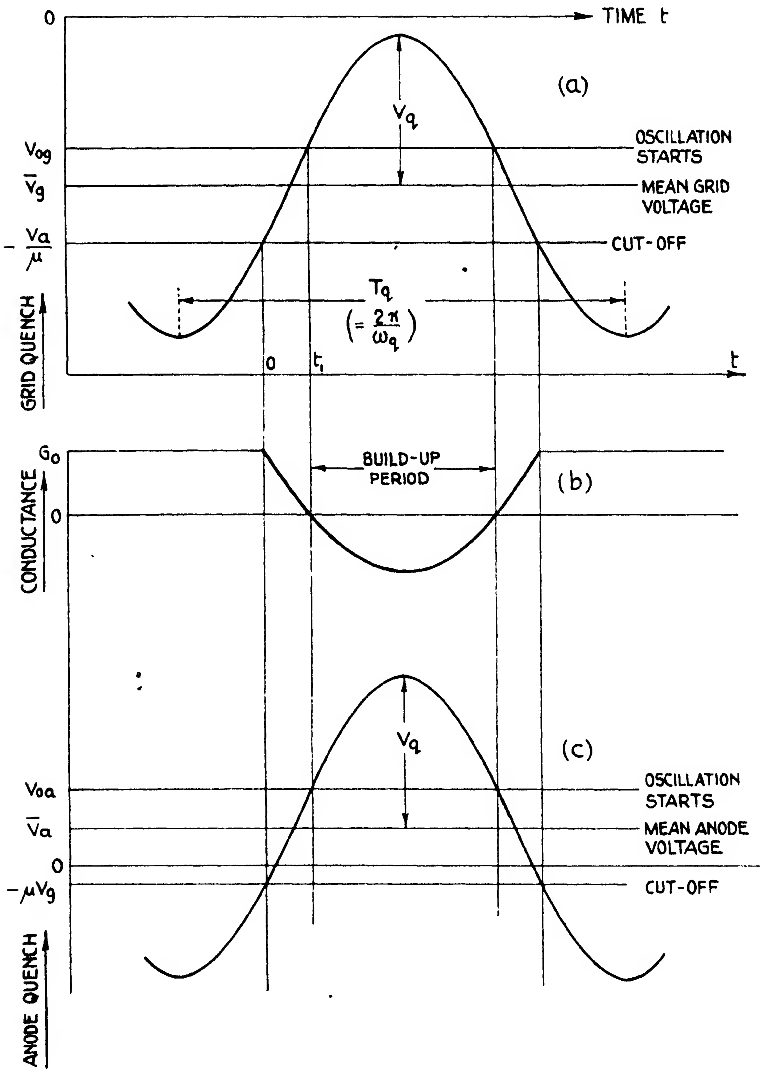


Fig. 5·4. Formation of conductance cycle (*b*) from *either* grid quench (*a*) *or* anode quench (*c*) illustrating the meaning of symbols used in the text.

5·4. Design formulae for sinusoidal quench

The properties of a super-regenerative receiver depend upon the conductance variation during the quench cycle, which is a complicated function of the voltages applied to the electrodes of the oscillator valve. We are not interested so much in predicting a precise value for the gain or band-width of the receiver, under a particular set of conditions, as in showing the trend of these properties as one or other of the parameters is varied. It is, therefore, necessary to choose the formulae carefully so that they may be applicable to a practical design problem with a minimum amount of rearrangement.

There are three voltages capable of variation:

(i) The quench voltage.

(ii) The mean voltage of the electrode to which the quench is applied (e.g. grid for grid quench).

(iii) The voltage of the other electrode (e.g. anode for grid quench).

It is rarely necessary to consider the variation of more than two of these in a design problem. For example, with grid quench it is usual to fix the anode voltage and to adjust the receiver to the desired operating condition by changing the quench voltage and the mean grid bias (i.e. (i) and (ii)). The quench voltage is nearly always one of the variables. It seems, therefore, that our formulae should be arranged in terms of two alternative sets of parameters, corresponding either to (i) and (ii) or to (i) and (iii).

Table 5

Grid quench	Anode quench
$y_g=\dfrac{V_q}{V_{0g}-\bar{V}_g}$	$y_a=\dfrac{V_q}{V_{0a}-\bar{V}_a}$
$K_g=\dfrac{(V_a/\mu)+\bar{V}_g}{V_{0g}-\bar{V}_g}$	$K_a=\dfrac{\mu V_g+\bar{V}_a}{V_{0a}-\bar{V}_a}$
$z_g=\dfrac{V_q}{(V_a/\mu)+V_{0g}}$	$z_a=\dfrac{V_q}{\mu V_g+V_{0a}}$
$P_g=\dfrac{(V_a/\mu)+\bar{V}_g}{(V_a/\mu)+V_{0g}}$	$P_a=\dfrac{\mu V_g+\bar{V}_a}{\mu V_g+V_{0a}}$
where $\sin\theta=\dfrac{K_g}{y_g}=\dfrac{P_g}{z_g}$	where $\sin\theta=\dfrac{K_a}{y_a}=\dfrac{P_a}{z_a}$
and $\sin(\omega_q t_1-\theta)=\dfrac{1}{y_g}=\dfrac{1-P_g}{z_g}$	and $\sin(\omega_q t_1-\theta)=\dfrac{1}{y_a}=\dfrac{1-P_a}{z_a}$

The voltages in (i), (ii) and (iii) appear in the formula for the conductance in the combinations shown in table 4 for $\sin\theta$ and $\sin(\omega_q t_1 - \theta)$. For this reason it is not possible to plot the conductance variation or receiver gain against one of the electrode voltages without some rearrangement. To do this we define the quantities given in table 5.

Thus we can write the formulae for sinusoidal quench in terms of either y and K (corresponding to (i) and (iii) above), or z and P (corresponding to (i) and (ii)). The utility of these particular quantities is evident when we come to derive design data for the dependence of super-regenerative gain upon the electrode voltages.

5·4·1. *Conductance slope*

The value of the conductance slope $G'(t_1)$ at time t_1 during the sensitive period of the quench cycle is an important quantity in the formulae for the slope-controlled state (table 2) which apply to a receiver using sinusoidal quench. Differentiating equation (15*a*), we get

$$G'(t) = -\frac{\omega_q G_0}{2}\sqrt{\left[\frac{\sin(\omega_q t_1 - \theta) + \sin\theta}{\sin(\omega_q t - \theta) + \sin\theta}\right]}\,\frac{\cos(\omega_q t - \theta)}{\sin(\omega_q t_1 - \theta) + \sin\theta} \tag{16}$$

and

$$G'(t_1) = -\frac{\omega_q G_0}{2}\,\frac{\cos(\omega_q t_1 - \theta)}{\sin(\omega_q t_1 - \theta) + \sin\theta}. \tag{17}$$

In terms of K and y, this becomes

$$G'(t_1) = -\pi f_q G_0 \sqrt{(y^2 - 1)}/(K+1), \tag{18a}$$

or, in terms of P and z,

$$G'(t_1) = -\pi f_q G_0 \sqrt{\{z^2 - (1-P)^2\}}, \tag{18b}$$

where y, K, z and P carry the subscripts a or g for anode or grid quench respectively.

5·4·2. *Receiver gain*

In §3·5 it is shown that the receiver gain has two parts, the slope gain μ_0 and the super-regenerative gain μ_s.

The slope gain, which is usually small, is given by

$$2\sqrt{\pi}(b/b_s) \quad \text{(table 2, Chapter 3)}, \tag{19}$$

where b is the band-width of the quiescent circuit at -3 db. and b_s is the super-regenerative band-width.

The slope gain is easily computed as the ratio of the measured (or calculated) band-widths, and it is not usually necessary to consider it separately. However, a useful alternative expression can be obtained from

$$\mu_0 = G_0\sqrt{\left[\frac{\pi}{C\,|G'(t_1)|}\right]} \tag{20}$$

or

$$N_0 = \tfrac{1}{2}\log_e\left[\frac{\pi G_0^2}{C\,|G'(t_1)|}\right] \text{nepers} \quad \text{(equations (44), (45), Chapter 3).}$$

Substituting for $G'(t_1)$ from equation (18 *a*) we get

$$N_0 = \tfrac{1}{2}\log_e(G_0/Cf_q) - \log_e\frac{\sqrt{(y^2-1)}}{K+1}\ \text{nepers,} \tag{21 a}$$

or, from equation (18 *b*),

$$N_0 = \tfrac{1}{2}\log_e(G_0/Cf_q) - \log_e\sqrt{[z^2-(1-P)^2]}\ \text{nepers.} \tag{21 b}$$

The super-regenerative gain in nepers (1 neper ≑ 8·7 db.) is, from table 2,

$$N_s = -\frac{1}{2C}\int_{t_1}^{t_2} G(x)\,dx$$
$$= a^-/2C, \tag{22}$$

where a^- is the area under the negative portion of the conductance-time curve. For sinusoidal quench

$$N_s = -\frac{G_0}{2\omega_q C}\int_{\omega_q t_1-\theta}^{\pi-\omega_q t_1+\theta}\left[1-\sqrt{\left\{\frac{\sin X+\sin\theta}{\sin(\omega_q t_1-\theta)+\sin\theta}\right\}}\right]dX\ \text{nepers.}$$

Integrating, this becomes in terms of y and K

$$H = \frac{\omega_q C}{G_0}N_s$$
$$= -[\tfrac{1}{2}\pi - \sin^{-1}(1/y)] + 2\sqrt{\left(\frac{2y}{1+K}\right)}\left\{E\left[\sqrt{\left(\frac{y+K}{2y}\right)},\phi\right]\right.$$
$$\left.-\left(\frac{y-K}{2y}\right)F\left[\sqrt{\left(\frac{y+K}{2y}\right)},\phi\right]\right\}_{K\leqslant y}, \tag{23 a}$$

where $\cos\phi = \sqrt{\left(\frac{1+K}{y+K}\right)}$ and

$$H = \frac{\omega_q C}{G_0}N_s$$
$$= -[\tfrac{1}{2}\pi - \sin^{-1}(1/y)] + 2\sqrt{\left(\frac{K+y}{1+K}\right)}E\left[\sqrt{\left(\frac{2y}{y+K}\right)},\phi_1\right]_{K\geqslant y}, \tag{23 b}$$

where $\cos 2\phi_1 = 1/y$, or, in terms of z and P,

$$H = \frac{\omega_q C}{G_0} N_s$$
$$= -\left\{\tfrac{1}{2}\pi - \sin^{-1}\left(\frac{1-P}{z}\right)\right\} + 2\sqrt{(2z)}\left\{E\left[\sqrt{\left(\frac{z+P}{2z}\right)}, \psi\right] - \left(\frac{z-P}{2z}\right)F\left[\sqrt{\left(\frac{z+P}{2z}\right)}, \psi\right]\right\}_{P\leqslant z}, \quad (23c)$$

where $\cos\psi = \sqrt{\left(\frac{1}{z+P}\right)}$ and

$$H = \frac{\omega_q C}{G_0} N_s$$
$$= -\left\{\tfrac{1}{2}\pi - \sin^{-1}\left(\frac{1-P}{z}\right)\right\} + 2\sqrt{(z+P)}E\left[\sqrt{\left(\frac{2z}{z+P}\right)}, \psi_1\right]_{P\geqslant z}, \quad (23d)$$

where $\cos 2\psi_1 = \frac{1-P}{z}$.

F and E are elliptic integrals of the first and second kinds respectively.

The factor multiplying N_s in the formulae above may be written

$$\frac{\omega_q C}{G_0} = \frac{f_q}{b}. \quad (24)$$

It is the ratio of the quench frequency to the band-width of the quiescent circuit. The right-hand side of equation (23) is a function only of the electrode voltages and amplification factor of the oscillator valve. Thus we are enabled to plot the super-regenerative gain against electrode voltages by substituting the appropriate values of y and K or z and P from table 5.

Design curves based upon equation (23) are included in the design data for sinusoidal quench which are presented in Appendix 2. The gain curves for sinusoidal quench are particularly useful in the design of systems for automatic gain-control, which are discussed in Chapter 7.

5·4·3. *Band-width*

The frequency response of a slope-controlled receiver is approximately an error curve with a band-width b_s at one neper down given by

$$b_s = \frac{1}{\pi}\sqrt{\left[\frac{|G'(t_1)|}{C}\right]} \quad \text{(see table 2).} \quad (25)$$

Substituting for $G'(t_1)$ from equation (18) we get

$$b_s = \sqrt{\left[\frac{f_q G_0}{\pi C}\frac{\sqrt{(y^2-1)}}{K+1}\right]}, \qquad (26a)$$

or

$$b_s = \sqrt{\left\{\frac{f_q G_0}{\pi C}\sqrt{[z^2-(1-P)^2]}\right\}}. \qquad (26b)$$

Design curves based upon these formulae are included in Appendix 2. They are useful in showing how the band-width is affected by varying the electrode voltages. In conjunction with the design data for receiver gain they also show the effect of variation of gain upon the frequency response.

The band-width curves are presented in another useful form, which helps us to choose the correct relationship between quench amplitude and mean voltage of the quench electrode. When applying quench to, say, the oscillator grid, there is an infinite choice of values of V_q and $\bar{V}_g$ to give the same receiver gain. The bias voltage $\bar{V}_g$ may be very large. Then V_q must be correspondingly great. If $\bar{V}_g$ is small (though numerically greater than V_{0g}) then a small quench amplitude will achieve the same value of gain. This is illustrated in fig. 5·5. In this figure two widely different values of the quench amplitude have been chosen. The appropriate adjustment of $\bar{V}_g$ has made the area a^- and, consequently, the super-regenerative gain, the same in both cases. It is evident, however, that the conductance slope at time $t = t_1$, which determines the band-width, differs quite considerably from one case to the other.

We may represent this effect in the design curves by plotting the band-width against z (representing quench voltage) as variable whilst adjusting the parameter P (representing the mean electrode voltage) for constant gain. For this purpose it is necessary to neglect the variation in slope gain which is, in any case, small compared with the super-regenerative gain. Curves calculated on this basis are also included in Appendix 3.

A similar effect is produced when the gain is held constant by adjusting the voltage of the non-quenched electrode. This case has no very great practical interest, and the corresponding curve is not included.

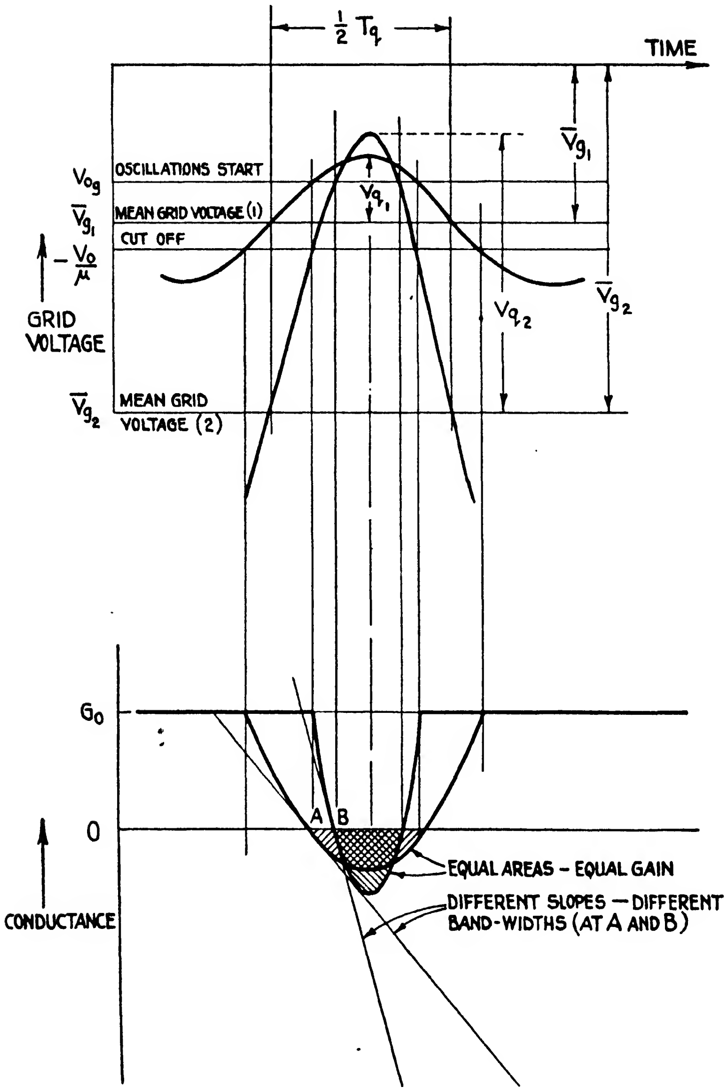


Fig. 5·5. Dependence of band-width on quench amplitude for constant gain.

5·4·4. *Output pulse of oscillations*

The envelope of the super-regenerative oscillations in a slope-controlled receiver is a Gaussian error-curve having a width τ at $1/e$ of the peak amplitude, given by

$$\tau = 4\sqrt{\left[\frac{C}{|G'(t_2)|}\right]} \quad \text{(equation (42), Chapter 3).} \qquad (27)$$

Because of the symmetrical nature of the sinusoidal quench cycle

$$G'(t_2) = G'(t_1). \qquad (28)$$

Thus, making the substitution for $G'(t_1)$ from equation (18),

$$\tau = 4\sqrt{\left[\frac{2C(K+1)}{\omega_q G_0 \sqrt{(y^2-1)}}\right]}$$

$$= 4\sqrt{\left[\frac{2C}{\omega_q G_0}\frac{1}{\sqrt{\{z^2-(1-P)^2\}}}\right]} \qquad (29)$$

for sinusoidal quench.

Similarly, the width of the output spectrum of a linearly rectified pulse at $1/e$ down (equation (26), Chapter 4) is

$$B = \frac{1}{2\pi}\sqrt{\left[\frac{|G'(t_2)|}{C}\right]}$$

$$= \frac{1}{2\pi}\sqrt{\left[\frac{\omega_q G_0}{2C}\frac{\sqrt{(y^2-1)}}{K+1}\right]}$$

$$= \frac{1}{2}\sqrt{\left[\frac{f_q G_0}{\pi C}\frac{\sqrt{(y^2-1)}}{K+1}\right]} = \frac{1}{2}\sqrt{\left[\frac{f_q G_0}{\pi C}\sqrt{\{z^2-(1-P)^2\}}\right]}. \qquad (30)$$

Equation (30) also refers to the band-width of the envelope of the frequency spectrum of a succession of pulses building up from signal or noise. Thus the frequency spectrum is precisely similar in shape to the frequency-response curve of the receiver. The distribution of frequencies in the receiver output may therefore be examined using the data given in Appendix 3 relating to the band-width b_s. This is only true of a receiver using a quench function which is symmetrical so that equation (28) is true.

5·4·5. *Criteria for slope-control*

In the analysis of sinusoidal quench we have assumed that the operation of the receiver is slope-controlled. Let us see to what extent the assumption is justified. The condition for slope-controlled operation is

$$\frac{G'(t_1)}{4C}t_1^2 > 3 \quad \text{(equation (23), Chapter 3).} \tag{31}$$

Let us suppose, as we did in §3·5·1, that the conductance variation is linear during the sensitive period, so that

$$G'(t_1) = G_0/t_1 \quad \text{or} \quad t_1 = G_0/G'(t_1). \tag{32}$$

Then the condition in (31) becomes

$$\frac{G_0^2}{4CG'(t_1)} > 3 \quad \text{or} \quad G'(t_1) < G_0^2/12C. \tag{33}$$

Substituting from equation (18) the value of $G'(t_1)$ appropriate to sinusoidal quench, we get

$$\pi f_q G_0 S^2 < G_0^2/12C \quad \text{or} \quad f_q < \frac{G_0}{12\pi C S^2},$$

where $$S = \sqrt{\left[\frac{\sqrt{(y^2-1)}}{K+1}\right]} = \sqrt[4]{[z^2-(1-P)^2]}. \tag{34}$$

But $G_0/2\pi C$ is the band-width b of the quiescent circuit thus making the condition

$$f_q < b/6S^2 \quad \text{or} \quad f_q < f_0/6QS^2 \quad (b = f_0/Q). \tag{35}$$

Equation (35) provides a formula for the upper limit to the quench frequency, beyond which the operation of the receiver is likely to depart from slope-control. It is particularly useful in describing the limiting quench frequency in terms of the natural frequency and Q-factor of the tuned circuit. Through the factor S, the limiting frequency also depends upon the quench amplitude relative to the steady electrode potentials applied to the oscillator valve.

5·4·6. *Condition for adequate damping*

Departure from slope-control is one factor deciding the upper limit to the quench frequency. There is, however, another limitation on the permissible value to which the quench frequency may be raised. It is imposed by the requirement that the oscillations from one quench cycle must die away before the next one starts.

For sinusoidal quench it can operate at either a lower or higher frequency than the deviation from slope-control, according to the relative adjustment of electrode voltages and quench amplitude.

The approximate condition for adequate damping, given in §3·5·4, is

$$N_d > N_0 + N_s + 3 \text{ nepers,} \tag{36}$$

where N_d is the damping loss in nepers, N_0 is the slope gain in nepers, N_s is the super-regenerative gain in nepers. But the logarithmic decrement of the tuned circuit when the valve is cut off is π/Q, and the damping ratio, in nepers, is therefore

$$N_d = n\pi/Q, \tag{37}$$

where n is the number of cycles of frequency f_0 occurring in the damped oscillation. For the purpose of this argument we shall say that the damping occupies one-half of the quench cycle. Thus

$$n = f_0/2f_q \quad \text{and} \quad N_d = f_0\pi/2f_qQ. \tag{38}$$

The condition for freedom of interference between adjacent cycles now becomes

$$f_0\pi/2f_qQ > N_t + 3$$

or

$$f_q < f_0/[(2Q/\pi)(N_t + 3)]. \tag{39}$$

As an example let us evaluate this for a typical receiver. If the total gain is 5×10^5 (13·1 nepers), then

$$N_t + 3 = 16{\cdot}1 \text{ nepers} \tag{40}$$

and, approximately,

$$f_q < f_0/10Q.$$

This criterion must be obeyed if it is desired to avoid coherent operation, with consequent loss of sensitivity.

The criteria of equations (35) and (40) are both approximate. For a certain value of S ($S = 1{\cdot}29$) they are identical. This means that an oscillator, quenched in such a way that $S < 1{\cdot}29$, is slope-controlled for all permissible values of the quench frequency up to the onset of coherence between adjacent pulses of oscillation. In practice S sometimes exceeds this value, but the corresponding departure from slope-control is not great and only occurs for quench frequencies near the higher limit.

Thus a super-regenerative receiver using sinusoidal quench nearly always operates in the slope-controlled state.

5·4·7: *Choice of quench frequency*

The upper limit to the quench frequency is determined by the onset of coherence or by the departure from slope-control, if that state of operation is desired. The lower limit is determined by the highest component frequency in the modulation envelope which is to be reproduced. Between these limits is a range of frequencies over which the receiver will operate satisfactorily.

The nature of the output spectrum (§4·4·5) shows that the quench frequency must be more than twice the highest component frequency of the modulation. How much more than this depends upon the efficacy of the output filter in rejecting the side-bands of the adjacent line. In other words it is desirable to use an efficient low-pass filter after the detector if the lowest possible quench frequency is required. In most practical cases the quench frequency f_q will have to be at least four times the highest modulation component f_m, i.e.

$$f_q \geqslant 4f_m. \tag{41}$$

We are now in a position to derive a simple approximate relationship between the modulation band-width, the quench frequency and the super-regenerative band-width, b_s. It appears later that the latter is necessarily several times the quench frequency, and it is consequently desirable, in most cases, to choose the conditions to achieve a minimum value of b_s.

The formula for the super-regenerative band-width for sinusoidal quench is, from equation (26),

$$b_s = \sqrt{\left[\frac{f_q G_0}{\pi C} S^2\right]}, \tag{42}$$

where $$S = \sqrt[4]{[z^2 - (1-P)^2]}.$$

Now fig. 6, Appendix 3, shows that the minimum value of S for a given gain occurs when $P = 1$, i.e. for S equal to $\sqrt{z}$. This condition represents the state in which oscillation starts on the axis of the sinusoidal quench. This is not quite permissible in practice because the damping condition is not usually fulfilled, but it is very near to the practical limit and adequate for our purpose here.

Substituting $\sqrt{z}$ for S in equation (42) gives

$$b_{s\,\text{min.}} = \sqrt{\left[\frac{f_q G_0}{\pi C} z\right]} = \sqrt{(2bf_q z)}, \tag{43}$$

where $b = G_0/2\pi C$ is the quiescent band-width.

But z is related to the super-regenerative gain by equations (23) (c) and (d). Only a graphical approximation to these equations is possible. If $H = (f_q/b)N_s$ is plotted against z according to equation (23) for $P = 1$, it becomes evident that, over the practical range of values of super-regenerative gain, H is proportional to z. Quantitatively, in fact,

$$H \doteqdot \tfrac{1}{3}z$$

and

$$N_s = (b/f_q)H \doteqdot bz/3f_q. \tag{44}$$

We may now substitute for bz in equation (43)

$$b_{s\,\text{min.}} \doteqdot f_q\sqrt{(6N_s)}. \tag{45}$$

Although approximate, this formula is an invaluable guide to the minimum band-width achievable with a given gain and quench frequency. For a gain of 10^6 (13·8 nepers)

$$b_{s\,\text{min.}} \doteqdot 9f_q. \tag{46}$$

It should be noted that the minimum band-width is independent of the radio frequency but is proportional to the quench frequency. Equation (45) shows that it only varies very slowly with super-regenerative gain, and there is consequently little advantage in operating the receiver at a low gain in order to obtain a smaller band-width.

Using the relationship in equation (41) we see that

$$b_{s\,\text{min.}} \doteqdot 36f_m, \tag{47}$$

provided that the *lowest possible* quench frequency is used consistent with receiving the highest component of modulation. Even under the most favourable conditions, therefore, the radio-frequency or super-regenerative band-width must be many times the width of the band of modulation frequencies to be reproduced. The super-regenerative receiver is consequently most useful when the radio-frequency band-width is necessarily wider than the modulation band-width for reasons such as frequency drift encountered at ultra-high frequencies.

It is shown in §4·5 that considerations of noise need have no influence on the choice of quench frequency, for the amount of noise appearing within a given post-detector band is independent of the quench frequency.

Table 6 provides a few examples calculated from the simple formulae derived above. They illustrate the factors which determine the choice of quench frequency. The lower limit of quench frequency is based upon the reception of audio-frequency-modulated signals within the band 0–5 kc./sec. From equation (41) this places a lower limit of 20 kc./sec. on the quench frequency. There is, of course, nothing to prevent the operation of a receiver at any lower quench frequency, provided the restriction of permissible modulation frequencies can be tolerated. The figures for circuit Q-factor and oscillator stability represent a reasonable compromise, but it is realized that wide deviations can be expected in individual cases.

Table 6

Note. Data relate to a receiver having a gain of 10^6 in all cases.

Radio-frequency (Mc./sec.)	Q	Estimated stability of super-regenerative oscillator (Mc./sec.)	Range of possible quench frequencies (kc./sec.)	Range of band-widths available (Mc./sec.)	Corresponding modulation band-width (kc./sec.)
20	100	0·05	20–20	0·18–0·18	5–5
45	75	0·15	20–60	0·18–0·54	5–15
100	75	0·25	20–133	0·18–1·2	5–34
200	75	1·0	20–266	0·18–2·4	5–66
500	200	1·0	20–250	0·18–2·25	5–62
1,000	200	1·0	20–500	0·18–4·5	5–125
3,000	300	2·0	20–1000	0·18–9·0	5–250
10,000	300	3·0	20–2000	0·18–27	5–500

The first line of table 6 represents the minimum radio-frequency (20 Mc./sec.) at which super-regenerative reception of audio-frequency-modulated signals is easily possible, using a slope-controlled linear receiver with sinusoidal quench. Here, the minimum super-regenerative band-width is adequate to allow for the frequency instability of the oscillator. At 45 Mc./sec., however, the minimum band-width corresponding to a quench frequency of 20 kc./sec. is 180 kc./sec., which is hardly enough to include the likely variations of oscillator frequency. At this and higher frequencies, therefore, it is advisable to raise the quench frequency until the super-regenerative band-width is adequately wide. It should also be remembered that the band-width for a given quench frequency may be increased by increasing the quench amplitude, except near the onset of coherence.

A higher maximum value of quench frequency than that quoted in table 6 may be obtained by deliberately damping the tuned circuit, or cavity, thus decreasing the Q-factor.

In interpreting the figures quoted in table 6 it must be remembered that the skirts of the frequency response of a slope-controlled receiver are very steep and that the band-width is measured at -1 neper. This causes the super-regenerative receiver to be much more immune to interference from signals on adjacent frequencies than other types of receiver having the same nominal band-width.

The design procedure for a super-regenerative receiver with sinusoidal quench may be summarized as follows:

(i) Decide the highest modulation frequency to be reproduced and choose the quench frequency at least four times higher (or twice as high if a good low-pass filter is used with cut-off frequency $\frac{1}{2}f_q$). Confirm that this quench frequency does not violate the conditions for coherence and slope-control.

(ii) Estimate the frequency stability of transmitting and receiving oscillators and increase the quench frequency above the value given by (i) until the super-regenerative band-width is adequate to include the probable drift.

(iii) Use the lowest possible quench amplitude consistent with the desired gain if the lowest ratio of super-regenerative band-width to modulation band-width is required.

5·5. Rectangular quench

5·5·1. *The conductance cycle*

The theory of the step-controlled state deals with a conductance function having a discontinuity or step at the start of the cycle, but not necessarily a constant value of conductance during the super-regenerative period. In practice, however, the quench function for step-control will usually have a *rectangular* form, and this is the only example of step-control which merits special consideration.

The theory of Chapter 3 has shown that the properties of the step-controlled state depend upon the extreme values G_0 and G_1 (fig. 3·5) of the effective shunt conductance in the equivalent circuit of the super-regenerative oscillator. G_0 usually depends only upon the properties of the quiescent circuit* and may be expressed,

* This assumes that the oscillator valve is biased beyond cut-off during the damping period t_d, and the whole of the shunt conductance is due to the losses in the quiescent circuit. If, however, the valve is still permitted to conduct during the damping period, so that the effective conductance is positive but less than the quiescent value G_0, then G_0 must be replaced in the formulae by a value computed from the valve and circuit parameters in the same way as G_1. The added complication is not introduced here.

if desired, in terms of the band-width b or the Q-factor, as in §2·2,

$$G_0 = 2\pi Cb = \omega_0 C/Q. \tag{48}$$

The value of the negative conductance G_1 depends upon the mutual conductance g_m of the oscillator valve corresponding to the quench voltage applied to it during the build-up period t_b. It is given by

$$G_1 = G_0 - kg_m,$$

where k depends upon the feed-back coupling of the oscillator (see §2·4). As before we may write k in terms of the minimum mutual conductance required to produce oscillation, g_{m0},

$$G_1 = G_0[1 - g_m/g_{m0}],$$

or in terms of the actual electrode voltages (as in equation (11))

$$G_1 = G_0\left[1 - \sqrt{\left\{\frac{\bar{V}_g + V_q + (V_a/\mu)}{V_{0g} + V_a/\mu}\right\}}\right] \tag{49a}$$

for grid quench, or

$$G_1 = G_0\left[1 - \sqrt{\left\{\frac{\bar{V}_a + \mu V_g + V_q}{V_{0a} + \mu V_g}\right\}}\right] \tag{49b}$$

for anode quench. The symbols have the same meaning as in §5·3.

Having computed the values of G_0 and G_1 from the formulae applicable to a particular problem, the general formulae of the step-controlled state may be used to predict the properties of the receiver. Although this process is straightforward, we shall examine briefly the trend of these properties as the proportions of the quench function are varied.

5·5·2. *Receiver gain*

The super-regenerative gain is

$$N_s = G_1 t_b/2C \text{ nepers,} \tag{50}$$

where t_b is the duration of the build-up period and C is the total circuit capacitance. Thus a given gain can be achieved with a variety of values of G_1, by altering the duration of the period t_b so that the product $G_1 t_b$ is constant. The damping loss

$$N_d = G_0 t_d/2C \text{ nepers} \tag{51}$$

must, however, be held at a value somewhat greater than N_s in order to avoid coherent operation and its undesirable properties described in §3·2.

It is interesting to examine the change in the other properties of the receiver, particularly in the frequency response, when the proportions of the conductance function are altered whilst retaining a constant super-regenerative gain.

5·5·3. *Frequency response*

The frequency response of the step-controlled receiver is

$$A(f) = \frac{(G_0/4\pi C)(G_1/4\pi C)}{\sqrt{\{[(f-f_0)^2+(G_0/4\pi C)^2][(f-f_0)^2+(G_1/4\pi C)^2]\}}}, \quad (52)$$

of which the band-width half-way down is approximately

$$b_s \doteqdot \frac{G_0 G_1}{\pi C(G_0+G_1)} \quad \text{(equation (87), Chapter 3).} \quad (53)$$

The response curve is drawn in fig. 5·6 for various values of the ratio $R = G_1/G_0$. For $G_1 \gg G_0$ the response near resonance is largely controlled by the term in G_0 and approximates to that of the quiescent circuit. The band-width is then

$$b_s \doteqdot G_0/2\pi C \quad \text{at } -3\text{ db.}$$

This cannot be deduced from (53), which is only valid for values of the ratio R close to unity (e.g. $\frac{1}{10} < G_0/G_1 < 10$). The response curve has the shape corresponding to a single-tuned circuit ($R = \infty$ in fig. 5·6). The half-width of this response is used as a unit of frequency.

As the ratio R decreases, the shape of the frequency response changes gradually until, for $R = 1$ ($G = G_0$) it becomes characteristic of a pair of similar loosely coupled circuits, with a band-width

$$b_s = G_0/2\pi C \quad \text{this time at } -6\text{ db.}$$

The skirts of the response curve are steepest under these conditions, when the conductance cycle is symmetrical about the horizontal axis.

When R is decreased below unity ($G_1 < G_0$), the width of the response curve continues to decrease, but the shape changes back towards that of a single-tuned circuit. From the symmetry of equation (52) it is evident that the curve has the same shape for $R = 4$, say, as for $R = \frac{1}{4}$. It tends to a single-tuned circuit response for $R \gg 1$ and $R \ll 1$, and to a pair of similar coupled circuits for $R = 1$.

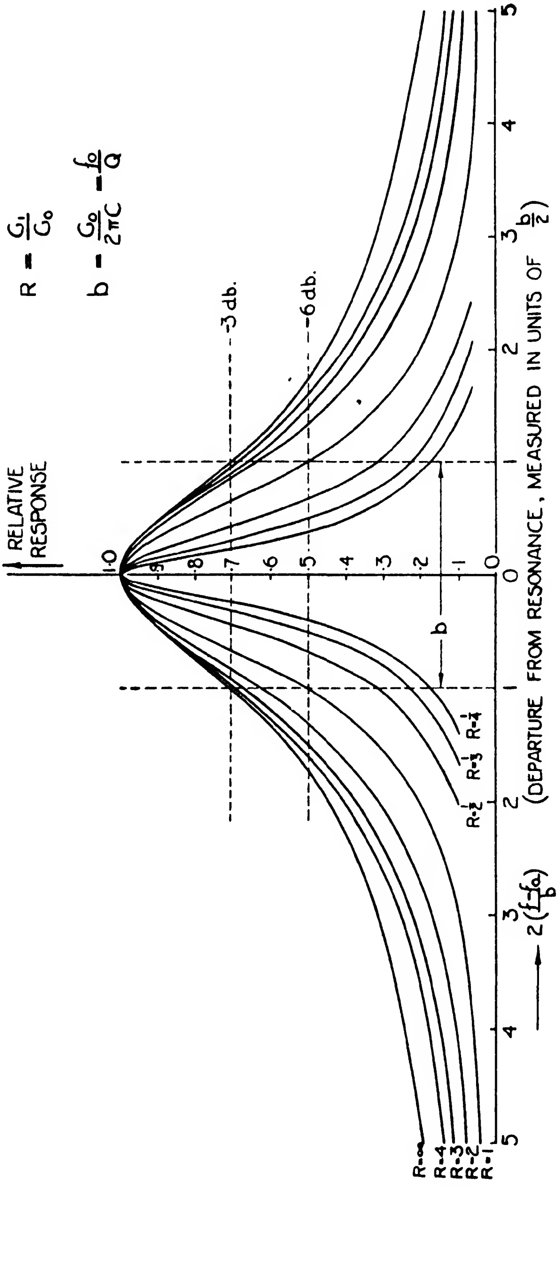


Fig. 5·6. Dependence of frequency response upon the ratio $R=G_1/G_0$.

5·6. Other kinds of quench

There is considerable scope, in the design of linear super-regenerative receivers, for achieving desirable properties by means of special quench or conductance variations. Although sinusoidal and rectangular quench have great practical importance, if only because of the ease with which they are produced, there are occasions when the achievement of a special result may justify added complication in the quench generator.

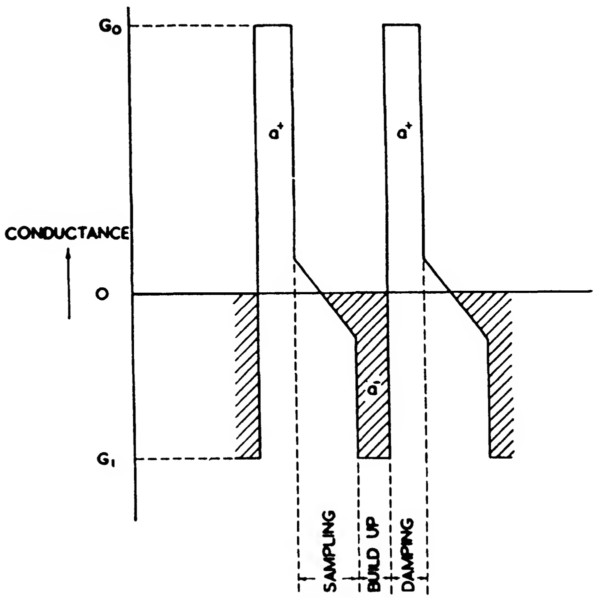


Fig. 5·7. Conductance cycle of minimum duration.

A disadvantage of the super-regenerative receiver is the high ratio of the radio-frequency band-width to the useful range of modulation frequencies. One important reason for using special kinds of quench is to decrease this ratio and thus to increase the effective signal-to-noise ratio of the receiver.

In the slope-controlled state the frequency response is determined by the conductance slope during the sampling period during which the self-oscillations are initiated. The period of slope must be of sufficient duration so that, at its end, the self-oscillations have built up to several times the level of the incoming signal. If t_1 is the period of slope, the condition for slope-control is

$$t_1 > 12C/G_0 \quad \text{(equation (35), §3·5).}$$

Provided this condition is satisfied, it is an advantage to speed up the remainder of the quench cycle by causing the build-up and decay of the oscillations to occur in the least possible time. This argument leads ideally to a conductance cycle of the type shown in fig. 5·7, in which each of the processes fundamental to super-regeneration, namely, sampling, build-up and damping, is carried out with maximum efficiency.

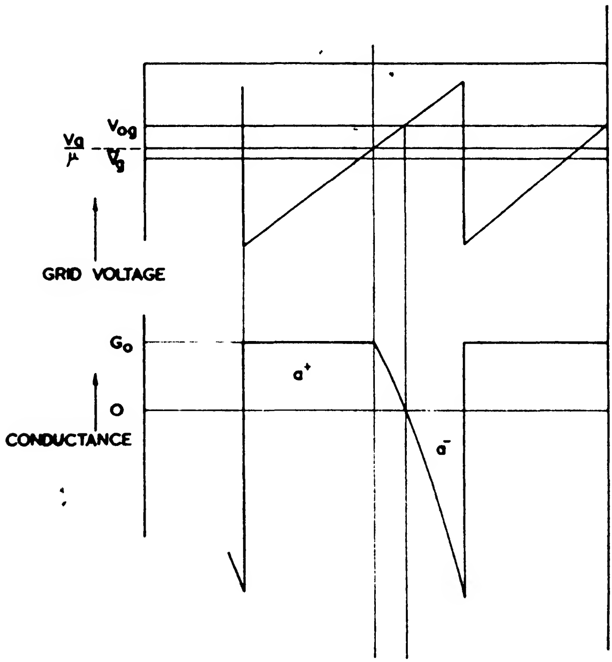


Fig. 5·8. Formation of conductance cycle: saw-tooth quench.

In practice it is relatively easy to increase the rate of damping by connecting a diode across the oscillatory circuit and causing it to conduct during the period T_d. The rate at which the oscillations build up is limited by the maximum available mutual conductance of the oscillator valve, and the duration of the build-up period cannot, therefore, be reduced below a certain value, corresponding to the desired gain. It is possible by these means to obtain an *improvement by a factor two, or even three, in the ratio of super-regenerative* band-width to quench frequency over that obtaining for sinusoidal quench.

A saw-tooth quench function has some of the advantages mentioned above, whilst being a relatively easy variation to produce. The formation of the conductance cycle for this type of quench is illustrated in fig. 5·8. The operation is slope-controlled, giving a frequency response similar to that for sinusoidal quench. The limiting quench frequency, before the operation departs from slope-control, is, however, somewhat greater.

Chapter 6

THE LOGARITHMIC MODE

6·1. Introduction

A super-regenerative receiver is said to operate in the logarithmic mode when the oscillations on each quench cycle are allowed to build up to a limiting value before they are damped. The limitation of oscillation amplitude is due to curvature in the valve characteristic and has been discussed in Chapter 2.

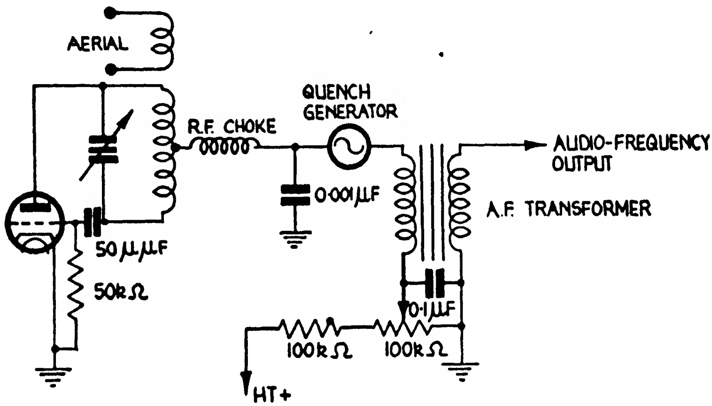
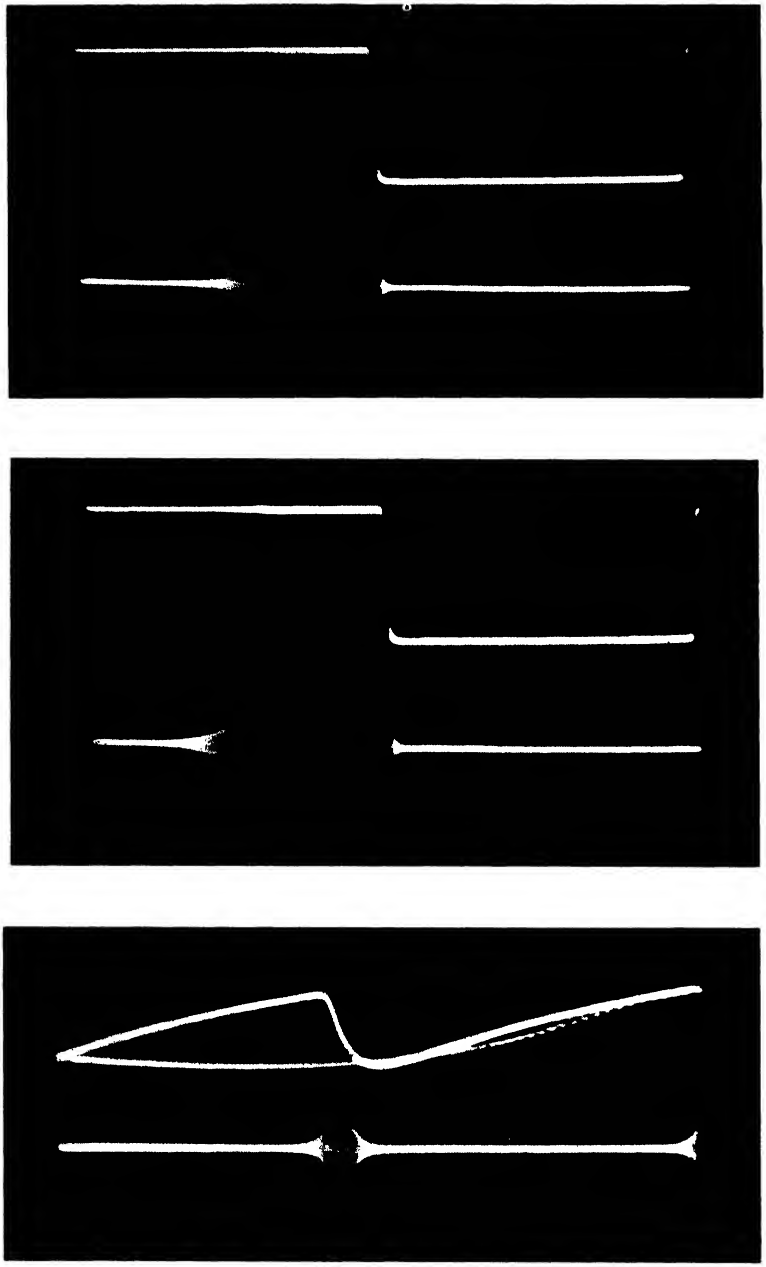


Fig. 6·1. Super-regenerative detector: logarithmic mode.

The super-regenerative oscillations build up from signal or noise in the resonant circuit. A receiver operates truly in the logarithmic mode when the oscillations build up to saturation from noise alone. It is possible, however, to adjust the quench so that the operation of the receiver is linear for small signals and logarithmic for large signals. A typical receiver intended for operation in the logarithmic mode is shown in fig. 6·1.

A receiver operating in the linear mode and having an appreciable gain can be transferred to the logarithmic mode by applying a large enough signal, or by increasing the quench amplitude. The transition from linear to logarithmic mode is illustrated in fig. 6·2 for a slope-controlled (sinusoidal) quench cycle.

Fig. 6·2 is instructive in showing the process of limitation of the oscillation amplitude and the time relationship between the con-



(a)

(b)

(c)

Fig. 6·3. Oscillatory output of super-regenerative receiver in relation to grid voltage. (*a*) Logarithmic mode: noise only. (*b*) Logarithmic mode: large c.w. signal. (*c*) Self-quenching.

ductance cycle and the oscillation amplitude. The shape of the conductance-time curve is explained below. When the oscillations build up to an equilibrium amplitude from noise alone, the effect of applying a signal is to advance the leading edge of the output pulse. That is to say, the limiting amplitude is reached at an earlier time. We see later that the amount of the advance depends logarithmically on the signal amplitude.

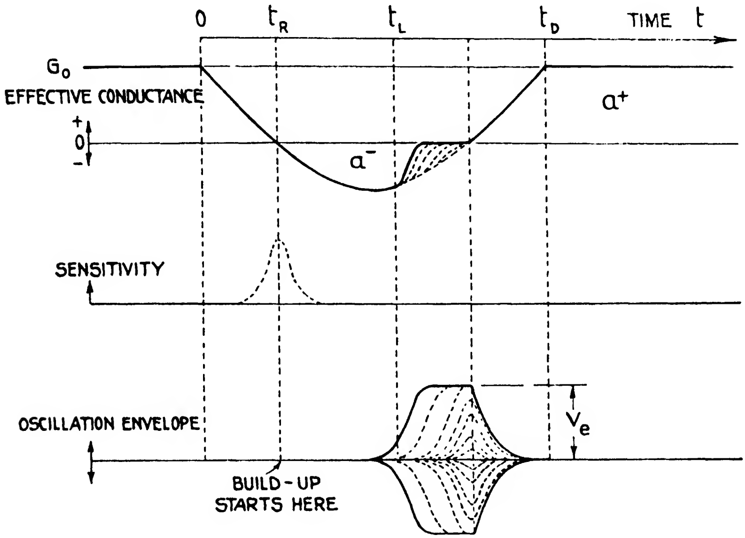


Fig. 6·2. Time relationships in a quench cycle: logarithmic mode.

Fig. 6·3 is an oscillogram of the oscillatory output of a super-regenerative receiver in the logarithmic mode. Each picture represents a number of successive pulses superimposed. The leading edge of the pulse in (*a*), due to noise alone, is blurred, showing the variable time of growth from a random noise voltage. The pulse in (*b*) shows how the build-up curve is advanced due to a signal much greater than noise, so increasing the area under the pulse envelope.

When the signal is amplitude-modulated, the time taken for the oscillations to achieve a given amplitude depends upon the sample of the signal taken at the start of the build-up. The gradual variation in the area under the pulse envelope, over a number of quench cycles, causes a corresponding variation of anode current in the super-regenerative oscillator. This occurs in a similar way to leaky grid detection if a grid condenser and leak are used, or to anode-bend detection if they are not. A super-regenerative oscillator operating in the logarithmic mode is, therefore, self-detecting because the anode current carries a component at the modulation frequency. A suitable output filter separates this from the quench and carrier-frequency components. For the purpose of analysis we shall separate the functions of super-regenerative amplification and detection and regard the incremental area under the oscillation envelope as the signal at the output.

The build-up or super-regenerative period is the same for the linear and logarithmic modes until the oscillations rise to an amplitude large enough for limitation to begin. The analysis carried out in Chapters 3, 4 and 5 for the linear mode holds, therefore, up to time such as t_L in fig. 6·2. Analysis of the logarithmic mode can, therefore, be made by using the formulae for the linear mode to represent the voltages at time t_L, and then considering the subsequent effect of amplitude limitation. Thus, for instance, the frequency response of the receiver is determined in the same way for the linear and logarithmic modes. In the latter mode, however, it is modified by the non-linearity of the amplification. In many ways it is convenient to regard the logarithmic super-regenerative receiver as a linear receiver followed by a logarithmic detector.

6·2. The quench cycle: logarithmic mode

Let us consider in more detail the events in a single quench cycle. For an approximate analysis we can still use the equivalent circuit of fig. 3·1, provided that we recognize the limitation of oscillation amplitude by a suitable change in the conductance function $G(t)$.

The sequence of events in a single cycle of sinusoidal quench is shown in fig. 6·2. The initial part of the cycle is the same as for the linear mode (fig. 3·3).

Departure from linearity begins as soon as there is appreciable variation of effective conductance during a cycle of oscillation. The mechanism of this is discussed in §2·9. The net effect of such variation, as the oscillation amplitude increases, is as though the mean conductance G steadily decreased to zero. On reaching zero it remains there until the end of the super-regenerative period, during which time the oscillations are maintained at their equilibrium amplitude. At the end of the cycle they are damped out, in the normal way, before the start of the next cycle.

Now we already know in great detail the dependence of the voltage V_1 on the amplitude and frequency of an incoming signal. So we shall start by assuming this knowledge and go on to examine the effect of the amplitude limitation upon the useful output of the receiver.

6·3. Approximate analysis of the logarithmic mode

Fig. 6·4 shows the build-up envelope, after time t_L, in greater detail. The two curves represent the build-up from two voltages V_1 and V_2. V_1 is the voltage reached at time t_L by the oscillations building up from a sample of noise in the sensitive period, earlier in the cycle. V_2 is due to signal plus noise.

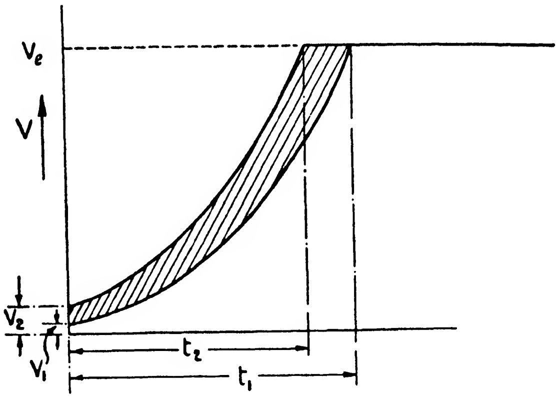


Fig. 6·4. Idealized curves of build-up and limitation of oscillation amplitude in the logarithmic mode.

Although we are considering the events in a single quench cycle we shall ultimately require the results to apply to a practical receiver, i.e. to a large number of cycles. Thus, although the noise voltage V_1 is indeterminate for a single cycle, its r.m.s. value is given by

$$V_1 = \sqrt{(\overline{V_n^2})}\mu_L. \tag{1}$$

Similarly

$$V_2 = \sqrt{(\overline{V_n^2} + V_s^2)}\mu_L, \tag{2}$$

where $\overline{V_n^2}$ is the mean square noise at input, V_s is the signal voltage at input, and μ_L is the voltage gain up to the time at which V_1 and V_2 are measured.

For simplicity, let us take the build-up curves to be truly exponential in form until they are suddenly limited at the equilibrium value V_e.* As we are starting with the values of the voltage at time $t = t_L$, we shall henceforth measure time from this point and not from the beginning of the true build-up period.

We shall use this simple model as a basis for calculating the incremental area due to the addition of the input signal to the noise. If V_2 is less than one-tenth of the equilibrium amplitude V_e (as we can make it by taking our initial measurement far enough back), then the difference in the areas to the left of the vertical axis is negligible. The incremental area to the right of the axis (shaded in fig. 6·4) represents the increase in output due to the application of the signal. It is a measure of the signal amplitude after detection. The conversion of the oscillation pulse into useful output is dealt with later. The gain of the system depends upon the method adopted for doing this. Consideration of the gain factor at this stage is, therefore, not profitable. In spite of this we can discover a good deal about the properties of the logarithmic mode by calculating the incremental area in arbitrary units.

Referring to fig. 6·4 we have

$$V_e = V_1 e^{at_1} = V_2 e^{at_2}, \tag{3}$$

where $1/a$ is the time constant of build-up, i.e.

$$t_1 = \frac{1}{a}\log_e(V_e/V_1) \quad \text{and} \quad t_2 = \frac{1}{a}\log_e(V_e/V_2). \tag{4}$$

Hence the incremental area is given by

$$\begin{aligned} \Delta A &= V_2 \int_0^{t_2} e^{at}\,dt + V_e(t_1 - t_2) - V_1 \int_0^{t_1} e^{at}\,dt \\ &= \frac{1}{a}[V_e \log_e(V_2/V_1) - (V_2 - V_1)]. \end{aligned} \tag{5}$$

For signals greater than noise, the first term in the bracket is of the order of the limiting voltage V_e, whilst the second is the much smaller initial voltage from which build-up in the logarithmic mode begins. Thus we can write

$$\Delta A \doteqdot (V_e/a)\log_e(V_2/V_1). \tag{6}$$

* In spite of this simplification the results apply quite accurately to practical build-up curves such as that in §2·9·1.

Substituting for V_1 and V_2 from equations (1) and (2),

$$\Delta A \doteqdot (V_e/a)\log_e\left[\frac{\sqrt{[(\overline{V_n^2})+V_s^2]}}{\sqrt{(\overline{V_n^2})}}\right]. \tag{7}$$

For signals much larger than noise this becomes

$$\Delta A \doteqdot (V_e/a)\log_e[V_s/\sqrt{(\overline{V_n^2})}]. \tag{8}$$

This shows why we call the mode 'logarithmic'. The incremental area under the output envelope is proportional to the logarithm of the signal-to-noise ratio at the input.

The noise voltage appears, even in the approximate expression, because it determines the initial pulse envelope, before the signal is applied. The fact that we always have to consider the signal in relation to noise has some curious effects, particularly upon the frequency response of the receiver.

6·3·1. *Frequency response*

The approximate equation

$$\Delta A \doteqdot (V_e/a)\log_e(V_2/V_1)$$

derived above for large signals refers to a signal at the resonant frequency. For signals near resonance the value of V_2 in a slope-controlled receiver is

$$V_2 \doteqdot V_s\mu_L \exp[4(f-f_0)^2/b_s^2],$$

in which V_s is the signal voltage, μ_L is the total gain up to the time t_L at which V_2 is measured, and b_s is the super-regenerative bandwidth in the linear mode (equation (50), Chapter 3), i.e.

$$b_s = \frac{1}{\pi}\sqrt{\left[\frac{|G'(t_1)|}{C}\right]}.$$

Also $$V_1 = \mu_L\sqrt{(\overline{V_n^2})}.$$

Substituting these values in equation (6), we get

$$\begin{aligned}\Delta A &= (V_e/a)\log_e\{[(V_s/\sqrt{(\overline{V_n^2})}]\exp[-4(f-f_0)^2/b_s^2]\}\\ &= (V_e/a)\log_e\{[V_s/\sqrt{(\overline{V_n^2})}]-[4(f-f_0)^2/b_s^2]\}.\end{aligned}$$

This equation is only true for signals much greater than noise $[V_s \gg \sqrt{(\overline{V_n^2})}]$. That is to say, it only describes the peak of the response curve, not the skirts. The response falls off in parabolic manner from the maximum value

$$(V_e/a)\log_e[V_s/\sqrt{(\overline{V_n^2})}].$$

The width of the response curve for ΔA to be equal to half its value at the peak depends upon the signal-to-noise ratio V_2/V_1. The 'band-width' at half-value is

$$b_L = 2(f-f_0) = b_s\sqrt{\{\tfrac{1}{2}\log_e[V_s/\sqrt{(\overline{V_n^2})}]\}}. \quad (9)$$

The response curve is plotted in fig. 6·5, for several values of the signal-to-noise ratio, compared with the frequency response of the

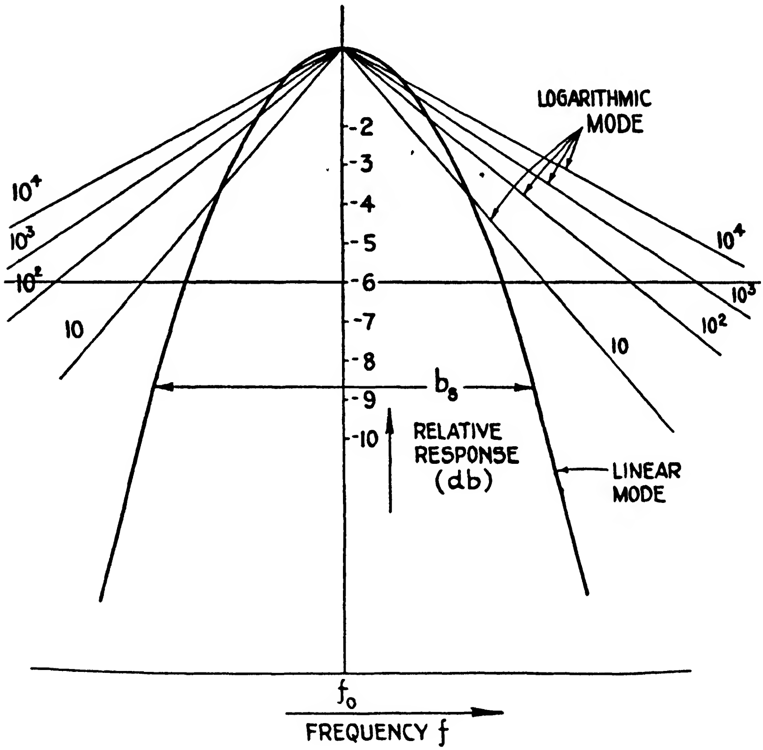


Fig. 6·5. Frequency response in the logarithmic mode compared with frequency dependence of oscillation amplitude earlier in the super-regenerative period, i.e. in the linear mode.

same receiver in the linear mode. It should be noted that the curves for the logarithmic mode relate to the variation of ΔA with frequency and are all adjusted to the same peak value. The curve for the linear mode shows the dependence of the output voltage in that mode upon the signal frequency.

6·3·2. *The effect of amplitude modulation**

Let us revert to equation (8) and investigate the effect of amplitude modulation on the signal voltage V_s. An amplitude-modulated signal has the form

$$V_s = \hat{V}_s(1 + M\cos\omega_m t). \tag{10}$$

Equation (8) therefore becomes

$$\begin{aligned}\Delta A &= (V_e/a)\log_e[\hat{V}_s(1 + M\cos\omega_m t)/\sqrt{(\overline{V_n^2})}] \\ &= (V_e/a)[\log_e \hat{V}_s + \log_e(1 + M\cos\omega_m t) - \log_e\sqrt{(\overline{V_n^2})}] \quad (|M| < 1).\end{aligned} \tag{11}$$

The term carrying the modulation frequency is independent of the carrier amplitude V_s to this degree of approximation. Thus perfect automatic volume control action is inherent in the operation of the receiver at all signal levels well above circuit noise. There is, however, considerable distortion of the modulation envelope.

In order to examine the distortion as a function of modulation depth Strafford expands $\log_e(1 + M\cos\omega_m t)$ into a harmonic converging series for $|M| < 1$, as follows. Use is made of a related series, namely,

$$\log_e(1 + 2a\cos\theta + a^2)_{a^2<1} \equiv 2[a\cos\theta - (\tfrac{1}{2}a^2)\cos 2\theta + (\tfrac{1}{3}a^3)\cos 3\theta - \text{etc.}]. \tag{12}$$

But $$\log_e(1 + 2a\cos\theta + a^2) \equiv \log_e\left[(1 + a^2)\left(1 + \frac{2a\cos\theta}{1 + a^2}\right)\right]$$

$$\equiv \log_e(1 + a^2) + \log_e\left(1 + \frac{2a\cos\theta}{1 + a^2}\right). \tag{13}$$

The second term of the expression is equal to $\log_e(1 + M\cos\theta)$, if $M = 2a/(1 + a^2)$. This gives

$$a = \frac{1}{M} \pm \sqrt{\left(\frac{1}{M^2} - 1\right)}. \tag{14}$$

To satisfy the necessary condition that $a^2 < 1$ we take the solution

$$a = \frac{1}{M} - \sqrt{\left(\frac{1}{M^2} - 1\right)} \tag{15}$$

as the valid one for the expansion.

* The treatment in this section closely follows a paper by F. R. W. Strafford, *J. Instn Elect. Engrs*, **93**, Part III, p. 23 (1946).

Therefore

$$\log_e(1+M\cos\omega_m t) = 2\left\{\left[\frac{1}{M}-\sqrt{\left(\frac{1}{M^2}-1\right)}\right]\cos\omega_m t - \frac{1}{2}\left[\frac{1}{M}-\sqrt{\left(\frac{1}{M^2}-1\right)}\right]^2\cos 2\omega_m t + \frac{1}{3}\left[\frac{1}{M}-\sqrt{\left(\frac{1}{M^2}-1\right)}\right]^3\cos 3\omega_m t + \text{etc.}\right\} - \log_e\left\{\left[\frac{1}{M}-\sqrt{\left(\frac{1}{M^2}-1\right)}\right]^2+1\right\}. \quad (16)$$

The output at the fundamental modulation frequency is thus proportional to

$$V_{\omega m} \propto \left[\frac{1}{M}-\sqrt{\left(\frac{1}{M^2}-1\right)}\right]\cos\omega_m t. \quad (17)$$

At first sight the non-linear relationship between output and modulation depth appears to be very poor, but there exists a fair degree of linearity up to about $M = 0{\cdot}6$ (60% depth). Above $M = 0{\cdot}6$ there is evidence of considerable 'volume expansion'. The super-regenerative detector is thus not capable of high-quality reproduction when receiving programme modulation at normal broadcasting levels. This is very obvious in practice. Deep peaks of modulation on speech and music give rise to a 'bark' in the headphones.

As a result of experiments, good speech intelligibility, for communication purposes only, is obtained if the peak modulation is not allowed to exceed about 80%.

From equation (16), as M approaches 1, the harmonic output approaches the value

$$2(\cos\omega_m t - \tfrac{1}{2}\cos 2\omega_m t + \tfrac{1}{3}\cos 3\omega_m t + \ldots). \quad (18)$$

The total harmonic percentage relative to the fundamental is given by

$$n\% = \sqrt{\left(\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\right)} \times 100\%. \quad (19)$$

The sum to infinity is a known result and gives

$$n\% = \sqrt{\left(\frac{\pi^2}{6}-1\right)} \times 100\% \doteqdot 80\%. \quad (20)$$

This result confirms the 'breaking up' of speech intelligibility during deep modulation.

6·3·3. *The damping period: maximum quench-frequency*

In the slope-controlled state the damping required to reduce the oscillations from one cycle below noise level before the next cycle starts is independent of signal amplitude, or receiver gain. This is because the oscillations always die away from the limiting amplitude V_e. The oscillations decay according to the law

$$V(t) = V_e \exp(-G_0 t/2C) \tag{21}$$

during the period of positive conductance G_0. The condition for the voltage to reach r.m.s. noise level $\sqrt{(\overline{V_n^2})}$ is

$$\sqrt{(\overline{V_n^2})} = V_e \exp(-G_0 t/2C).$$

But $$G_0/2C = \pi f_0/Q \quad \text{(see Chapter 2)}.$$

Hence $$\sqrt{(\overline{V_n^2})} = V_e \exp(-\pi f_0 t/Q).$$

The time allowed for decay is usually about half the quench period, i.e.

$$t = \frac{1}{2f_q}. \tag{22}$$

Therefore $$\sqrt{(\overline{V_n^2})} = V_e \exp(-\pi f_0/2Qf_q),$$

or $$\frac{\pi f_0}{2Qf_q} = \log_e \frac{V_e}{\sqrt{(\overline{V_n^2})}}, \quad f_q = \frac{\pi f_0}{2Q \log_e (V_e/\sqrt{(\overline{V_n^2})})}, \tag{23}$$

where $\sqrt{(\overline{V_n^2})}$ is the r.m.s. noise voltage at input. Strafford* derived this formula without the factor 2 in the denominator. He carried out an experimental investigation by measuring the quench frequency at which the noise just began to vanish, due to interference between adjacent quench cycles. The measured quantities were:

$$V_e = 2{\cdot}5 \text{ V.,} \qquad Q = 100,$$
$$V_n = 20{\cdot}0 \times 10^{-6} \text{ V.,} \quad f_0 = 30 \times 10^6 \text{ c./sec.}$$

These values predict a limiting quench frequency of 40 kc./sec. when substituted in equation (23) (80 kc./sec. in Strafford's formula). The measured value was 50 kc./sec., which agrees very well with the theory. Equation (23) is a useful guide to the maximum quench frequency permissible in the logarithmic mode.

* *loc. cit.*

6·3·4. *Reception of small signals*

In order to examine the growth of signal out of noise we have to go back to the equation (5), which is

$$\Delta A = (1/a)\,[V_e \log_e (V_2/V_1) - (V_2 - V_1)]. \qquad (24)$$

If $V_e \gg (V_2 - V_1)$, the first term in the brackets is much greater than the second, provided that $\log_e(V_2/V_1)$ is not too small. Now

$$\log_e(V_2/V_1) = \log_e \sqrt{[1 + (V_s^2/\overline{V_n^2})]} \quad \text{(using equations (1) and (2))}$$

$$\doteqdot V_s/\sqrt{(\overline{V_n^2})} \quad \text{for} \quad V_s \ll \sqrt{(\overline{V_n^2})}. \qquad (25)$$

If the ratio $V_e/(V_2 - V_1)$ is equal to 100 then $V_s/\sqrt{(\overline{V_n^2})}$ must be equal to $\frac{1}{100}$ for the two terms inside the brackets in (24) to be equally important. Thus we may neglect the second term, even for signals less than noise, and the equation

$$\Delta A = (V_e/a) \log_e [\sqrt{(\overline{V_n^2} + V_s^2)}/\sqrt{(\overline{V_n^2})}] \qquad (26)$$

still represents the incremental area due to small signals.

It is, however, adequate to use the expression

$$\Delta A = (V_e/a) \log_e [V_s/\sqrt{(\overline{V_n^2})}]$$

when the signal-to-noise ratio is greater than three.

6·3·5. *The super-regenerative detector*

The mean anode current of a logarithmic super-regenerative receiver increases during the burst of oscillation on each quench cycle. We have seen that the duration of the oscillations is varied by the signal modulation. For this reason the anode current has a component at the modulation frequency. So, therefore, has the voltage across an impedance in the anode circuit (such as the telephones in fig. 6·1). A suitable filter circuit ensures that only the modulation-frequency component appears across this impedance. The unwanted carrier and quench frequencies are largely removed. The modulation-frequency voltage is the useful signal at output. It is due to the contribution ΔA from each quench cycle, given by the modulation term of equation (11). Consequently it is proportional to ΔA and to the quench frequency, i.e.

$$V_m \propto f_q \Delta A \propto (V_e f_q/a)\,[\log_e (1 + M \cos \omega_m t)], \qquad (27)$$

for signals well above the level of circuit noise. This formula is useful to illustrate the trend of receiver gain due to alteration of the circuit parameters. The time constant a is approximately

$$a = G_1/2C, \tag{28}$$

where G_1 is the value of the effective negative conductance during build-up. Equation (27) now becomes

$$V_m \propto (2V_e f_q C/G_1)\,[\log_e(1 + M\cos\omega_m t)]. \tag{29}$$

Thus a slow rate of building up, corresponding to a low value of negative conductance, gives a greater gain. This is to be expected from the dependence of the output signal upon the incremental area under the build-up envelope. It is the opposite condition to that which holds in the linear mode.

6·4. Self-quenching receivers

There is a further, widely used, type of super-regenerative receiver in which the quench wave-form is supplied by relaxation oscillations in the super-regenerative circuit itself. In other words the super-regenerative receiver is a squegging oscillator. The arrangement of the oscillator differs from the circuit of fig. 6·1 only in the absence of a separate quench generator, and in an increased time constant in the grid circuit due to the grid leak and associated condenser. The squegging oscillator has been briefly discussed in §2·9·3, sufficiently to show the importance of the components in the grid circuit in determining whether or not squegging takes place.

In a squegging oscillator, just as in one that is separately quenched, the oscillations build up from the level of the oscillatory voltage existing in the quiescent circuit. Usually this voltage is supplied by thermal and shot noise. In the super-regenerative receiver it is supplied by an applied signal. We have already seen how the build-up of oscillations in a separately quenched receiver is modified by the presence of a signal. It is reasonable to expect a similar modification in the case of a self-quenching oscillator, enabling it also to act as a receiver. A detailed examination of the events during a squegging cycle show that this is so.

6·5. The self-quenching cycle

The initiation of the squegging process has been described in §2·9·3. After the oscillator has been switched on for a short time the relaxation process settles down so that the mean grid voltage is always negative and fluctuates approximately in the manner shown in fig. 6·6. It is in this 'steady' state that we are interested. Let us follow the events in a squegging cycle and observe the effect of introducing a signal voltage into the oscillatory circuit.

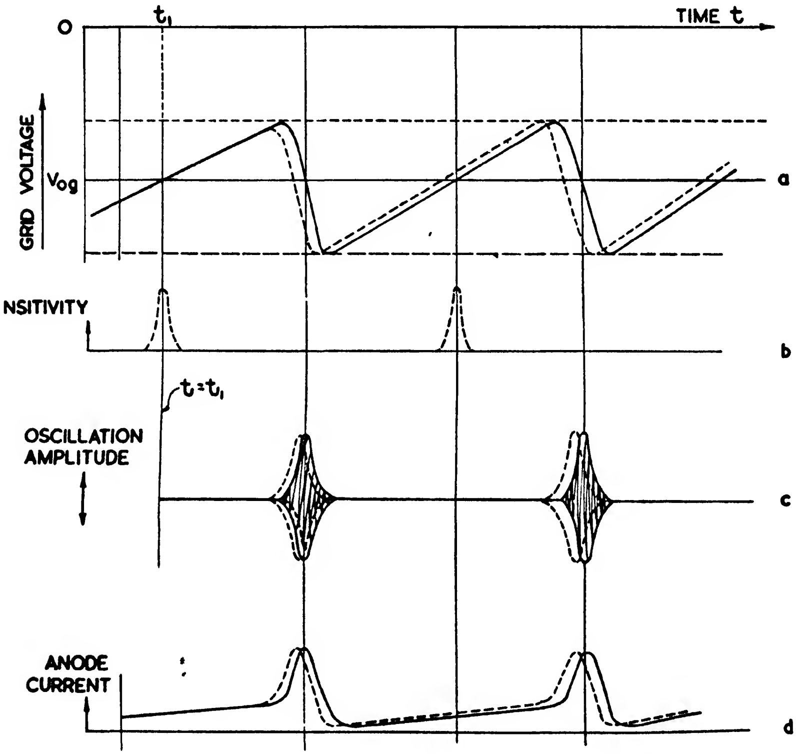


Fig. 6·6. Sequence of operations in a self-quenching receiver.

We shall begin our examination of the self-quench cycle when the oscillatory circuit is quiescent and the grid voltage is recovering on the time-constant in the grid circuit. Time $t = 0$ is some arbitrary time after the oscillations from the preceding cycle have died away. As the grid voltage decreases towards zero a time $t = t_1$ is reached at which the effective circuit conductance is zero, corresponding to the grid voltage V_{0g}. From this point oscillations build up from the level of circuit noise (solid lines). Initially, the oscillation amplitude is small and has no influence on the continued rise of the grid voltage.

The corresponding conductance variation is slow, because it is governed by the slow recovery of the grid potential. For this reason *the self-quenched receiver is always likely to operate in the slope-controlled state.*

Using the same arguments as previously, for the linear mode, the receiver is sensitive for a short period about the time $t = t_1$. The sample of noise or signal taken during this period determines the amplitude of the oscillations at a later time, before they reach their limiting value.

As the oscillations grow, grid current begins to flow on the positive peaks. At this stage it is necessary to imagine an oscillatory voltage superimposed on the mean grid voltage and anode current variations pictured in fig. 6·6 (*a*) and (*d*). It has been omitted for the sake of clarity. The grid current charges the grid condenser and causes the mean negative grid voltage to increase, in an attempt to follow the oscillation envelope. Eventually the oscillations reach a limiting amplitude at which the valve can only just supply sufficient energy to maintain oscillations. The oscillations have now reached their peak value. The grid voltage, due to the persisting grid current, continues its negative trend. This reduces the duration of the synchronous bursts of anode current which maintain the oscillations. The effect is cumulative and the oscillations die away. Grid current no longer flows, and it only remains for the charge on the grid to leak away in readiness for the next cycle.

When a signal voltage exists in the circuit about time $t = t_1$, the oscillations build up from this new level. Build-up starts at exactly the same grid voltage, namely, V_{0g}.* However, the oscillations reach a given amplitude at an earlier time, as shown by the dotted line. The point at which grid current begins to flow is correspondingly advanced, and the grid voltage starts its negative excursion before it has recovered quite so far as it did in the absence of a signal.

Fig. 6·3 (*c*) shows the oscillation pulse of an actual receiver in relation to the grid-voltage wave-form.

All the succeeding events in the cycle are advanced by a corresponding amount. The oscillations reach their peak earlier and the sensitive period in the next quench cycle consequently occurs at an

* This is emphasized because most descriptions of the operation of a self-quenching receiver suggest that the effect of the incoming signal is to change the value of V_g at which oscillations commence. That is not so.

earlier time. The effect of applying a steady c.w. signal, therefore, is to decrease the interval between successive oscillation peaks, thus increasing the quench frequency. This is illustrated by the dotted line in fig. 6·6.

The variation of mean anode current during the cycle is also shown in fig. 6·6. At $t = 0$ it has a value determined by the grid voltage at that time. It increases gradually as the negative value of the grid voltage is reduced. During the oscillations, however, the mean anode current is greatly increased, because it represents the average of bursts at the oscillation frequency. It falls with the oscillation amplitude. By this time, however, the grid potential is more negative, and the anode current drops to a lower value than it had before oscillations began. On the receipt of a signal, the pulses of anode current become more frequent, and the value of the current, averaged over a number of quench periods, increases with the signal amplitude. If the signal is modulated, this mean anode current varies at the modulation frequency. The receiver is thus self-detecting.

6·6. Approximate analysis of self-quenching

The determination of the amount of advance of the oscillation envelope is a similar problem to that illustrated in fig. 6·4 for the logarithmic mode. We shall use this figure again in the simple analysis of self-quenching. Let V_1 and V_2 again represent the noise and noise-plus-signal voltages respectively at some time during the early build-up period. We know, then, that

$$V_2/V_1 = \sqrt{(\overline{V_n^2} + V_s^2)}/\sqrt{(\overline{V_n^2})}, \tag{30}$$

where $\overline{V_n^2}$ is the mean square noise voltage at input, and V_s is the signal voltage at input.

The advance in the oscillation envelope due to the signal is

$$\Delta t = t_1 - t_2 = \frac{1}{a}[\log_e(V_e/V_1) - \log_e(V_e/V_2)]$$

$$= \frac{1}{a}\log_e(V_2/V_1), \tag{31}$$

in which $1/a$ is the time constant of build-up and V_e is the peak amplitude of the oscillations, as before.

Now let f_q be the squegging frequency before the signal is applied and $f_q + \Delta f_q$ the frequency with the signal. Then

$$f_q = 1/T_q \quad \text{and} \quad f_q + \Delta f_q = 1/(T_q + \Delta t), \tag{32}$$

where T_q is the quench period with no signal. The change in quench frequency due to the signal is

$$\Delta f_q = 1/T_q - 1/(T_q + \Delta t) \doteqdot \frac{\Delta t}{T_q} f_q. \tag{33}$$

The mean anode current of the oscillator, due to a series of similar pulses of current, is proportional to the frequency with which these pulses arrive. The change in the current due to the presence of a signal is, consequently, proportional to the incremental quench frequency, Δf_q. So is the voltage across an impedance in the anode circuit of the oscillator. If we call this incremental voltage ΔV, then

$$\Delta V \propto \Delta f_q A, \tag{34}$$

where A is the area under the voltage-time envelope representing a single pulse at output. Substituting from equations (31) and (33) we get

$$\Delta V \propto \frac{f_q^2}{a} A \log_e (V_1/V_2), \tag{35}$$

and, for signals much greater than noise,

$$\Delta V \propto \frac{f_q^2}{a} A \log_e V_s/\sqrt{(\overline{V_n^2})}. \tag{36}$$

This has a similar form to equation (8) for the logarithmic mode. It shows that a self-quenching receiver is also logarithmic in operation. With the exception of the multiplying factor all the equations of §6·3, relating to frequency response, modulation distortion, etc., apply equally to a self-quenching receiver. In spite of the fundamental difference in the methods of operation, the properties of the two types of receiver are very similar. For this reason the self-quenching receiver is often chosen in preference to one that is separately quenched, for reasons of economy, when logarithmic operation is desired. By its nature the self-quenching receiver cannot be operated in the linear mode, for which purpose a separate quench oscillator is always necessary.

Chapter 7

AUTOMATIC GAIN-STABILIZATION

7·1. Introduction

The gain of a linear super-regenerative receiver depends exponentially upon the total circuit capacitance, the quiescent conductance and, approximately so, upon the voltages applied to the electrodes of the oscillator valve. Adjustment of the gain is consequently critical, and the receiver is very sensitive to fluctuations of supply voltage or other parameters, e.g. aerial loading.

It is often desirable to use an auxiliary circuit to stabilize the receiver sensitivity in the absence of signal and to provide automatic volume control when the signal is received. This process is known as automatic gain-stabilization (a.g.s.). The name is a little unfortunate, because it is the detector output voltage, and not the receiver gain, that is held nearly constant.

If a super-regenerative circuit is tuned by a variable capacitance the gain varies rapidly with tuning because of the relationship

$$N_s = 8{\cdot}7\, a^- / 2C \,\text{db}. \quad \text{(equation (43), Chapter 3).}$$

If some form of output stabilization is not used the manual gain control requires adjustment with each tuning change. In a search receiver, intended to cover a wide band of frequencies, a.g.s. is, therefore, essential.

The critical dependence of super-regenerative gain upon the circuit parameters makes it extremely difficult to adjust a super-regenerative receiver to have a reasonably high output, whilst still maintaining operation in the linear mode. There is a considerable risk of approaching limiting amplitude of oscillations with consequent transition to the logarithmic mode. If it is desired to maintain linear operation over a wide range of signal amplitudes, it is usually necessary to apply a.g.s.

In the logarithmic mode the relationship between gain and the relevant circuit parameters is roughly linear, and a.g.s. is, consequently, not necessary. The description here is confined to its application to receivers in the linear mode.

7·2. The principle of automatic gain-stabilization

Automatic gain-stabilization is used to maintain the receiver output relatively constant in spite of variations in the circuit parameters. This applies particularly in the absence of signal, in which case the a.g.s. circuits operate on circuit noise and the sensitivity is maintained at a constant level. When a c.w. signal is received, a powerful automatic volume control action is usually, but not necessarily, arranged to occur. The stabilization can be arranged to operate about a gain level pre-set by the receiver manual gain control.

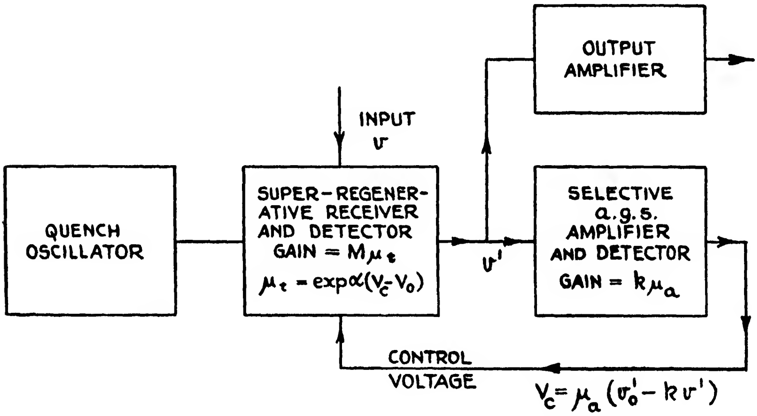


Fig. 7·1. Block diagram of super-regenerative receiver with a.g.s.

It was shown earlier (Chapter 4) that a considerable fraction of the output power from the detector following a super-regenerative receiver is at quench frequency and its harmonics. In order readily to obtain a high gain in the a.g.s. amplifier it is usual to tune it to quench frequency. That is not essential and in some cases is, in fact, a disadvantage. These cases are rare and are mentioned briefly later. Until then the description is confined to quench-tuned a.g.s. systems.

Various types of a.g.s. system are described in detail below, but, for the moment, the operation of a typical circuit is outlined with reference to a block diagram.

A block diagram of a typical a.g.s. system is given in fig. 7·1. The output from the detector, in addition to being fed to the audio- or video-output circuits, is also applied to the input of an amplifier tuned to quench frequency. The output from this amplifier is rectified and provides a d.c. control voltage. Provided that the amplifier and detector are arranged to operate on a linear portion of their combined characteristic, this voltage varies linearly with the input to the super-regenerative receiver, but in the opposite sense. The linear relationship is only strictly true if the input signal to the receiver retains the same frequency distribution (e.g. noise or c.w. signal), which is assumed, for the time being, to be so. The voltage obtained in this way is used to control the super-regenerative gain, in the appropriate direction, so that any tendency for the receiver output to change is strongly opposed. The control voltage may be applied in several ways. A usual way, and probably the simplest, is to provide bias to the grid of the receiver oscillator, thus changing the effective circuit conductance by altering the mutual conductance of the valve. Another method is to control the quench amplitude by arranging the a.g.s. bias level to determine the gain of a quench amplifier, from the output of which quench is supplied to the receiver.

7·3. Simple theory of automatic gain-stabilization

In either of the above methods of applying the voltage V_c to control the receiver gain, the relationship between this voltage and the gain in decibels may be regarded as linear over a restricted range. This is apparent on examination of the gain curves in Appendix 3, i.e.

$$N_t = \alpha(V_c - V_0), \tag{1a}$$

or

$$\mu_t = \exp\left[\alpha(V_c - V_0)\right], \tag{1b}$$

where N_t is the total receiver signal gain in nepers, μ_t is the total receiver signal gain expressed as a voltage ratio, and α represents the slope of the curve at the point about which stabilization is to occur.

Fig. 7·2 shows a typical curve and illustrates the meaning of α and V_0.

If we are considering the amplification of noise alone, the super-regenerative gain, measured as the ratio of *mean output* to r.m.s. noise input, is effectively less than μ_t, because some of the noise energy goes into producing fluctuations of the output. The amount of the reduction depends upon the probability distribution of amplitudes of the output pulses, which in turn depends upon the detector law. The factor by which it is necessary to multiply μ_t is 0·71 for a square-law detector and 0·88 for a linear detector, for reception of random noise alone. When a small signal is introduced and gradually increased, this factor itself increases, until it is approximately unity for signals greater than about four times noise.

In the following theory, the deviation of this factor from unity is neglected and the signal gain μ is used throughout. This does not affect the theory and involves little numerical error. The behaviour of the super-regenerative receiver with a.g.s. at low signal-to-noise ratios is discussed later when this factor is again mentioned.

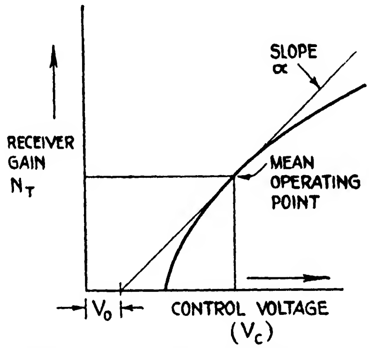


Fig. 7·2. Variation of receiver gain with control voltage.

The characteristics of the a.g.s. tuned amplifier and detector (usually linear) can be lumped together to give a curve such as that in fig. 7·3 for an input signal tuned to the peak of the frequency-response curve (i.e. to quench frequency). Here, again, the portion of the curve used is assumed to be linear,

$$V_c = \mu_a(v_0' - kv'), \tag{2}$$

where μ_a represents the amplifier gain at quench frequency. It is assumed that the band-width of the amplifier is small compared with quench frequency. v' is the *mean* amplitude of the output from the signal detector. μ_a and v_0' may be obtained from the characteristic curve, shown in fig. 7·3. The factor k requires some explanation. It has to be introduced because we are selecting only the quench frequency component of the receiver output, and it represents the ratio of the amplitude of the quench-frequency component of the output spectrum to the mean amplitude of the output pulses. It depends only upon the relationship between quench frequency and the band-width of the spectrum of the receiver output pulses.

The amplitude A_1 of the component frequency f_q in the output spectrum of a receiver in the slope-controlled state is obtained by substituting $n = 1$ in equation (27), §4·4·2. Thus

$$A_1 = A_0\left(\frac{f_q}{\sqrt{\pi}B}\right)\exp\left(-\frac{f_q^2}{B^2}\right), \tag{3}$$

where A_0 is the mean amplitude of the output pulses, and B is the width of the spectrum of a single pulse at $1/e$ of the peak.

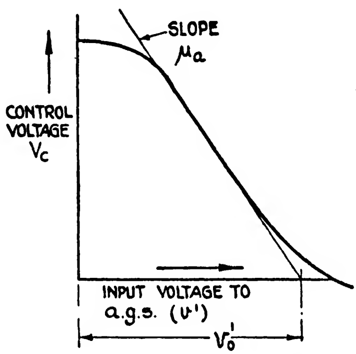


Fig. 7·3. Variation of control voltage with input to a.g.s.

The factor k is therefore

$$k = \left(\frac{f_q}{\sqrt{\pi}B}\right)\exp\left(-\frac{f_q^2}{B^2}\right). \tag{4}$$

But for a sinusoidal quench function

$$B = \tfrac{1}{2}b_s. \tag{5}$$

That is to say, the spectral width is one-half of the super-regenerative band-width.

If the ratio $f_q/B = \frac{1}{5}$, which is a typical value, then $k = 0{\cdot}11$.

For an input voltage v (which may represent either a c.w. signal or r.m.s. noise at input), the mean amplitude v' of the output pulses is given by

$$v' = M\mu_i v, \tag{6}$$

where M is a factor which includes the losses in the quiescent circuit and also the efficiency of the detector which is assumed to be linear. From equations (1), (2) and (6), the following relationship

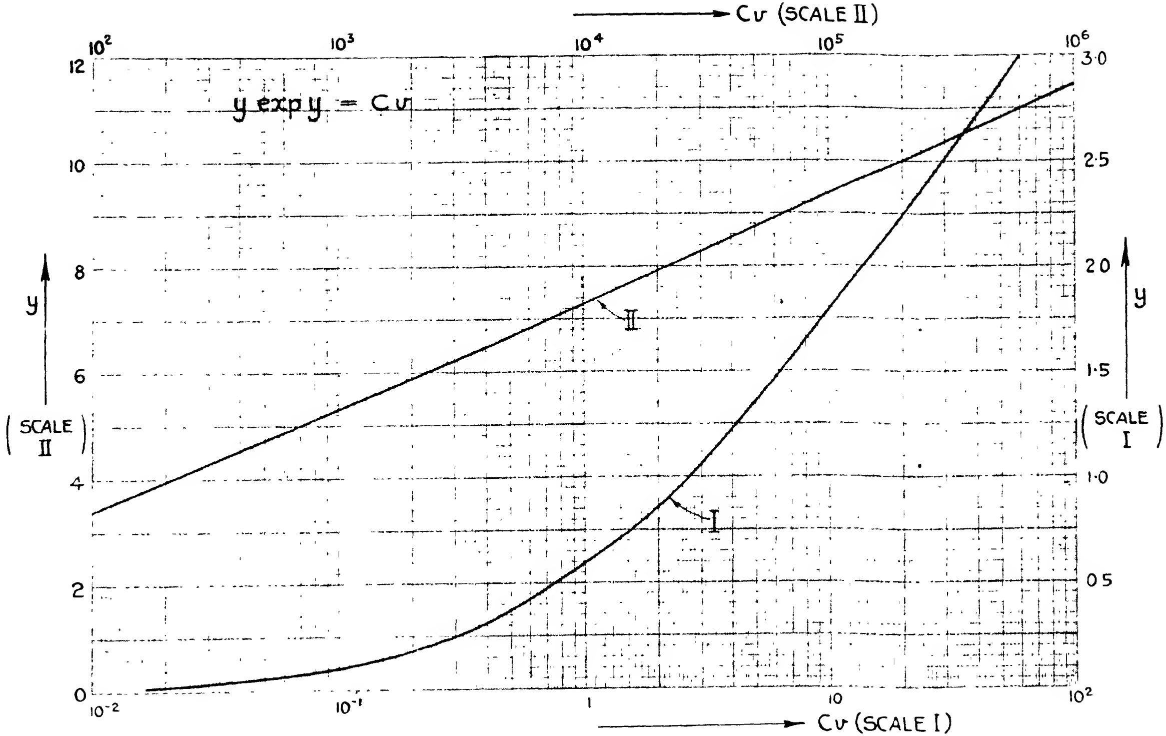


Fig. 7·4. Design curve for a.g.s.: linear detector.

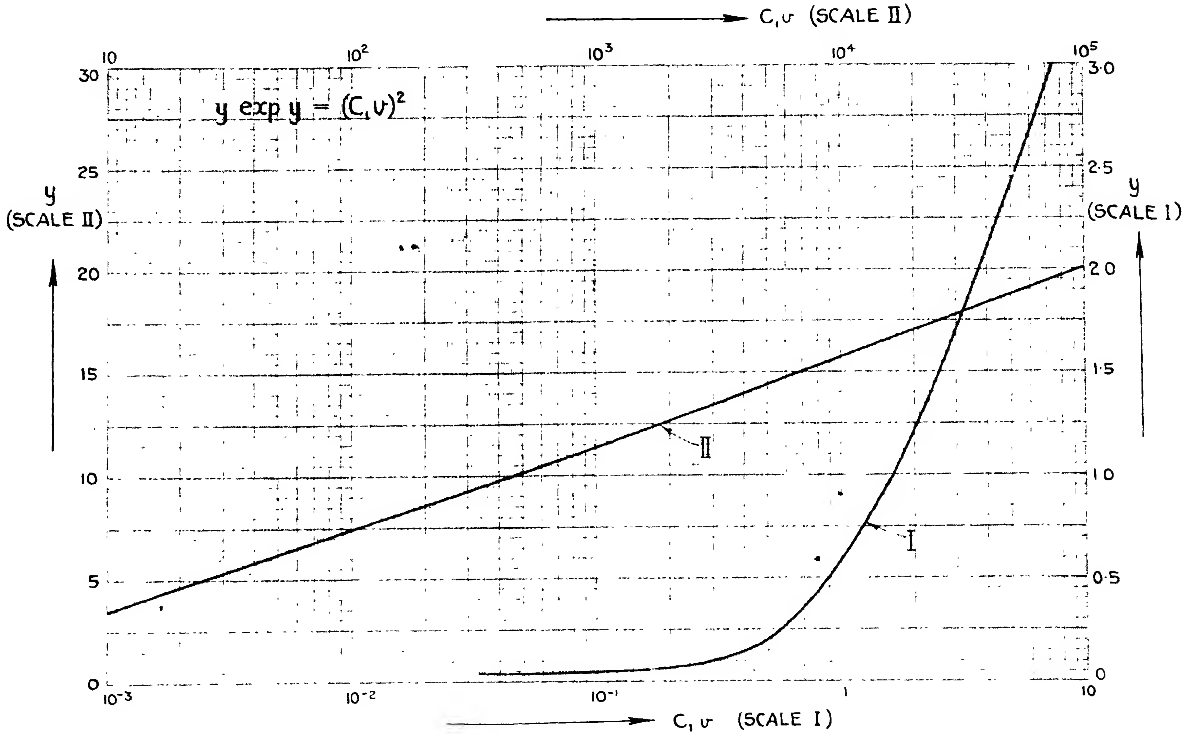


Fig. 7·5. Design curve for a.g.s.: square-law detector.

between the input and output voltages of the stabilized circuit is obtained:

$$v' = Mv \exp [\alpha(V_c - V_0)]$$
$$= Mv \exp \{\alpha[\mu_a(v_0' - kv') - V_0]\}.$$

This may be written

$$\alpha k\mu_a v' \exp (\alpha k\mu_a v') = M\alpha k\mu_a v \exp [\alpha(\mu_a v_0' - V_0)],$$

$$\left.\begin{aligned} &\text{i.e.} & y \exp y &= Cv, \\ &\text{where} & y &= \alpha k\mu_a v' \\ &\text{and} & C &= M\alpha k\mu_a \exp [\alpha(\mu_a v_0' - V_0)]. \end{aligned}\right\} \quad (7)$$

The expression in (7) only holds, of course, subject to the limitations of linearity of valve characteristics and truly exponential dependence of receiver gain on the control voltage V_c. That is closely enough true over a reasonable range in most practical circuits. y is plotted against Cv in fig. 7·4 over a wide range of values.

If the detector output is small the detector characteristic might be represented more nearly by a parabola through the origin. Equation (6) then becomes

$$v' = R(\mu_t v)^2, \quad (8)$$

where R is a constant depending upon the detector characteristic and on the voltage ratio of gain or loss in the quiescent circuit.

In this case equation (7) becomes

$$y \exp y = C_1^2 v^2,$$

where $y = 2\alpha k\mu_a v'$,

and $C_1^2 = R\alpha k\mu_a \exp [2\alpha(\mu_a v_0' - V_0)].$

Fig. 7·5 shows how y varies with $C_1 v$.

This simple theory demonstrates how the receiver output is maintained relatively constant in spite of large variations in circuit parameters and (in the relevant case) of output signal. The numerical example below demonstrates the order of magnitude of the changes involved.

7·4. Numerical example

To illustrate the application of the above theory, a practical example, based on the circuit on fig. 7·9, is given here.

Figs. 7·6 and 7·7 show the measured characteristics of an actual receiver. Fig. 7·6 gives the magnitude of the receiver gain and its

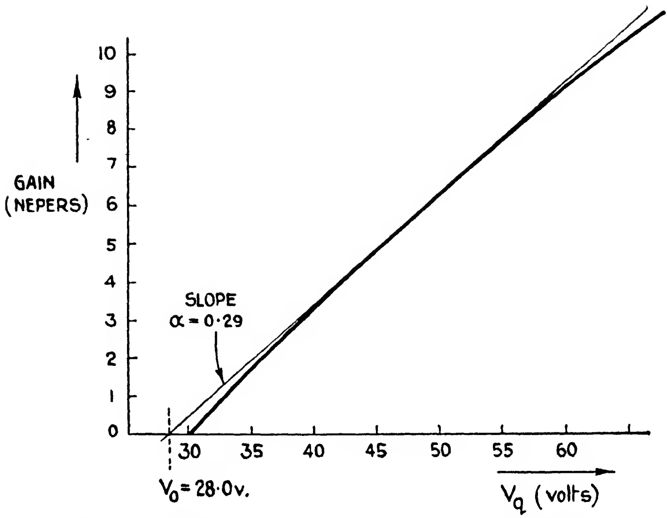


Fig. 7·6. Variation of receiver gain with quench amplitude. Practical example.

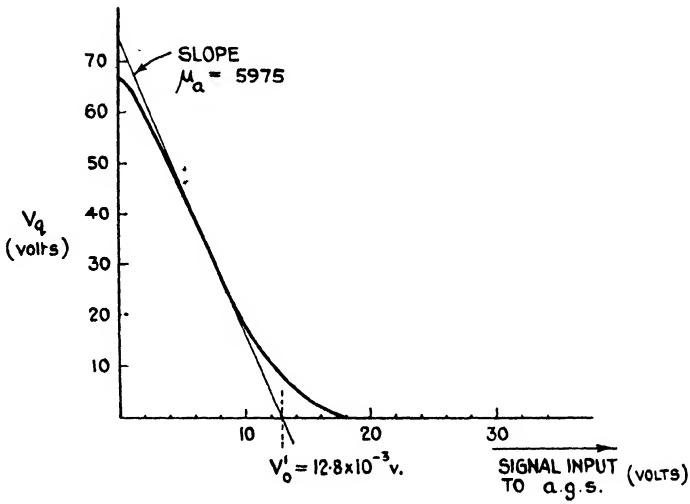


Fig. 7·7. Variation of quench amplitude with signal input to a.g.s. Practical example.

dependence on the quench amplitude, thus providing the constants α and V_0 for substitution in equation (1 *a*). The measured receiver gain includes the losses in the 'quiescent' circuit and the detector efficiency. The detector is assumed to be operating over the square-law portion of its characteristic. By removing the quench supply to the oscillator, these losses were measured independently, thus allowing that part (μ_t) of the gain depending on quench amplitude to be plotted alone in fig. 7·6.

Fig. 7·7 shows the variation of quench amplitude with the input to the a.g.s. circuit at quench frequency. This provides the values of the constants μ_a and v_0' for substitution in equation (2).

The following are the values given for these four constants by the receiver characteristics of figs. 7·6 and 7·7:

$$\alpha = 0{\cdot}29\,\text{nepers/V.}, \qquad v_0' = 12{\cdot}8 \times 10^{-3}\,\text{V.},$$
$$V_0 = 28{\cdot}0\,\text{V.}, \qquad \mu_a = 5975.$$

The values of v_0' and μ_a refer to a specified setting of the manual gain control, maintained throughout the measurements.

The following measured values are also required:

Quench frequency $f_q = 460$ kc./sec.,
Radio frequency $f_0 = 500$ Mc./sec.
Super-regenerative band-width $b_s = 4{\cdot}0$ Mc./sec.

We know that the width B of the frequency spectrum is equal to $\frac{1}{2}b_s$, so that

$$B = 2{\cdot}0\,\text{Mc./sec.},$$

and the ratio

$$\frac{f_q}{B} = \frac{460 \times 10^3}{2 \times 10^6} = 0{\cdot}23.$$

The ratio of the amplitude of the quench-frequency component of the output to the mean amplitude of the peaks of the output pulses is therefore

$$k = \frac{f_q}{\sqrt{\pi}B}\exp\left(-\frac{f_q^2}{B^2}\right) = \frac{0{\cdot}23}{\sqrt{\pi}}\exp[-(0{\cdot}23)^2] = 0{\cdot}123.$$

Let us assume first an input of noise only. In this case, for a square-law detector, it is known that half the noise energy of the output goes into the spectrum lines and half into the continuous spectrum (§4·4). For this reason the effective gain, measured as the ratio,

(mean amplitude of output)/(r.m.s. noise at input) is less than the signal gain by a factor $\sqrt{2}$. The final reduction factor is therefore

$$k/\sqrt{2} = 0{\cdot}087.$$

We can now substitute in equation (7) (putting $R = 1$ because it is included in the ordinates of fig. 7·7)

$$\begin{aligned} C_1^2 &= [(\alpha k \mu_a)/\sqrt{2}] \exp [2\alpha(\mu_a V_0 - v_0')] \\ &= 0{\cdot}087 \times 0{\cdot}29 \times 5975 \exp [2 \times 0{\cdot}29 (5975 \times 12{\cdot}8 \times 10^{-3} - 28)] \\ &= 151 \exp (28{\cdot}1), \\ C_1 &= 12{\cdot}3 \exp (14{\cdot}05) \\ &= 15{\cdot}2 \times 10^6. \end{aligned}$$

If the r.m.s. noise voltage in the tuned circuit is, say 10 μV., then

$$C_1 v = 152.$$

From fig. 7·5, the corresponding value of y is

$$y = 8{\cdot}0,$$

and the mean detector output voltage

$$v' = y/(2\alpha k \mu_a) = 8{\cdot}0/427 = 18{\cdot}7 \text{ mV}.$$

Now let us examine the effect of applying a signal 100 times greater than noise. In this case the factor $\sqrt{2}$, in the formula for C_1^2, disappears because the proportion of energy in the noise is now negligible. The new value of C_1 is thus

$$C_1 = 18{\cdot}1 \times 10^6.$$

For a signal of 1 mV, $\quad C_1 v = 18{\cdot}1 \times 10^3.$

From fig. 7·5, the corresponding value of y is

$$y = 17,$$

and the amplitude of the receiver output pulses is

$$v' = y/(2\alpha k \mu_a) = \tfrac{17}{427} = 39{\cdot}8 \text{ mV.},$$

compared with 18·7 mV. for noise alone.

This example is not intended to demonstrate a design procedure, but should be sufficient to indicate the essential factors in a.g.s. calculations. It is based on the theory of sinusoidal quench and the formulae for the slope-controlled state. The argument applies equally to any super-regenerative receiver operating in the linear mode, provided we use formulae appropriate to the state of operation.

7·5. Automatic gain-stabilization circuits

From the preceding paragraphs it is apparent that there are alternative ways of applying satisfactory a.g.s. In either of the two main types of linear super-regenerative receiver, namely, grid-quenched and anode-quenched, the a.g.s. bias voltage can be used to control: (i) the mean d.c. level of the oscillator grid, (ii) the mean d.c. level of the oscillator anode or (iii) the quench amplitude.

The method chosen for any particular purpose depends upon circumstances. For example, where economy in components is essential (iii) would not be chosen because it usually involves several extra tuned circuits.

A description is given of three representative circuits, and these should indicate clearly the application of the principles outlined above.

7·5·1. *Grid quench*

Fig. 7·8 shows a super-regenerative receiver with grid quench and a.g.s. controlling the oscillator grid bias. V_1 is the quench oscillator which provides quench voltage to the grid of the oscillator V_2 through an attenuating network. A diode detector is used, the output from which is amplified, if necessary, to give the desired receiver output level. The detector output is also fed to the grid of the first a.g.s. amplifier V_4, the anode circuit of which is tuned to quench frequency. The response curve is shown in fig. 7·10(*b*). The quench-frequency component of the output is thereby selected. The resulting voltage is rectified, and it is in this part of the circuit that the time constant of the a.g.s. system is determined according to requirements. V_6 is a d.c. amplifier with its load in the cathode circuit. Because of this, the a.g.s. time constant is increased by the Miller effect and becomes $\mu C_1(R_1+R_2)$, where μ is the working amplification factor of V_6. In this way the use of excessively large condensers is avoided even for very long a.g.s. time constants. The smoothed voltage obtained at the cathode of V_6 is used to provide bias for the grid of the oscillator V_2. Due to the fairly high mean voltage obtained at this point when V_6 is working on a suitably linear part of its characteristic, it is necessary to raise the voltage of V_2 cathode also. This is done by bleeding a small current from the high-tension line through a fairly large cathode load. The cathode circuit of V_2 is, in fact, a very convenient place for final adjustment of the a.g.s. circuit to the middle of its characteristic.

It is, of course, necessary to prevent direct leakage of quench to the a.g.s. circuits. For this reason it is advisable to include a screen between the oscillator and detector, and to keep the detector and a.g.s. circuits physically apart from the quench oscillator.

No provision is made for manual gain control on this circuit, but such provision could readily be made. The method of doing this is indicated in the description of the next circuit.

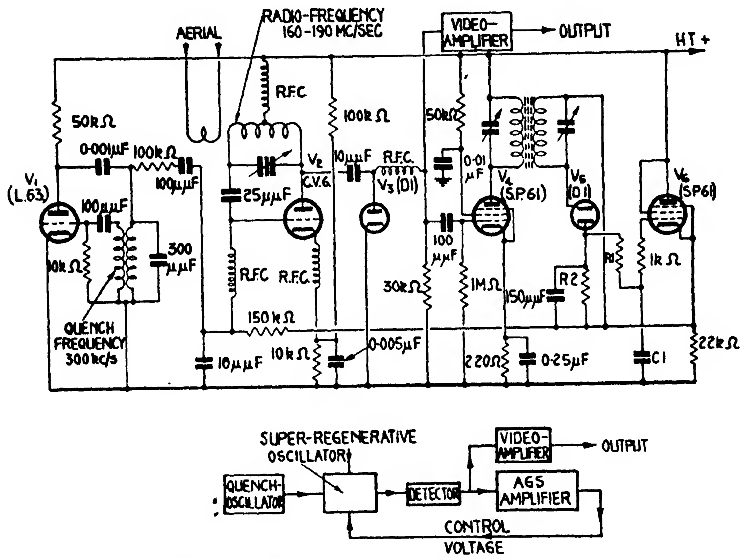


Fig. 7·8. A.g.s. circuit using grid-bias control.

A disadvantage of this type of a.g.s. circuit is the degeneration of the a.g.s. bias voltage necessarily obtained in the cathode load of the oscillator (V_2). This can be overcome by using an auxiliary power supply of good regulation (such as a battery) to provide the bias voltage for V_2.

7·5·2. *Anode quench*

Fig. 7·9 shows the circuit of a super-regenerative receiver using anode quench, the amplitude of which is determined by the a.g.s. circuit. The circuit values correspond to a radio frequency of about 500 Mc./sec. In this case, the video-output wide-band amplifier is also used as part of the a.g.s. circuit. To avoid complicating the diagram, all circuit details of this straightforward amplifier are

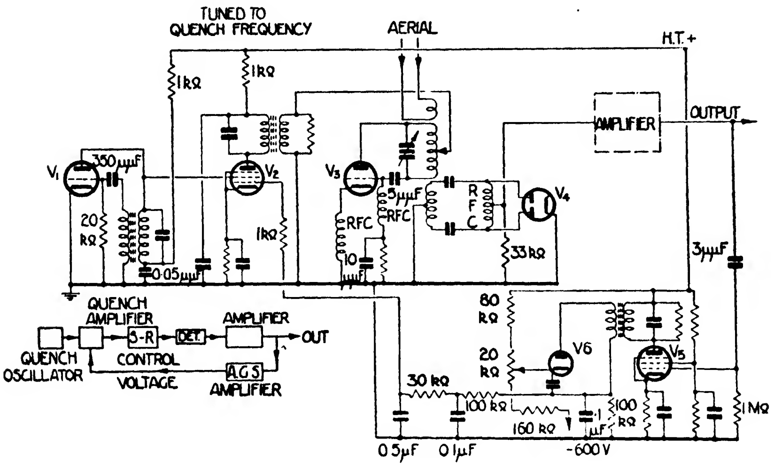


Fig. 7·9. A.g.s. circuit using quench amplitude control.

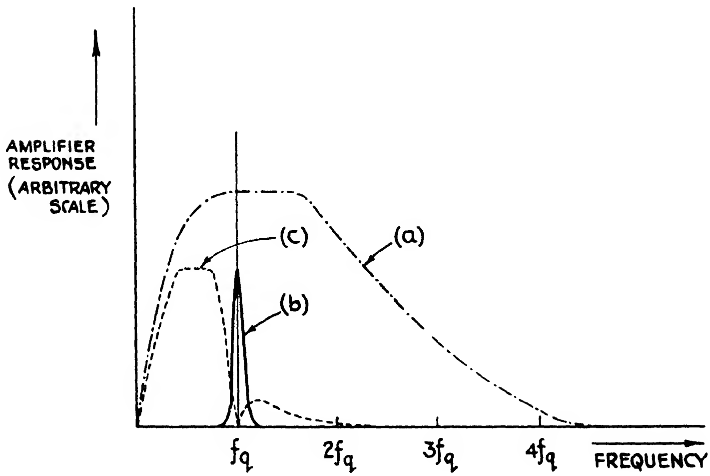


Fig. 7·10. A.g.s. amplifier frequency response. (*a*) Video amplifier of fig. 7·9. (*b*) A.g.s. amplifier of fig. 7·8. (*c*) Quench-rejection a.g.s. amplifier of fig. 7·11.

omitted. Its approximate frequency characteristic is shown in fig. 7·10(*a*).

The quench-frequency component of the detector output is amplified in the video amplifier and further in a quench-tuned circuit similar to that described in § 7·5·1. A diode rectifier is again used, but in this case the cathode potential of the diode is varied in order to provide manual gain control. The smoothed output from the detector is used as bias for the quench amplifier V_2. Quench voltage from the oscillator V_1 is applied to the screen of the quench amplifier, and the voltage appearing across the anode circuit is controlled by the a.g.s. bias on V_2 grid. The secondary of the transformer in the anode circuit of the quench amplifier provides quench directly to the super-regenerative receiver anode circuit.

As before, under normal working conditions, an equilibrium exists between the detector output voltage, the quench amplitude and the super-regenerative gain. The gain control serves to change the equilibrium conditions in the following way.

When the cathode of V_6 is at zero (earth) potential, the entire output from the a.g.s. amplifier V_5 is effective in controlling the receiver gain. When the cathode of V_6 is made positive to earth by adjusting the gain control, the output voltage of V_5 has to reach a certain level before it is effective as an a.g.s. control voltage, and the receiver gain necessary to maintain equilibrium is therefore greater than before. Thus, raising the positive voltage on the cathode of V_6 increases the receiver gain, whilst still maintaining the same degree of stabilization. If a considerable reduction of gain is desired (as it often is in practice), it is necessary to arrange for the cathode of the diode to be taken negative. When this is done the diode conducts and the grid of V_2 is biased back directly and the receiver gain reduced. In this way a gain control range of about 7 nepers (60 db.) can be readily obtained. It must be noted that the *positive* bias on V_6, when at maximum gain, must be less than the peak output of V_5, otherwise instability will result, due to the lack of a.g.s. control.

7·5·3. *Automatic gain-stabilization system not tuned to quench frequency*

A characteristic of an a.g.s. circuit tuned to quench frequency is the strong automatic-volume-control action obtained on a c.w. signal. That is because all the output energy from such a signal is at quench frequency and its harmonics. The effect can be avoided if required, whilst still providing adequate stabilization on noise, by using an a.g.s. system not tuned to quench frequency. Such a

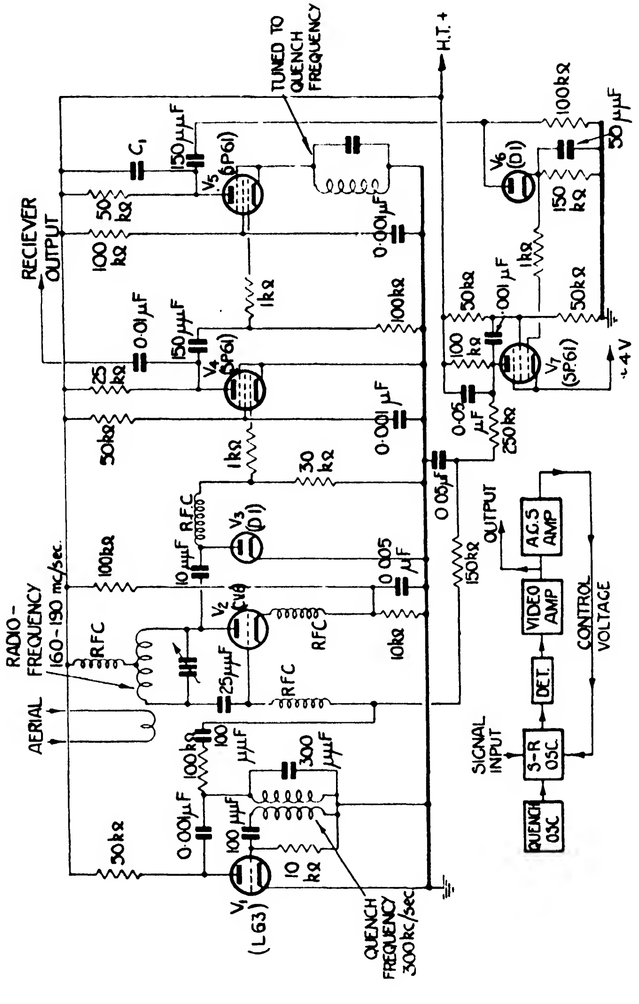


Fig. 7·11. A.g.s. system not tuned to quench frequency.

circuit is shown in fig. 7·11, which differs from fig. 7·8 only in the a.g.s. part of the circuit. A valve V_4 is included to provide extra amplification to compensate for the a.g.s. gain sacrificed when a sharply tuned stage is not used. This valve also serves as an amplifier for the output signal. The amplifier V_5 has a quench rejector in the cathode circuit, and it is this stage that determines the response curve of the a.g.s. system. The nature of this response is shown in fig. 7·10(*c*). It can be seen that the circuit, whilst rejecting any input at quench frequency, accepts some energy at the second harmonic. It is necessary to do this to ensure stability when a c.w. signal is received, by providing some input to the a.g.s. circuit under these conditions, when all the output energy is at quench frequency and its harmonics.

The amount of energy accepted at the second harmonic of quench frequency is conveniently determined by the value of the condenser C_1, which serves, in conjunction with the normal frequency cut-off of the preceding amplifier, to allow a small but controlled amount of energy at this latter frequency to pass.

The subsequent a.g.s. detector and d.c. amplifier (V_6, V_7) are variants of those shown in fig. 7·8. The diode is inverted and the output taken from the anode instead of the cathode of V_7. Smoothing and the determination of the a.g.s. time constant are carried out in the anode circuit of V_7, and the d.c. bias voltage is supplied to the oscillator grid to control the receiver gain.

Chapter 8

SUPER-REGENERATIVE CIRCUITS

8·1. Introduction

The super-regenerative circuit was first described by Armstrong (1922). Some of the limitations associated with the logarithmic mode, such as distortion and excessive band-width, were soon evident, and the circuit received little attention until the early 1930's. In the few years before the war there was a good deal of literature* on the subject, mainly concerned with attempting to explain the phenomenon of super-regeneration (e.g. Ataka, 1935; Scroggie, 1936; and Frink, 1938), or with describing a new super-regenerative circuit (e.g. Lewis and Milner, 1936; Becker and Leeds, 1936). During this period the super-regenerative circuit came into widespread use by radio amateurs as an economical ultra-short-wave communications receiver. Descriptions of suitable circuits of self-quenching and logarithmic receivers were (and are still) common in the literature. Typical examples are given in §8·2.

The advantages of light weight and economy associated with the super-regenerative circuit were realized by the U.S. Army Signal Corps who, in 1933–6, developed a 'walkie-talkie' communications receiver on this principle. Similar receivers were used during the 1939–45 war.

The most important application of the super-regenerative receiver came with the advent of war. It was suggested, in 1938, that a super-regenerative receiver might be used as a pulse responder for receiving a radar pulse and transmitting (by virtue of its oscillatory nature) a pulse of greater strength for the purpose of identifying an aircraft or ship to which the responder was fitted. I.F.F.† responders were designed on this principle using an ingenious feed-back trigger circuit due to F. C. Williams. The addition of a.g.s. to the later versions of the responder eliminated the need for manual adjustment. The resulting fully automatic responders (I.F.F. Mark III and its variants) were fitted to every Allied ship and aircraft. More than 200,000 super-regenerative

* See Bibliography.
† Identification, Friend or Foe.

I.F.F. responders were produced in this country and in the U.S.A. The successful mass production of this enormous number of super-regenerative receivers was carried out within close tolerances (e.g. ± 5 db. on absolute sensitivity over a 30 Mc./sec. band). This should destroy the reputation for instability and unpredictable performance, always held by super-regenerative receivers. There is now no doubt that the super-regenerative receiver has earned the right to be considered for a given application on the same basis as the tuned radio-frequency or superheterodyne receiver.

The I.F.F. responder circuit was also used successfully in a large number of early radar beacons and in the paratroop beacon, known as 'Eureka'.*

It is interesting that the Germans also used a linear receiver with a.g.s. in one of their wartime radar equipments (§8·3·4).

The widespread use of the super-regenerative circuit in wartime had the inevitable effect of furthering understanding of the principle and of reawakening interest in its application to other problems. It also called attention to the possibilities of the linear mode, hitherto unexplored. The use of increasingly high radio-frequencies favours the use of super-regeneration for several reasons. The high ratio of radio-frequency band-width to the permissible range of modulation frequencies (§5·4·7) becomes much less of a disadvantage, because the excess band-width is often necessary to take up fluctuation in the frequency of system components such as the transmitter or the receiving oscillator. The comparison between a super-regenerative receiver and a tuned radio-frequency or superheterodyne receiver is increasingly favourable to the former because the efficiency of radio-frequency amplifiers is greatly reduced at higher frequencies.

Interest is also awakening in the use of super-regenerative circuits for frequency-modulation receivers. Loughlin (1947) has published a circuit of one broadcast receiver of this type, the Fremodyne,† which has been adopted for commercial use. This and other circuits for frequency modulation are discussed in §8·4.

* For further details see K. A. Wood, *J. Instn Elect. Engrs*, **93**, IIIA, 481 (1946).

† See Bibliography.

8·2. Super-regenerative reception of amplitude-modulated continuous wave signals

8·2·1. *Introduction*

The simplest application of the super-regenerative principle is to receivers for communication, in which the incoming signal is amplitude-modulated at audio-frequencies lying within a more or less restricted range. The self-quenching circuit, in spite of its attendant distortion, has frequently been used for this purpose; so has the separately quenched receiver operated in the logarithmic mode.

The linear receiver is, however, preferable where it is important to reproduce the modulation envelope faithfully. It is usually necessary to include an a.g.s. circuit to ensure consistent operation in the linear mode, and this adds to the complication and expense of the circuit. A radio-frequency amplifier should always precede the super-regenerative stage, to prevent radiation of the super-regenerative oscillations by the aerial, but it is by no means always included in practice. By the time these additions have been made the super-regenerative receiver approaches the superheterodyne in complication. A discussion of the relative advantages of the two types of receiver is included in §§ 5·4·7 and 4·5.

We have been concerned so far with super-regenerative radio-frequency amplifiers. It is possible, however, to make a superheterodyne receiver with a super-regenerative intermediate frequency amplifier and second detector. Circuits of this type are described below.

When considering the use of super-regenerative circuits for the reception of modulated signals it is necessary to bear in mind the fundamental limitation imposed upon the upper value of the modulation frequency by the interruption, at quench frequency, of the output oscillations. For average quality transmission of audio-frequencies the highest modulation side-band is of the order of 5 kc./sec. Thus the minimum usable quench frequency is about 20 kc./sec., *irrespective of the radio-frequency*. Because of the more or less fixed relationship between limiting quench and radio-frequencies (see Chapter 5) the minimum radio-frequency that can be used under these circumstances is about 10 Mc./sec. (30 m.). It is generally true that the super-regenerative receiver is only useful for communication on frequencies higher than this. It is only on *much* higher frequencies (e.g. 1000 Mc./sec.), where the efficiency of conventional radio-frequency amplifiers is very poor, that the use of the super-regenerative receiver becomes advantageous in terms of performance as well as economy.

8·2·2. *Self-quenching receivers*

The operation of the self-quenching receiver is described in Chapter 6. The circuit of a receiver of this type is given in fig. 8·1. The component values in the super-regenerative oscillator, so far as they are given, correspond to a radio-frequency of the order of 100 Mc./sec., and a no-signal quench frequency in the region of 30 kc./sec.

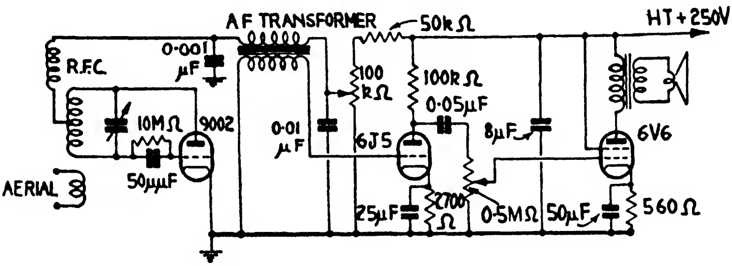


Fig. 8·1. Self-quenching receiver.

The signal from the aerial is coupled into the oscillator tank circuit L, C, chosen to tune over the desired range of frequencies. The oscillator is a modified Hartley circuit commonly used at high frequencies. Self-quenching is caused by the 50 $\mu\mu$F. grid condenser and 10-megohm grid leak. The grid leak is returned to a high potential in order to facilitate the initiation of squegging and to obtain a high self-quench frequency with a relatively long nominal time constant in the grid circuit.

The audio-frequency component of the oscillator anode current develops a voltage across the primary of an audio-frequency transformer. The resulting signal in the secondary is amplified by a conventional audio-frequency amplifier.

Control of super-regeneration is effected by varying the high-tension voltage supplied to the oscillator. At low voltages the mutual conductance of the valve is not sufficient to cause oscillations to build up. At high voltages the circuit may pass from squegging into a state of continuous oscillation, or, at least, a state of coherence in which each pulse of oscillations builds up from the tail of the one preceding it. Between these extremes there is a range of anode voltages over which self-quenching takes place correctly, and over which the receiver is sensitive to incoming signals. This state is recognized by a characteristic hiss in the loud-speaker.

In order to prevent radiation of the self-oscillations, and consequent interference with neighbouring equipment, it is desirable to use a radio-frequency amplifier as a buffer between the aerial and the super-regenerative stage. A suitable arrangement is shown in fig. 8·2. In this case the aerial is coupled to the tuned-grid circuit

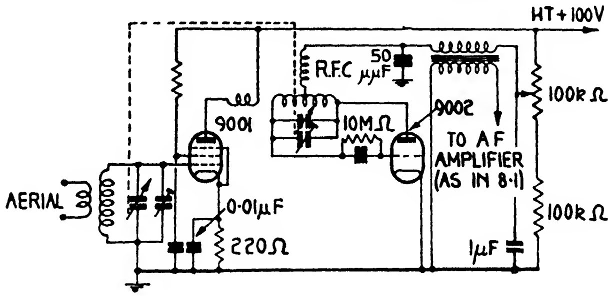


Fig. 8·2. Self-quenching receiver with radio-frequency stage.

of the radio-frequency amplifier. The signal is conveyed to the super-regenerative circuit by an aperiodic coupling in the anode circuit of the radio-frequency stage. We may summarize the benefits due to the addition of the radio-frequency stage as follows:

(i) It prevents radiation of the self-oscillations.

(ii) It usually gives some improvement in signal-to-noise ratio.

(iii) It may enable the notoriously poor selectivity of the self-quenching receiver to be improved in the tuned-grid circuit of the radio-frequency stage.

The disadvantages are:

(i) Added complication and expense.

(ii) It destroys what is probably the greatest single advantage of the super-regenerative receiver, namely, single-circuit tuning, by the addition of a second tuned circuit. (*Note:* This can be avoided by the use of a grounded-grid triode. An example is given in §8·2·4.)

The self-quenching receiver is not particularly critical in adjustment, although it is sometimes difficult to achieve the desired squegging frequency. Some idea of the quench frequency at which to aim may be obtained from the criteria for the slope-controlled linear mode, given in § 5·4·5. The values of the grid condenser and leak to produce interruption at that frequency are usually a matter for experimental determination.

This type of receiver is a convenient and economical way of reproducing a high radio-frequency signal, provided that the attendant distortion and lack of selectivity can be tolerated.

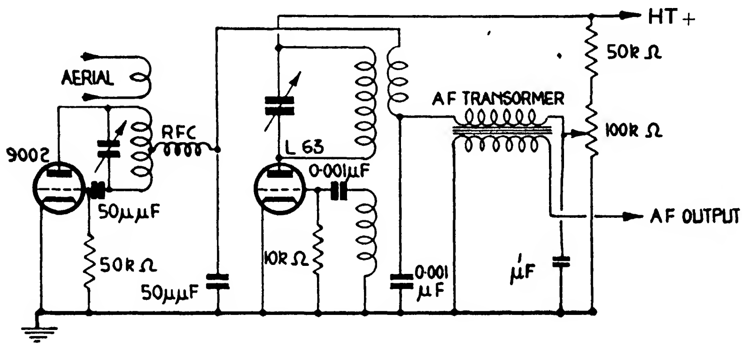


Fig. 8·3. Super-regenerative receiver: logarithmic mode.

8·2·3. *Receivers operating in the logarithmic mode*

There is nothing to choose in performance between a self-quenching receiver and a separately quenched receiver operating in the logarithmic mode. The provision of separate quench does, however, make the initial adjustment of the receiver much easier, because the amplitude and frequency of the quench are independent of the characteristics of the super-regenerative oscillator itself. For this reason a representative circuit is included here in fig. 8·3. It differs from that of fig. 8·1 only in the quenching arrangements. The oscillator grid leak is reduced in value and returned to earth, so that the oscillator is no longer self-quenching. A separate quench oscillator provides sinusoidal anode quench about a variable mean anode voltage. The potentiometer which supplies the anode voltage acts as the super-regeneration control as in the case of the self-quenching receiver.

Fig. 8·4 shows a hybrid circuit in which the super-regenerative oscillator acts as its own quench oscillator by virtue of a separate feed-back circuit at the quench frequency. This circuit is more controllable than that using the relaxation type of self-quenching, but is more expensive in components.

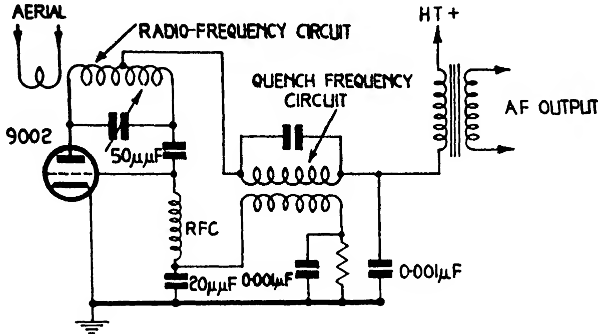


Fig. 8·4. Alternative arrangement: self-quenching.

8·2·4. *Linear receiver with automatic gain-stabilization*

When reasonably faithful reproduction is desired from a super-regenerative receiver it is necessary to maintain it in the linear mode. This mode has the added advantage of better selectivity and offers much greater scope to the designer because of its extreme flexibility. These advantages are offset by the necessity to use a.g.s.; at least where it is necessary to tune over an appreciable band of frequencies. In a serious attempt to compete with other types of receiver, on a basis of performance alone, the designer must inevitably employ the linear mode.

A circuit of a linear receiver with a.g.s. is given in fig. 8·5. No serious attempt at valve or component economy has been made, but the circuit illustrates the essentials of linear receiver design. The signal is injected into the cathode circuit of a grounded-grid triode amplifier, V_1, the anode of which is aperiodically coupled into the tank circuit of the oscillator valve V_2. This arrangement has the advantage of providing radio-frequency amplification and preventing radiation whilst retaining the desirable property of single-circuit tuning. The super-regenerative oscillations are rectified by V_3, but the time constant of the cathode circuit is short enough to pass the quench-frequency component of the rectified output, as well as the modulation components. V_4 amplifies both these components, but, thereafter, they are separated. The audio-frequency components in the anode voltage of V_4 pass to V_5, the power amplifier, and thence to the loud-speaker. The quench-frequency components are attenuated, in this channel, by the choke and condenser filter, but are amplified by V_8, which has a quench-tuned transformer in its anode circuit. The quench-frequency voltage thus obtained is rectified and smoothed to provide the control voltage for automatic

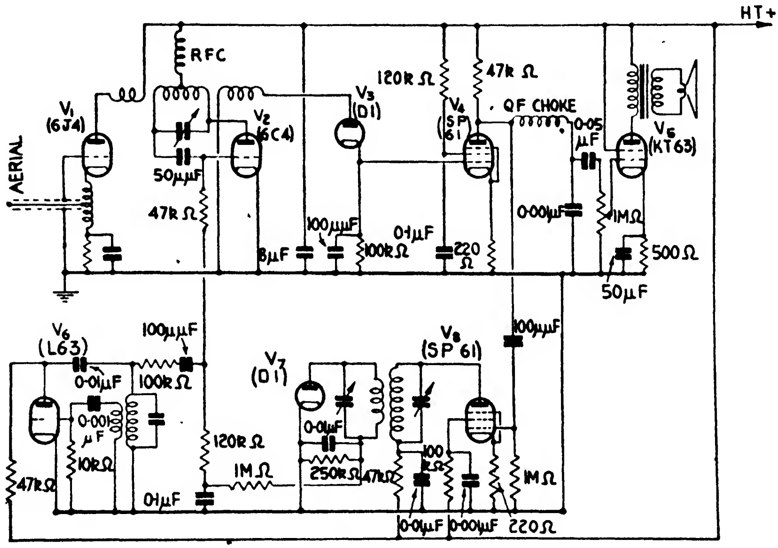


Fig. 8·5. Linear receiver with a.g.s.

gain stabilization. The a.g.s. time constant is about $\frac{1}{10}$ sec., which is usual for automatic volume control systems for audio-frequency modulated signal reception. Both the a.g.s. control voltage and the sinusoidal quench voltage from the quench oscillator V_6 are injected via the oscillator grid leak.

8·2·5. *The super-regenerative superheterodyne*

A super-regenerative stage may be used in place of the more conventional intermediate-frequency amplifier and second detector in a superheterodyne receiver.

By operating the super-regenerative circuit at a lower frequency the selectivity is improved, and radiation of super-regenerative oscillations by the aerial is no longer a problem. A block diagram of the arrangement is shown in fig. 8·6 and a suitable circuit arrangement in fig. 8·7, which refers to a receiver designed for a frequency of about 100 Mc./sec. The self-quenching super-regenerative intermediate-frequency amplifier and second detector operates on the intermediate frequency of 25 Mc./sec., the mean self-quenching

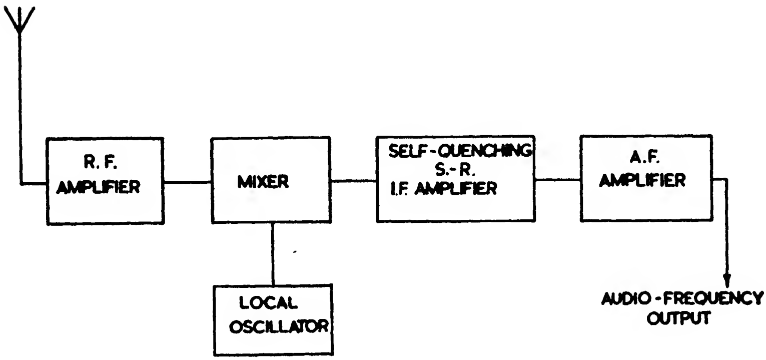


Fig. 8·6. Super-regenerative superheterodyne receiver: block diagram.

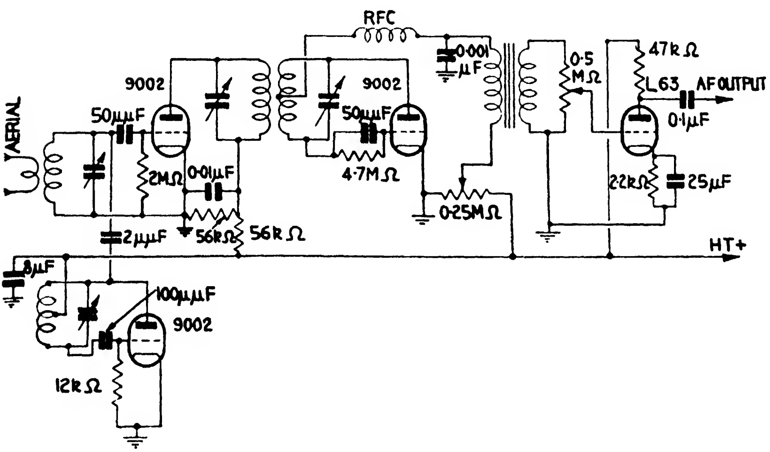


Fig. 8·7. Super-regenerative superheterodyne receiver.

rate being about 50 kc./sec. Super-regeneration is controlled by a potentiometer which alters the anode supply voltage to the super-regenerative stage.

This type of receiver is only useful for frequencies of 100 Mc./sec. and above. On lower radio-frequencies the intermediate frequency is also necessarily lower and the corresponding self-quenching rate too low for adequate reproduction of audio-frequency modulation.

8·2·6. *A super-regenerative receiver-transmitter for two-way communication*

The radiation of self-oscillations by a super-regenerative receiver is not always a disadvantage. Lewis and Milner (1936) have shown that it is possible to use a pair of super-regenerative receivers for simultaneous two-way communication over considerable distances, by amplitude-modulating the quench oscillators and synchronizing the quench frequencies.

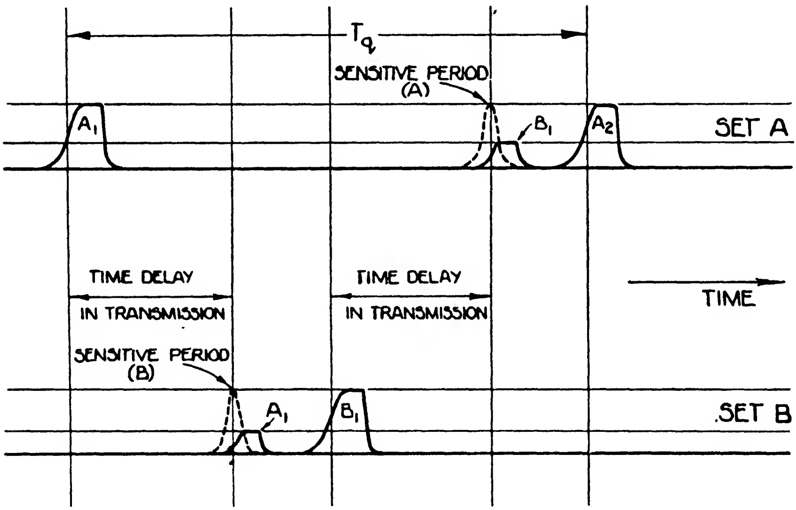


Fig. 8·8. Time relationships of transmission and reception in two-way communications by the method of Lewis and Milner.

The principle of operation is as follows. Slight modulation of the quench oscillation of a super-regenerative receiver, operated in the logarithmic mode, results in width modulation of the output pulse of oscillation.

If the quench oscillators of two receivers are operated on a common frequency it is possible to adjust the phase so that the radiated pulse from one receiver arrives during the sensitive period of the other, and vice versa. Not only that, but the beginning of the received pulse, where the modulation is effective, must lie within the sensitive period. For sensitive reception, therefore, the time relationship of transmission and reception must be as shown in fig. 8·8.

A pulse A_1 is transmitted from set 'A' and received by set 'B', some distance away, at a later time. The phase of the quench is so arranged that the rising edge of the pulse from A arrives during the sensitive period of set B. The pulse B_1, which builds up from the sample taken during this sensitive period, is transmitted and arrives

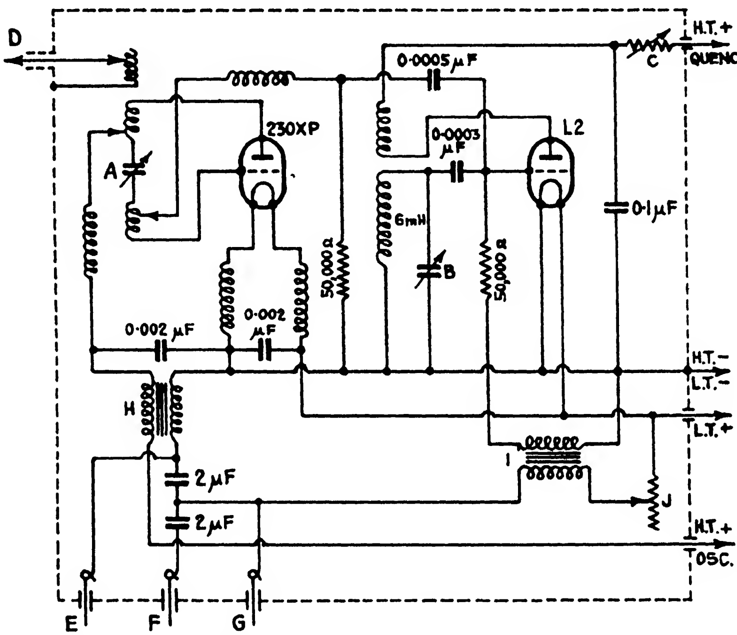


Fig. 8·9. Super-regenerative receiver transmitter (Lewis and Milner).

at A correspondingly later. Here it also arrives during a sensitive period if the quench frequency (and therefore the quench interval T_q) is chosen correctly. In this way modulation of the quench amplitude in either set is heard in the other.

It is clear that two-way communication is only possible with certain quench frequencies. Lewis and Milner found, using an experimental receiver on 3 m., that any quench frequency between 50 and 100 kc./sec. could be used for distances up to 200 yards between the sets. Beyond that distance the quench frequency had to be chosen most carefully, so that the beginning of each received pulse arrived during a sensitive period (as in fig. 8·8).

The circuit diagram of the receiver-transmitter is given in fig. 8·9. It is a conventional super-regenerative oscillator with grid quench applied from the tuned-grid circuit of the quench oscillator. The latter is, however, voice-modulated by a microphone coupled through the microphone transformer *I*. The mean quench amplitude is adjusted by means of the variable resistance *C* in series with the high-tension. This acts as a super-regeneration control. The received audio-frequency signal is obtained from transformer *H*, in series with the high-tension supply to the super-regenerative oscillator, in the manner described in §8·2·3 with reference to fig. 8·3.

In operation the quench oscillators of the two sets lock in synchronism. The mechanism of locking is complicated but depends upon suitable coupling between the quench and super-regenerative oscillators. Components of the received signal, at quench frequency and its harmonics, must be able to appear in the grid circuit of the quench oscillator, to act as a synchronizing signal.

8·3. Super-regenerative pulse receivers

8·3·1. *Introduction*

There are special problems in the use of a super-regenerative circuit for the reception of pulses of radio-frequency oscillation. The intermittency of the sampling process causes the output of the receiver to resemble the input less and less as the pulse duration is reduced. The quench frequency at which the signal is sampled can be no higher than that determined by the onset of coherence (§3·2). We shall see that the pulse must last for several quench periods if effective reproduction of the original envelope is desired.

Fig. 8·10 shows the time relationship between the received pulse and the output from a super-regenerative receiver. Only that part of the pulse which coincides in time with the sensitive period produces an effective output. The least possible delay occurs when the leading edge of the pulse arrives at the beginning of this period. Then the pulse at output is delayed beyond the signal pulse by a time corresponding to the build-up period T_b, the interval between peak sensitivity and peak output. The delay can never be less than T_b.

If the pulse arrives just after a sensitive period, almost a whole quench period elapses before the next sample is taken. Thus the delay is almost $T_q + T_b$.

The time relationship between received pulses and cycles of quench is random. The delay in the output pulses relative to the input therefore varies according to the instant of the quench cycle at which the pulse arrives. There is, consequently, an uncertainty of amount T_q in the time at which the output pulse begins. The 'jitter' on the receiver output is a fundamental disadvantage of the use of the super-regenerative circuit for receiving pulse signals.

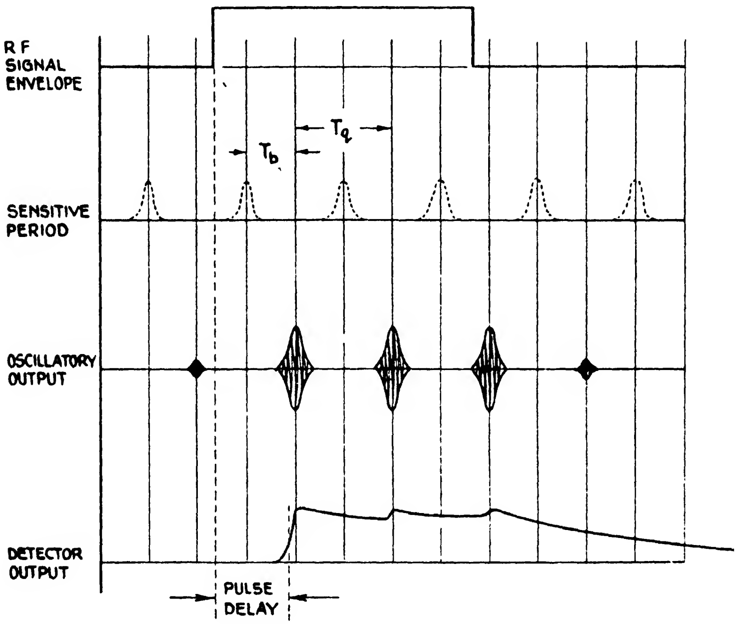


Fig. 8·10. Super-regenerative reception of radio-frequency pulse.

An incoming pulse which lasts for less than one quench period only produces an output when it coincides with a sensitive interval. The percentage of pulses in a very large number which produce more than a given amplitude at output depends on the relative durations of the signal pulse, the quench period and the effective sensitive period at the particular signal amplitude. It is apparent that the duration of the incoming signal must be greater than the quench period, T_q, if it is desired to produce an output pulse corresponding to every pulse received.

8·3·2. *Identification-friend-or-foe responders*

The purpose of an I.F.F. responder is to receive a radar pulse and to transmit, usually on the same frequency and with as little delay as possible, a pulse of much greater power and of distinctive form. In this way the ship or aircraft carrying the responder is identified as friendly to the remote, interrogating radar station. For airborne use it is essential that the responder be compact and economical and, for this reason, the super-regenerative receiver, capable of a very high gain in a single stage, is particularly suitable. We have seen that the output signal from a super-regenerative receiver takes the form of a series of short pulses of oscillation at the resonant frequency of the tank circuit. Thus the super-regenerative receiver retransmits for a short period on each quench cycle. It was originally hoped that the amount of retransmission might be adequate to permit the use of a super-regenerative receiver alone as a complete responder. Early experiments showed, however, that the amount of retransmission was hardly enough to ensure reliable responses from an I.F.F. equipment of this type.

It is usually possible to drive the oscillator valve used in a super-regenerative receiver to provide a much greater power output than that occurring in the normal course of super-regenerative reception, even of large signals. A great improvement is therefore made when the receiver output is detected, amplified and fed back to the oscillator valve, thus driving the valve to provide a transmitted pulse of adequate power, without the use of an elaborate modulating circuit. A circuit on these lines, due originally to Williams, has been the basis of all the I.F.F. responders used by the Allies during the late war.

8·3·3. *The basic responder circuit*

The circuit which forms the basis of all the super-regenerative responders is given in fig. 8·11 (*a*). The block diagram of fig. 8·11 (*b*) illustrates the operation of the circuit. A pulse signal from the aerial is developed across the tank circuit of a signal-frequency oscillator supplied with grid quench from a separate quench oscillator. The output of this super-regenerative receiver is rectified and amplified and fed back to the grid of the super-regenerative oscillator, thus increasing the mutual conductance of the valve and causing oscillations to build up further. The process is regenerative, and the oscillations quickly reach a limiting amplitude determined by the amount of power available under the conditions of drive. The oscillations are transmitted as a reply signal. The duration of the transmitted pulse depends upon the parameters in the feed-back circuit. The mechanism of its determination is discussed below with reference to the details of the circuit of fig. 8·11.

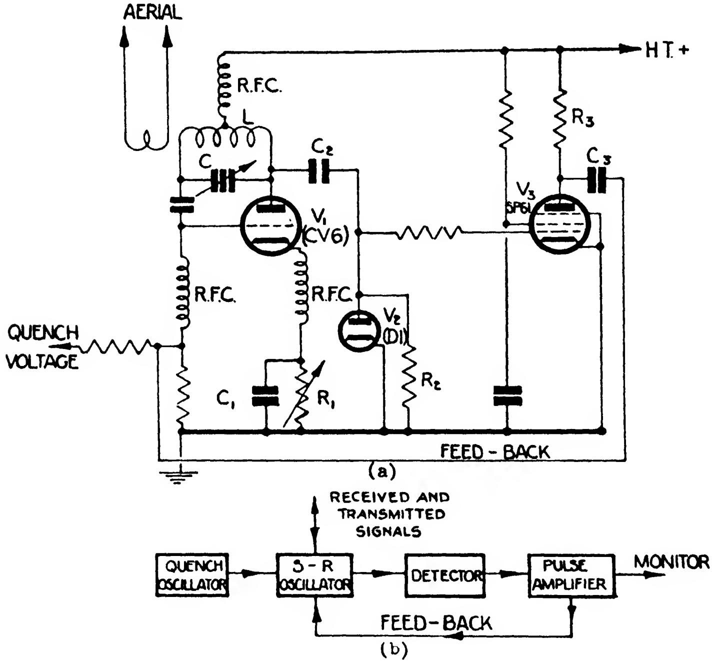


Fig. 8·11. Super-regenerative responder.
(*a*) Basic circuit. (*b*) Block diagram.

The super-regenerative oscillator in fig. 8·11 is a triode valve connected in a commonly used high-frequency circuit of the Hartley type which relies on the internal capacities of the valve for its operation. The value of the variable cathode-bias resistance R_1 is adjusted so that oscillation is prevented in the absence of quench. A sinusoidal quench voltage is applied to the grid of the valve. Its amplitude is such that weak self-oscillations in the circuit LC build up from noise on each quench cycle, when no signal voltage is present. Let us assume for the present that the feed-back link including C_3 is absent. The oscillations in the circuit are rectified by the diode V_2 and amplified by the pentode V_3. The amount of fluctuation on the anode of V_3 is therefore determined by the setting of the gain control resistance R_1 in the cathode circuit of the oscillator.

A pulsed radio-frequency signal arriving at the aerial is developed across the tank circuit LC. On the first quench cycle to follow the arrival of the pulse, self-oscillations build up from the level of the signal and to a proportionately greater amplitude. When the feed-back is connected, however, the amplified receiver output is applied to the grid of the oscillator. There, before producing any effect, it must overcome a bias voltage which is developed across the cathode bias resistance R_1. R_1 is adjusted so that the signal voltage must be several times noise in order to overcome this bias. When a large pulse signal arrives oscillations build up on the first quench cycle to follow its arrival. Owing to the regenerative circuit, these continue to build up instead of being damped out in the usual way. This is the start of the trigger action of the circuit and oscillations build up rapidly due to the increased drive from the pulse amplifier V_3. The triggering is necessarily arranged to occur when the oscillation amplitude is well below the level of saturation. Thus the properties of the receiver are determined when it is operating in the linear mode. Although regeneration ceases as soon as the amplifier V_3 is biased beyond cut-off, oscillations continue to build up because the oscillator V_1 has been brought well within its oscillating regime by removal of the negative grid-bias. The grid is, in fact, driven positive, and grid current flows into the feed-back condenser C_3. The negative bias on the oscillator is gradually increased by grid current entering C_3. Simultaneously the cathode current of V_1 charges the cathode condenser C_1 and the cathode voltage rises. After a time, depending upon the various circuit parameters involved, oscillation can no longer be maintained and the transmitted pulse ends.

The events which occur after the oscillation has stopped are important because they determine the length of time that must elapse before reception is again possible. Fig. 8·12 illustrates the behaviour of the various parts of the circuit both during and after the transmission. As a result of the high peak voltage generated across the tank circuit LC during transmission, C_2 is charged up to a high voltage which has to decay to a small fraction of a volt before current in the amplifier valve V_3 is back to its quiescent value. In reinstating

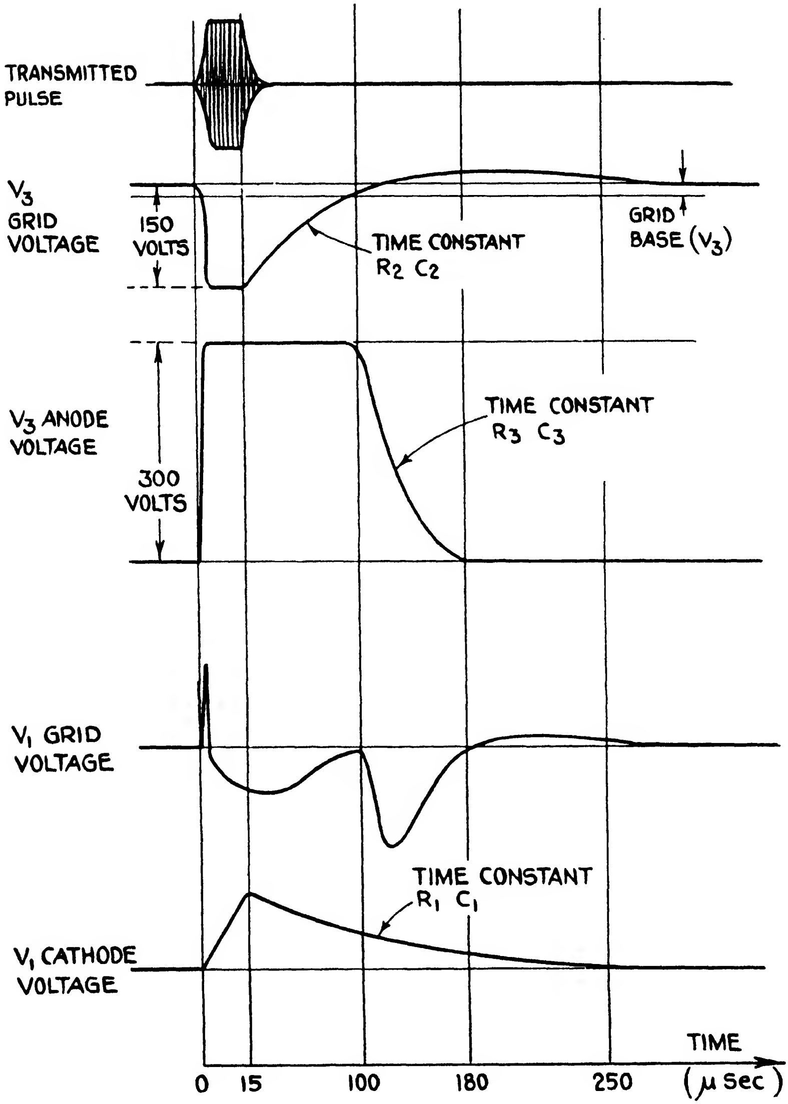


Fig. 8·12. Simplified oscillograms relating to the responder circuit in fig. 8·11 (superimposed quench voltage omitted).

the current in V_3, the grid of the oscillator V_1 is caused to swing negative, after which it recovers on the time constant determined by C_3 and the associated resistance network. During this time the cathode voltage of the oscillator is returning to normal with a time constant $R_1 C_1$. The grid and cathode recovery periods are adjusted to be roughly equal, and, together, they determine the recovery period of the receiver. Reception is not possible until the oscillator grid-cathode voltage has returned to within about 1 V. of normal, after which the sensitivity gradually increases to the value it had before the pulse arrived. This is also shown in fig. 8·12. The times and voltages quoted in this figure refer approximately to I.F.F. Mark I, although the circuit of fig. 8·11 is not accurately representative of that equipment.

Practical responders. The I.F.F. Mark III responders, which were used successfully in large numbers by the Allies for the greater part of the war, incorporated a number of improvements over the basic responder circuit described. An account of the interesting process of evolution of the final circuits would not be in place here. The circuit of a typical Mark III I.F.F. responder is briefly described below.

The circuit of the I.F.F. Mark III responder is given in fig. 8·13. Power supplies and pulse-width coding circuits are omitted. Improvements over the basic circuit of fig. 8·11 are:

(i) The use of a.g.s. to maintain the receiver sensitivity constant whilst sweeping over a wide band of frequencies (157–187 Mc./sec.).

(ii) Reception and transmission are carried out by separate oscillators which, however, use the same oscillatory circuit. This has the effect of freeing the grid of the receiving valve from the low-impedance connexions to the feed-back circuit, thus facilitating the application of quench and a.g.s. voltages. Regeneration takes place round the feed-back circuit exactly as before, due to the common oscillator circuit. Fig. 8·14 shows details of the common oscillator circuit and its coupling to the aerial.

(iii) A cathode follower is included in the feed-back circuit. This gives a greater drive to the transmitter valve, thus increasing the transmitted power and improving the pulse-shape.

(iv) Suppression circuits are included to prevent mutual interference between the I.F.F. responder and a radar equipment in the same aircraft. These need not concern us here.

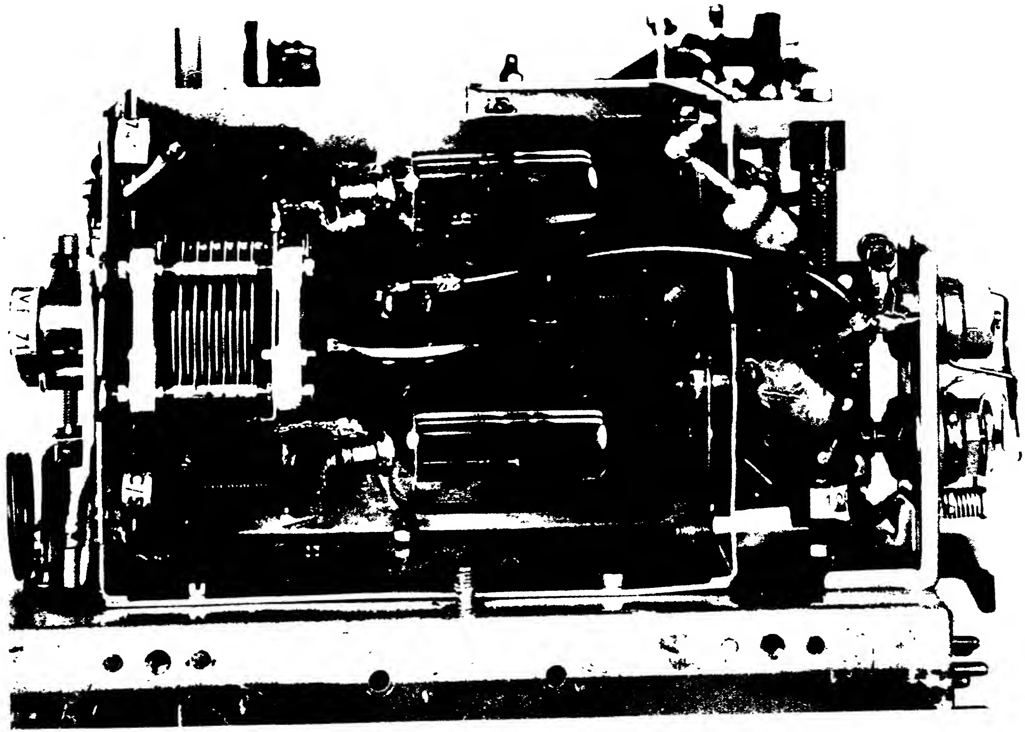


Fig. 8·14. Radio-frequency section of I.F.F. responder.

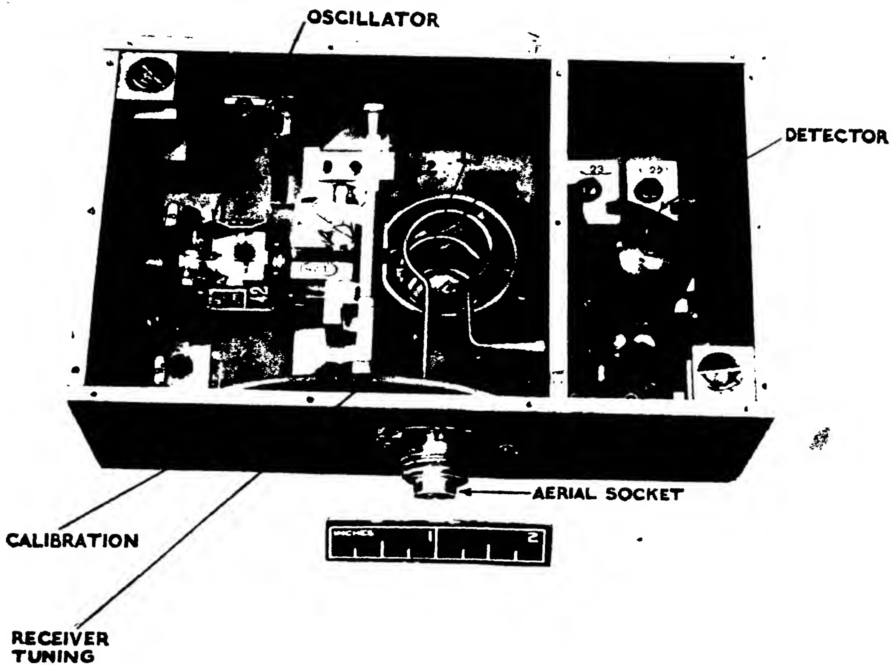


Fig. 8·15. Radio-frequency section of 'Lichtenstein' receiver.

Responders of this kind were also widely used for early radar homing beacons. More details of these are to be found in a paper by K. A. Wood,* prepared for the I.E.E. Convention on Radio-location.

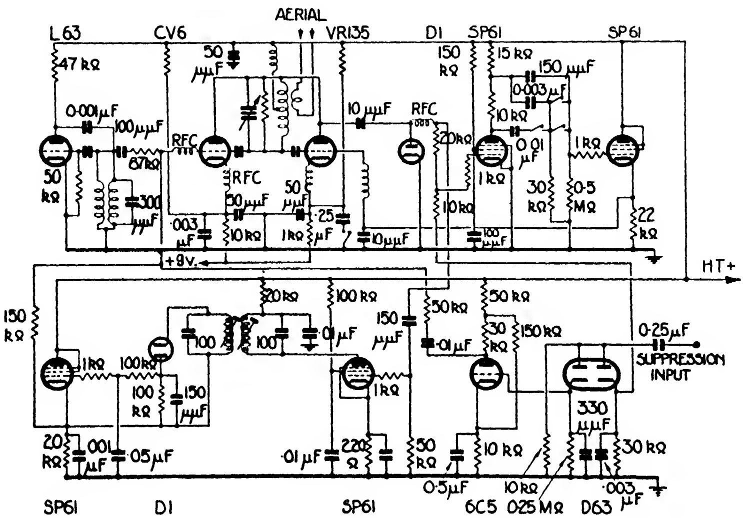


Fig. 8·13. I.F.F. Mark III responder circuit.

8·3·4. *The 'Lichtenstein' (German air interception)*†

It is worth brief mention here that one version of the German A.I. radar equipment, known as the 'Lichtenstein', used a linear super-regenerative receiver with automatic gain-stabilization. The receiver was intended to receive pulses of about 3 μsec. duration from the local transmitter, after reflexion from a remote aircraft.

Fig. 8·15 shows the lay-out of the radio-frequency section of the receiver. The oscillatory circuit consists of the inductance L_2 and a small condenser formed by two adjacent blocks of copper situated between L_2 and the oscillator valve. Tuning is accomplished by the rotation of a copper cylinder, mounted upon a ceramic rod, which forms, with the blocks of copper, a split-stator condenser. L_1 is the aerial coupling coil and L_3 the coupling to a full-wave diode detector. The radio-frequency is about 500 Mc./sec. and the quench frequency 600 kc./sec.

* See Bibliography.

† Air Interception = A.I., a term given to radar equipment fitted to night-fighter aircraft.

The circuit of the Lichtenstein receiver is similar to that described in §7·5·2 and drawn in fig. 7·9. The super-regenerative stage is operated at low gain (about 100×), and the additional gain required (about 2000×) is achieved in six stages of video amplification which are not shown in detail. This uneconomical arrangement helps to reduce the susceptibility of the receiver to deliberate interference by a m.c.w. signal.

8·3·5. *Strobe-quench receivers*

The fundamental nature of super-regenerative reception makes it possible to construct a receiver which is only sensitive to incoming signals for short, infrequent intervals which are time-related to some other occurrence, such as the transmission or receipt of a radar pulse.

This type of receiver has been suggested by Williams for use in an aircraft proximity warning device and has been used in the U.S.A. in beacon receivers.* The first application is described below.

The radar proximity indicator transmits a short pulse and emits a warning signal (e.g. lights a lamp) if an echo is received from an aircraft (or obstruction) within, say, half a mile. The super-regenerative receiver is sensitized for a short period following transmission, by means of a quench 'pulse' of suitable shape derived from the transmitter circuit. It is then quiescent until the next sensitizing pulse comes along. The shape of the quench 'pulse' can be varied to alter the duration of the sensitive period, on the principles described in Chapter 3.

When a signal is received the oscillatory output of the super-regenerative receiver is appreciable and can be used, after rectification, to close a relay circuit and thus to operate the desired warning device.

The duration of the transmitted pulse and of the sensitive period determine the range over which the instrument operates.

8·4. Super-regeneration and frequency modulation

The super-regenerative circuit has considerable possibilities as a frequency-modulation amplifier and detector. In the simplest case a self-quenching receiver will demodulate a frequency-modulated

* See, for instance, *Microwave Receivers*, edited by S. N. Van Voorhis, Radiation Laboratory Series, No. 23, Chapter 20, p. 567, McGraw Hill, 1948.

signal when slightly detuned from the centre carrier frequency. The parabolic form of the frequency response of a receiver in the logarithmic mode (including self-quenching) is quite suitable for this purpose and does not give rise to undue distortion (see fig. 8·16).

The circuit shown in fig. 8·1 is suitable for an elementary frequency-modulation receiver. The receiver is tuned in to the carrier in the ordinary way, but there are two tune points for

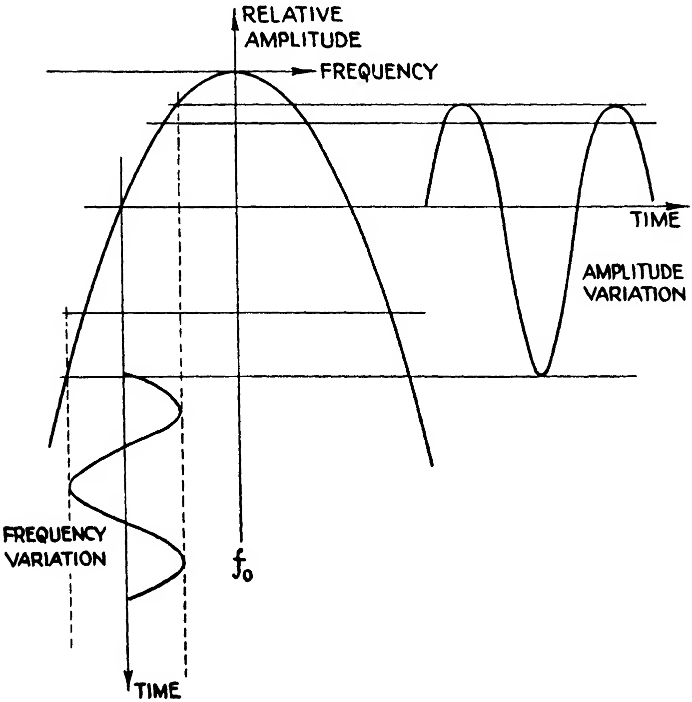


Fig. 8·16. Frequency-modulated detection by detuning a receiver operating in the logarithmic mode.

maximum undistorted signal. These lie either side of the mid-frequency. The procedure in deciding the optimum tuning point is as follows. The maximum suppression of noise occurs when the centre carrier frequency coincides with the mid-frequency response. The noise is at its greatest when the carrier lies outside the frequency response altogether. Between these two extremes lies a range in which a distinguishable and relatively distortionless signal is heard.

The optimum tuning point lies in this region. The setting is not usually very critical because of the gradual slope of the skirts of the response curve.

It is interesting to observe that the distortion in the super-regenerative detector when used for frequency-modulation is considerably less than that resulting from its use for amplitude-modulated signals.

The simple logarithmic circuit, although useful as a cheap frequency-modulation receiver, is not by any means ideally suited to the purpose. The limiting action is rather poor, in spite of the obvious limitation of oscillation amplitude during each super-regenerative pulse. This is adequately proved by the fact that the super-regenerative detector will detect amplitude-modulated signals.

Loughlin (1947) has used an off-tune super-regenerative circuit for the mixer, intermediate-frequency amplifier and discriminator in his 'Fremodyne' circuit described below.

8·4·1. *The 'Fremodyne' receiver*

The Hazeltine 'Fremodyne',* due to B. D. Loughlin, is a super-regenerative superheterodyne receiver for frequency-modulated signals. The basic circuit is given in fig. 8·17. The valve is a double triode (12 AT 7), one-half of which is used as the super-regenerative oscillator and the other half as a local oscillator in superheterodyne fashion. Super-regeneration takes place on the intermediate frequency of 21·75 Mc./sec. A frequency-modulated signal at a frequency of 108 Mc./sec. is mixed with a local oscillation at a frequency of 129·75 Mc./sec. on the grid of the super-regenerative detector, which consequently acts as mixer, intermediate-frequency amplifier and off-tune discriminator. The super-regenerative stage is self-quenching at a frequency of 20–30 kc./sec. by virtue of the grid leak R_1 which is returned to the high-tension supply. The audio-frequency component of the current in the super-regenerative valve is taken from the cathode circuit of that stage to an audio-frequency power amplifier (not shown).

The 'Fremodyne' is probably the most economical frequency-modulated receiver known. Its disadvantages are:

(i) Radiation of interfering oscillations. The frequency of these differs from the signal frequency and their intensity is small.

* Licensed by Hazeltine Electronics Corporation, Long Island, New York.

(ii) Limited audio-frequency band, due to the relatively low quench frequency.

(iii) Signal distortion due to the non-linearity of the frequency response curve.

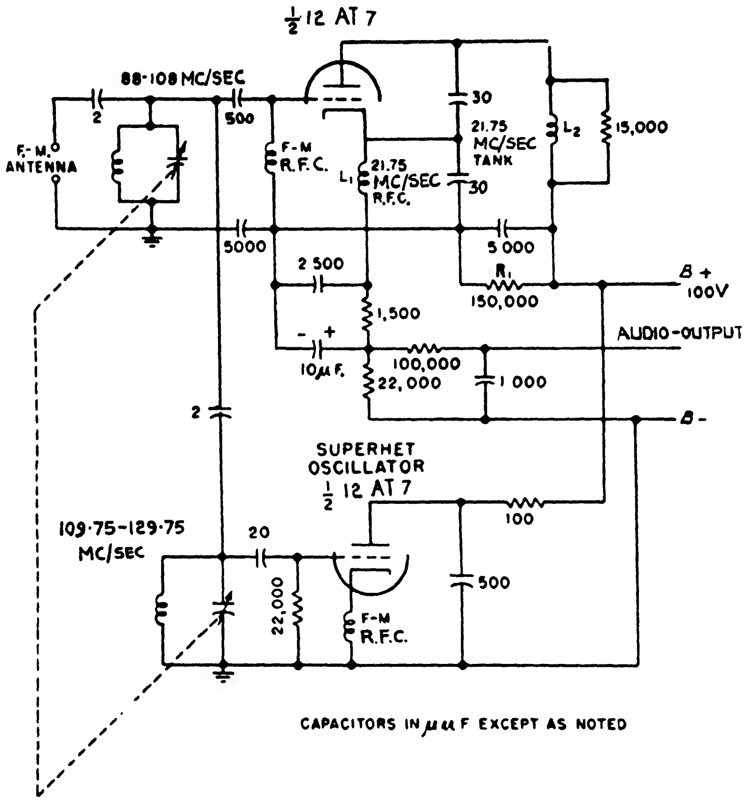


Fig. 8·17. The 'Fremodyne' receiver.

In spite of this there is no doubt that the receiver fulfils its aim of providing reception of frequency-modulated signals with the utmost economy of components.

8·4·2. *Super-regenerative amplification of frequency-modulated signals*

A more satisfactory, though more complicated frequency-modulated receiver can be made using a super-regenerative amplifier followed by a conventional discriminator. Before describing circuits for this purpose we will examine the behaviour of a super-regenerative receiver with a frequency-modulated signal at the input.

We have seen in Chapter 4 that the spectrum of the detected output from a super-regenerative receiver, due to a c.w. signal, consists of a series of lines separated by the quench frequency. This is also true of the output before detection, but the former spectrum is centred on zero frequency, whereas the latter is centred on a radio-frequency. These conclusions are correct whether we consider the linear or the logarithmic mode, for the different shapes of pulse only alter the envelope of the line spectrum and not its essential character.

The super-regenerative pulse that builds up from the incoming signal consists mainly of self-oscillations at an 'instantaneous' frequency equal to the natural frequency f_0 of the resonant circuit. These oscillations are, however, synchronized at their start to the signal frequency when self-oscillations and signal are comparable in amplitude. They are only coherent in so far as this synchronizing takes place. Kalmus (1944) has shown that this leads to the remarkable result that although the envelope of the spectrum of the pulses is centred on the frequency f_0 lines only appear at the signal frequency f and at frequencies $f \pm nf_q$.

The effect of modulating the signal frequency is to move the whole line spectrum by the amount of the modulation f_m. Thus each line is shifted by the same amount. It is this fact that makes it possible to use a super-regenerative amplifier for frequency-modulated signals.

A discriminator centred on one of the lines of the output spectrum (e.g. $f-f_q$, $f-2f_q$, etc.) demodulates the signal without distortion provided that the amount of frequency modulation is less than the quench frequency. The demodulation cannot, however, be conveniently carried out at the signal frequency, which is usually above 40 Mc./sec., and it is necessary to transpose the whole spectrum to a suitable intermediate frequency before demodulating.

Circuits due to Kalmus are given below in which a super-regenerative amplifier is used in this way. They are essentially super-regenerative superheterodyne receivers with the super-regenerative stage used either as a radio-frequency or intermediate-frequency amplifier.

8·4·3. *Super-regenerative superheterodyne receivers for frequency modulation*

Fig. 8·18 is the circuit of a superheterodyne receiver on a radio-frequency of 45 Mc./sec., using a self-quenching super-regenerative radio-frequency amplifier preceded by one tuned radio-frequency stage. The latter prevents radiation of super-regenerative oscillations. The amplified signal is mixed with local oscillations at a frequency of 36·7 Mc./sec. A single line of the transposed spectrum is selected by a circuit tuned to the intermediate frequency (8·3 Mc./sec.). The signal is finally passed to a limiter and a discriminator.

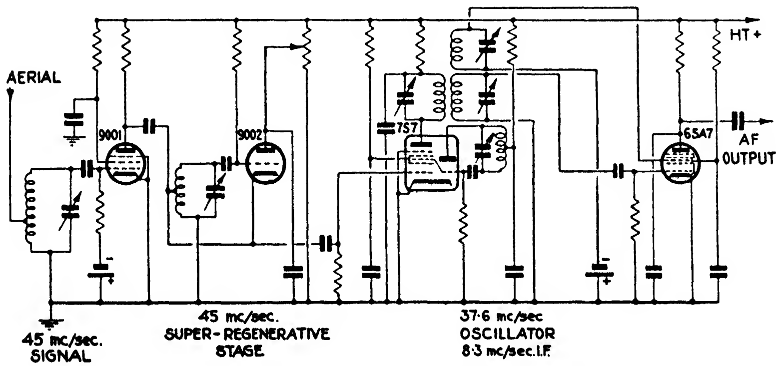


Fig. 8·18. Super-regenerative superheterodyne receiver for frequency-modulation.

The receiver is particularly insensitive to impulse interference (e.g. ignition noise) because amplitude limiting takes place in three successive stages, namely, the super-regenerative, mixer and limiter stages. The highest self-quenching frequency attainable at a signal frequency of 45 Mc./sec. does not, however, greatly exceed 100 kc./sec. We have seen that this must exceed the maximum deviation due to frequency modulation of the signal. This is inadequate for the 150 kc./sec. deviation used in wide-band frequency-modulated transmissions in the U.S.A., and it is preferable to operate the super-regenerative stage on a higher radio-frequency in order to increase the permissible quench frequency. Fig. 8·19 shows a circuit in which this is done. It is a double superheterodyne with a super-regenerative first intermediate-frequency stage. A 30 Mc./sec. local oscillation is mixed with the incoming signal (45 Mc./sec.) in a 9001 pentode mixer. The resulting signal with a centre frequency of 75 Mc./sec. is coupled into a self-quenching super-regenerative oscillator (9002). The super-regenerative oscillations are mixed with a 66·7 Mc./sec. local oscillation, and a single line of the transposed spectrum is selected by a circuit tuned to the second intermediate frequency, 8·3 Mc./sec. The conventional limiter and discriminator complete the circuit. The quench frequency in this receiver is about 200 kc./sec. which is adequate for the widest deviation likely to be used.

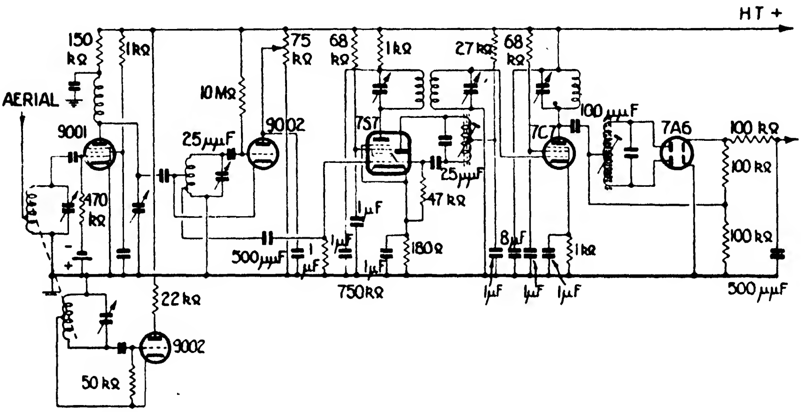


Fig. 8·19. Super-regenerative double superheterodyne receiver for frequency-modulation.

The use of a higher radio-frequency for frequency-modulated transmissions (e.g. B.B.C. transmissions on 90 Mc./sec.) makes the use of a double superheterodyne unnecessary.

8·5. Super-regenerative control circuits

The super-regenerative circuit, by virtue of its enormous amplification in a single stage, is very suitable as a control receiver for radio-controlled model ships or aircraft, or for other applications where light weight and current economy are important.

In the usual model-control arrangement the receiver is only required to distinguish between the 'signal' and 'no-signal' states. Reception of a signal closes a relay which operates a uni-selector or stepping switch. The operation required is thus selected by keying the signal on and off.

The change of anode current produced by a c.w. signal in a self-quenching super-regenerative receiver can be sufficient to operate a low-current relay. A simple control circuit is shown in fig. 8·20, in which the relay is in the anode circuit of the super-regenerative oscillator.

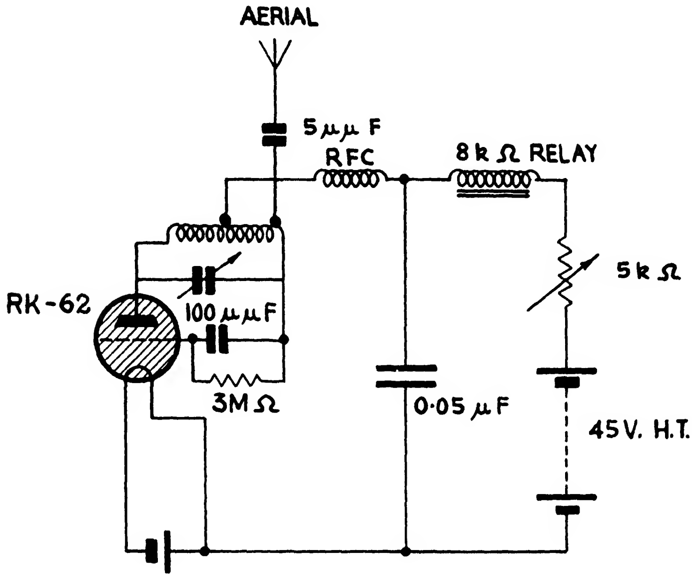


Fig. 8·20. Super-regenerative control circuit.

The circuit uses a special Raytheon valve (RK-62) which has been designed for the purpose. The precise operation of the circuit is somewhat obscure. The valve is self-quenching, but the current in *absence* of a signal is sufficient to close the relay. Reception of a signal increases the mean negative grid potential and turns off a large proportion of the anode current. The action of the circuit is, therefore, more certain, and the requirements for the relay are much less stringent.

Appendix 1

EXPRESSION FOR NOISE AT RECEIVER OUTPUT

Let us consider first of all the noise voltage during a quiescent period. The conductance G has then the constant value G_0. Let us assume that the noise is produced by a current generator giving a fluctuating current $i(t)$ in the interval $-\frac{1}{2}T<t<\frac{1}{2}T$. $i(t)$ now replaces $A\sin(\omega t)$ in the equivalent circuit (fig. 3·1).

Express $i(t)$ for $-\frac{1}{2}T<t<\frac{1}{2}T$ by its Fourier integral

$$i(t) = \int_{-\infty}^{+\infty} e^{j2\pi ft} N(f)\,df. \tag{1}$$

The circuit equation for fig. 3·1 now reads

$$C\frac{dV}{dt}+G_0 V+\frac{1}{L}\int V dt = \int_{-\infty}^{+\infty} e^{j2\pi ft} N(f)\,df. \tag{2}$$

Its solution is
$$V(t) = \int_{-\infty}^{+\infty} e^{j2\pi ft} N(f)\,A(f)\,df, \tag{3}$$

where
$$A(f) = j\frac{2\pi f}{C}\Big/\left[(2\pi f_0)^2-(2\pi f)^2+j\frac{G_0}{C}2\pi f\right]. \tag{4}$$

From Parseval's theorem, adapted for Fourier integrals, the mean-square value of $V(t)$ in the interval $-\frac{1}{2}T<t<\frac{1}{2}T$ is

$$\overline{|V(t)|_T^2} = \int_{-\infty}^{+\infty} \frac{|N(f)|^2}{T}|A(f)|^2 df. \tag{5}$$

Now we assume that the spectral density of a random noise fluctuation $|N(f)|^2$ is a slowly varying function of frequency which has no sharp peaks provided that the interval T is large. On the other hand, the function $|A(f)|^2$ has a single peak at the resonant frequency, which is very nearly f_0. Very little error will be made by taking $|N(f)|^2$ to be a constant and equal to $|N(f_0)|^2$; then the mean-square value of $V(t)$ for an infinite interval is

$$\begin{aligned}\overline{|V(t)|^2} &= \lim_{T\to\infty}\frac{|N(f_0)|^2}{T}\int_{-\infty}^{+\infty}|A(f)|^2 df\\ &= \lim_{T\to\infty}\frac{|N(f_0)|^2}{T}\frac{1}{4CG_0}.\end{aligned} \tag{6}$$

Let us now consider the build-up of pulses from noise in successive quench cycles. The interval T is now taken over $2n+1$ quench periods, and the noise fluctuation is again represented by a current generator

$$i(t) = \int_{-\infty}^{+\infty} e^{j2\pi ft} N(f)\, df.$$

The voltage at the end of the nth build-up period due to noise current $i(t)$ is readily obtained by following the steps leading to equation (27) (Chapter 3), with $A\sin(\omega t)$ replaced by $i(t)$. The resulting expression for $V_n(t_2)$ is

$$V_n(t_2) = \sqrt{\left[\frac{\pi}{C\,|\,G'(t_1)\,|}\right]} \exp\left(\frac{a^-}{2C}\right) B(t_n)\, e^{j\omega_0(t-t_1)}, \tag{7}$$

where
$$B(t_n) = \int_{-\infty}^{+\infty} e^{j2\pi ft_n} N(f)\, S(f)\, df \tag{8}$$

and
$$S(f) = \frac{f}{f_0} \exp\left[-\frac{4\pi^2 C(f-f_0)^2}{|\,G'(t_1)\,|}\right]. \tag{9}$$

The mean-square value of $V(t_2)$ averaged over $2n+1$ pulses in the interval T is

$$\overline{|\,V(t_2)\,|_T^2} = \frac{\pi}{C\,|\,G'(t_1)\,|} \exp\left(\frac{a^-}{C}\right) \overline{|\,B(t_n)\,|^2}, \tag{10}$$

where
$$\overline{|\,B(t_n)\,|^2} = \int_{-\infty}^{+\infty} e^{-j2\pi ft_n}\, G(f)\, df. \tag{11}$$

and
$$G(f) = \int_{-\infty}^{+\infty} N(u)\, \overline{\mathbf{N}}(u-f)\, S(u)\, \bar{\mathbf{S}}(u-f)\, du. \tag{12}$$

Now
$$t_n = nT_q \tag{13}$$

and
$$T = (2n+1)\, T_q.$$

Therefore by definition

$$\overline{|\,B(t_n)\,|^2} = \frac{1}{2n+1} \sum_{-n}^{n} |\,B(t_n)\,|^2$$
$$= \int_{-\infty}^{+\infty} \left[\frac{1}{2n+1} \sum_{-n}^{n} e^{-j2\pi f_n T_q}\right] G(f)\, df$$
$$= \int_{-\infty}^{+\infty} \left[T_q \sum_{-n}^{n} e^{-j2\pi f_n T_q}\right] \frac{G(f)}{T}\, df. \tag{14}$$

Now let us assume that $G(f)/T$ is a function which tends to a bounded function $H(f)$ as $T \to \infty$, i.e. $\lim_{T\to\infty} \frac{G(f)}{T} = H(f)$.

Then $$\lim_{T\to\infty} \frac{G(f)}{T} T_q \sum_{-n}^{n} e^{-j2\pi f_n T_q} = H(f)\,\delta\!\left(f-\frac{m}{T_q}\right), \tag{15}$$

where m is an integer and

$$\delta(f) = 0 \quad \text{for} \quad f \neq 0,$$
$$= \infty \quad \text{for} \quad f = 0,$$

and $$\int_{-\infty}^{+\infty} \delta(f)\,df = 1.$$

However, all the cases for which $m = 0$ arise solely because we have chosen exactly equal intervals of time between the successive pulses. Very slight variations in the interval would eliminate these cases. They are obviously spuriously introduced by our rigid condition and must be omitted. Equation (15) should therefore read

$$\lim_{T\to\infty} \frac{G(f)}{T} T_q \sum_{-n}^{n} e^{-j2\pi f t_n} = H(f)\,\delta(f). \tag{16}$$

If we now let $T\to\infty$ in equations (10) and (14) and use the last result, we get

$$\begin{aligned}\overline{|B(t_n)|^2} &= \int_{-\infty}^{+\infty} H(f)\,\delta(f)\,df \\ &= H(0) \\ &= \lim_{T\to\infty} \frac{G(0)}{T} \\ &= \lim_{T\to\infty} \int_{-\infty}^{+\infty} \frac{|N(f)|^2}{T} |S(f)|^2\,df. \end{aligned} \tag{17}$$

This is in the same form as equation (5). Once again we take out the spectral noise density from under the integral sign at its value where $S(f)$ is a maximum. From equation (9), this occurs when $f = f_0$. Therefore

$$\begin{aligned}\overline{|B(t_n)|^2} &= \lim_{T\to\infty} \frac{|N(f_0)|^2}{T} \int_{-\infty}^{+\infty} |S(f)|^2\,df \\ &\doteqdot \frac{1}{2}\sqrt{\left[\frac{|G'(t_1)|}{2\pi C}\right]} \lim_{T\to\infty} \frac{|N(f_0)|^2}{T}. \end{aligned} \tag{18}$$

We can now express $\overline{|V(t_2)|^2}$ in terms of $\overline{|V(t)|^2}$ of equation (6) by eliminating $\lim_{T\to\infty} \frac{|N(f_0)|^2}{T}$ from equations (6) and (18). Thus

$$\overline{|B(t_n)|^2} = G_0\sqrt{\left[\frac{2C\,|G'(t_1)|}{\pi}\right]}\,\overline{V^2}, \tag{19}$$

and, from equation (10) in the limit $T \to \infty$,

$$\sigma^2 = \overline{|V(t_2)|^2} = \overline{V^2} G_0 \sqrt{\left[\frac{2\pi}{C\,|\,G'(t_1)\,|}\right]} \exp\left(\frac{a^-}{C}\right). \tag{20}$$

From equation (58 *a*) (Chapter 3) this can be written

$$\sigma^2 = 2\overline{V^2}\,\frac{b_n}{b_{ns}} \exp\left(\frac{a^-}{C}\right). \tag{21}$$

Let us define $\overline{V_s^2}$ as the mean-square noise voltage within the energy band-width of the super-regenerative circuit. It is related to $\overline{V^2}$ in the following way:

$$\frac{\overline{V^2}}{\overline{V_s^2}} = \frac{b_n}{b_{ns}}. \tag{22}$$

Through this (21) may be written, using equation (61), (Chapter 3)

$$\sigma^2 = 2\overline{V_s^2}\left(\frac{b_n}{b_{ns}}\right)^2 \exp\left(\frac{a^-}{C}\right) \tag{23}$$

$$= \overline{V_s^2} \times \quad \text{(total power gain)}.$$

Thus the mean-square deviation of the output voltage is equal to the mean-square noise voltage within the *super-regenerative* band-width times the total power gain of the receiver.

Appendix 2

DESIGN DATA FOR SINUSOIDAL QUENCH

Figs. 1–6 are based upon the formulae of § 5·4. They relate the gain and frequency response of a super-regenerative receiver using sinusoidal quench to the electrode voltages applied to the super-regenerative oscillator. They are only valid in so far as the assumptions made in Chapter 5 are valid. The symbols are defined in table 5, page 81.

1. Super-regenerative gain

Figs. 1–3 relate the super-regenerative gain in nepers to the electrode voltages through the parameters y and K, or z and P, defined in table 5. They are derived from equation (23) in Chapter 5. Fig. 1 shows the trend of the gain as the quench voltage is varied, y being directly proportional to the quench voltage provided that the steady electrode potentials remain constant. Fig. 2 shows the dependence of gain upon the mean voltage of the electrode to which quench *is not* applied. Fig. 3 shows the dependence of gain upon the mean voltage of the electrode to which quench *is* applied.

Because of the nature of the formulae for K and P the curves of figs. 2 and 3 cannot be used directly to represent the variation of gain with electrode voltage. It is not possible to make the scale proportional to the voltage which is to be varied but only to P or K, each of which is linearly dependent upon that voltage. The values of the parameter (y or z) and the variable (P or K) must be calculated from the electrode voltages using the formulae of table 5.

The quantity representing super-regenerative gain on the ordinate axis of figs. 1–3 is

$$H = \frac{\omega_q C}{G_0} N_s = (f_q/b) N_s.$$

Thus each ordinate must be multiplied by the factor b/f_q before it represents the gain in nepers. This factor is constant for a given circuit arrangement.

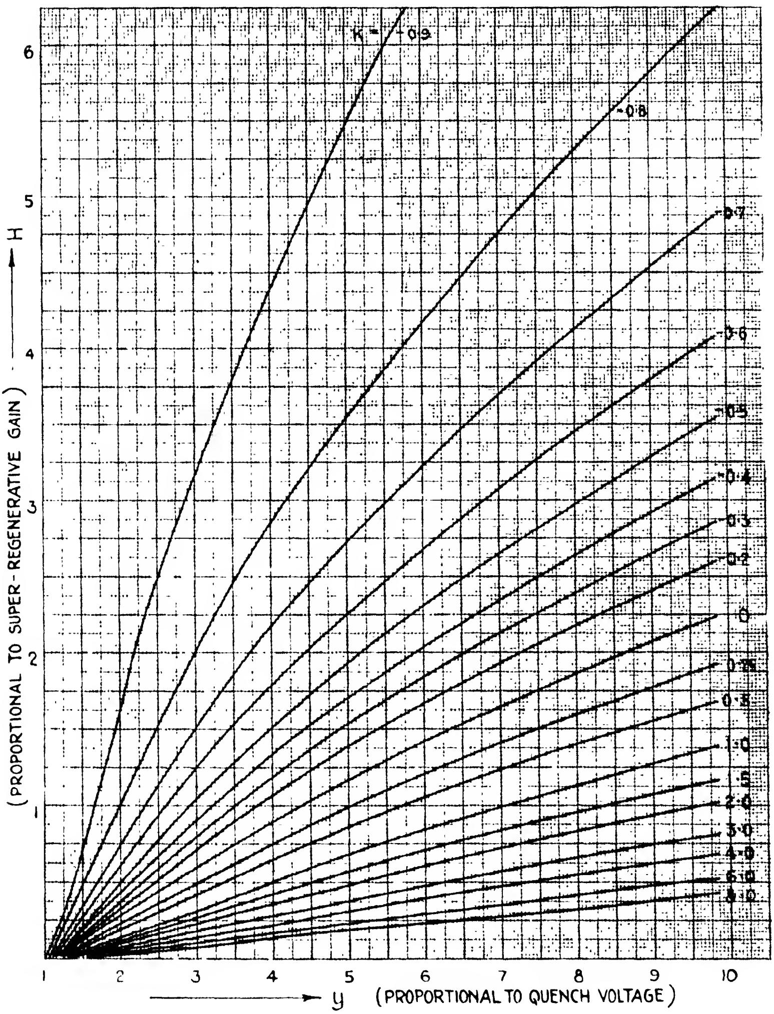


Fig. 1. Gain curves for sinusoidal quench (based on equation (23), Chapter 5).

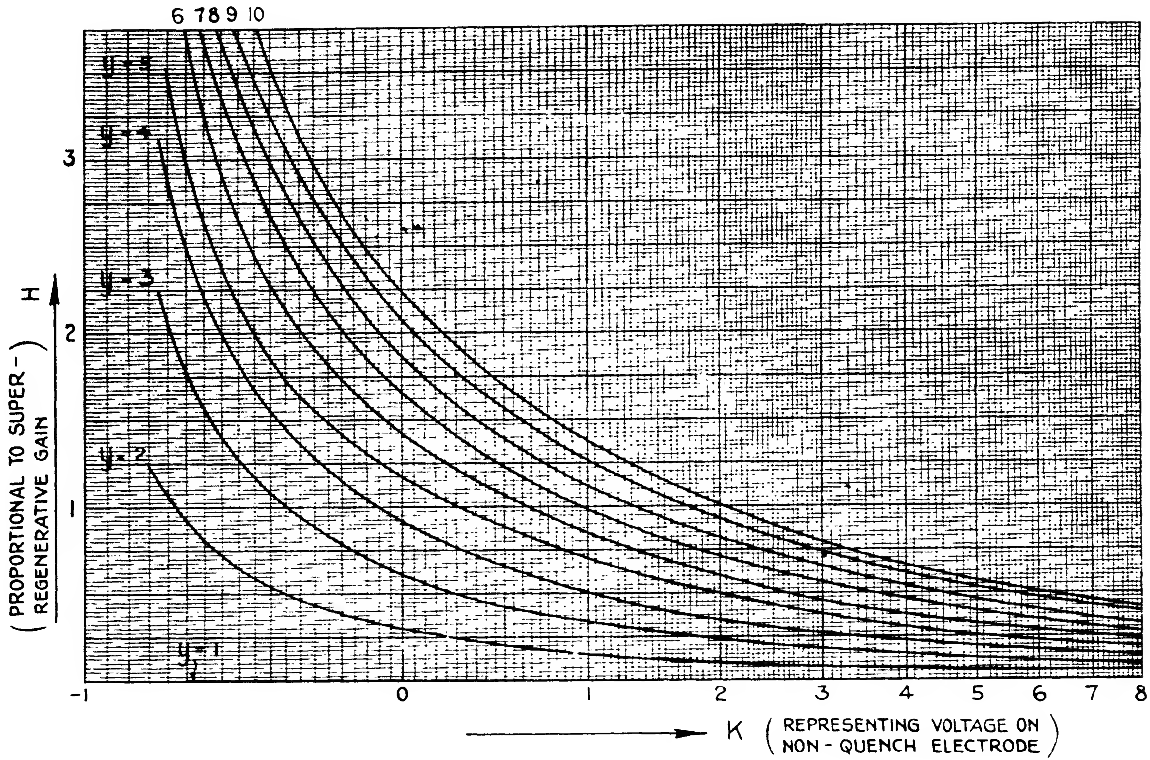


Fig. 2. Gain curves for sinusoidal quench (based on equation (23), Chapter 5).

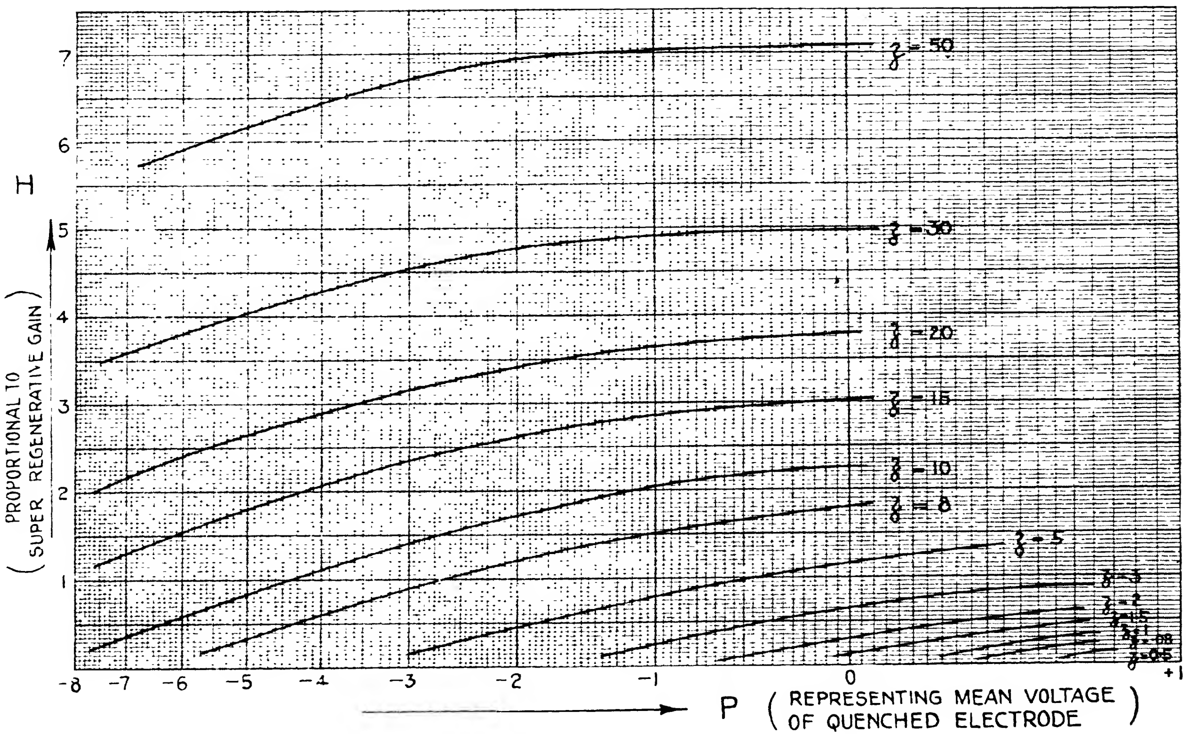


Fig. 3. Gain curves for sinusoidal quench (based on equation (23*d*), Chapter 5).

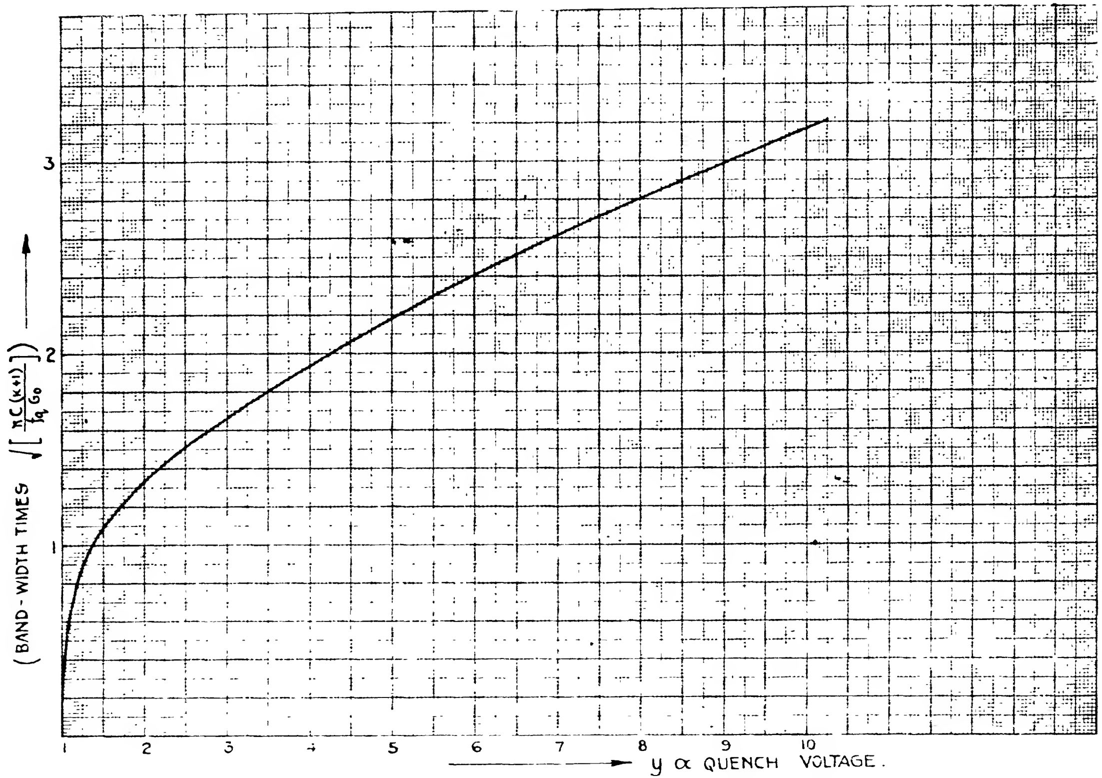


Fig. 4. Band-width of response curve for sinusoidal quench (based on equation (26), Chapter 5).

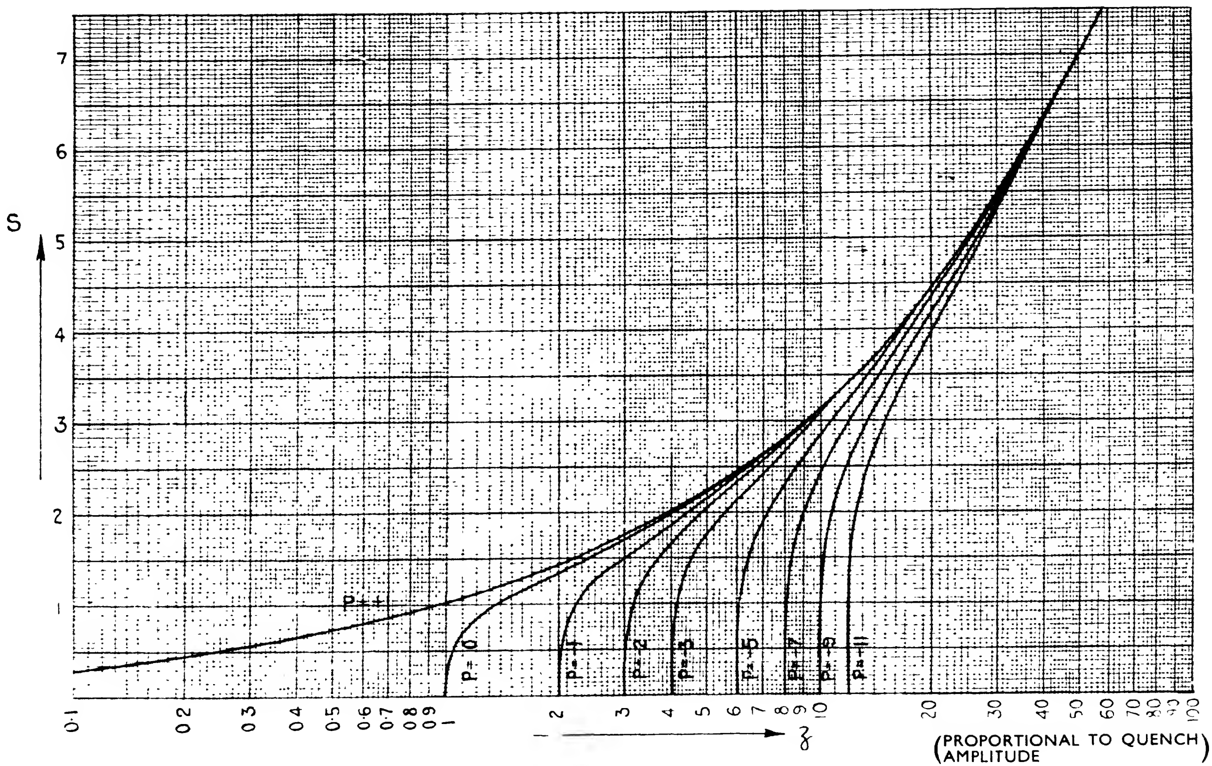


Fig. 5. Variation of band-width with quench amplitude: variable gain.

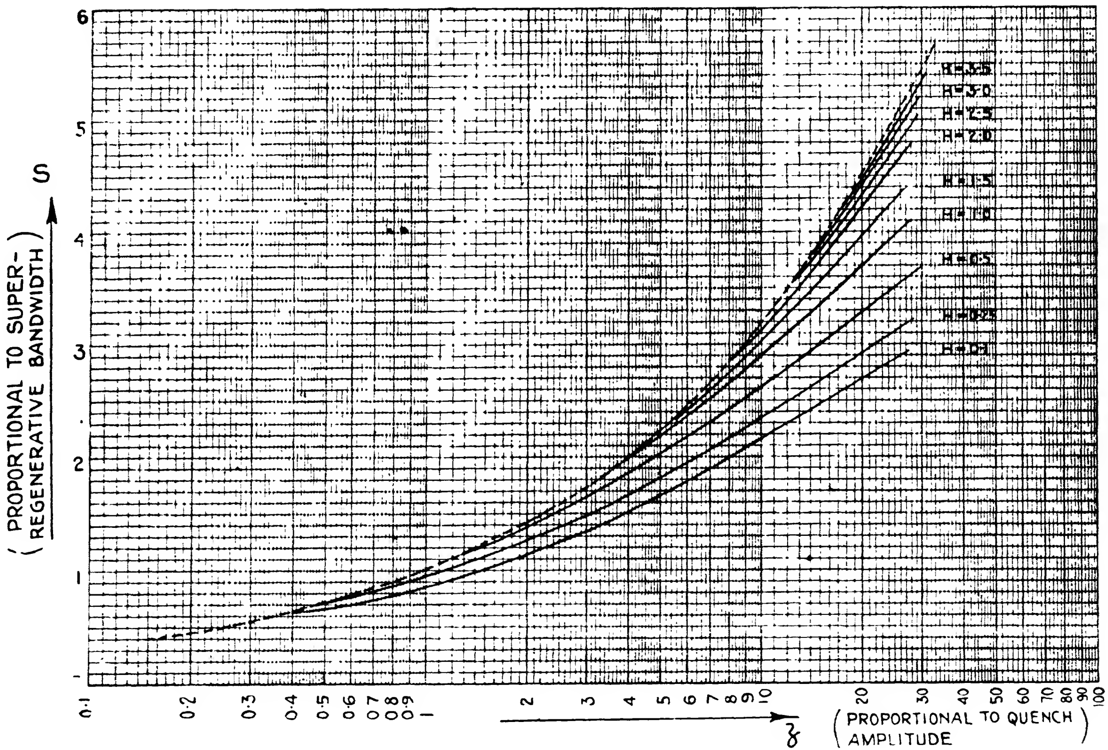


Fig. 6. Variation of band-width with quench amplitude: constant gain.

2. Band-width

Figs. 4 and 5 relate the super-regenerative band-width at 1 neper down with the electrode voltages through the parameters y and K or z and P. They are based upon equation (26), Chapter 5.

Fig. 6 shows how the band-width varies with quench amplitude when the super-regenerative gain is held constant by adjusting the mean voltage of the electrode to which quench is applied. The process is described in § 5·4·3. Fig. 6 is obtained by taking constant-gain cross-sections of fig. 3, thus relating z and P for various values of H, the gain parameter. This enables P, representing the mean voltage of the quench electrode, to be eliminated from the expression for super-regenerative band-width, which is then plotted against z for various values of H.

The quantity representing band-width in figs. 4–6 is

$$S = \sqrt{\left(\frac{\sqrt{(y^2-1)}}{K+1}\right)} = \sqrt[4]{[z^2-(1-P)^2]}$$
$$= \sqrt{\left(\frac{\pi C}{f_q G_0}\right)} b_s = \frac{b_s}{\sqrt{(2f_q b)}}.$$

Thus each ordinate must be multiplied by the factor $\sqrt{(2f_q b)}$ before it represents the super-regenerative band-width at -1 neper.

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LIST OF PRINCIPAL SYMBOLS

A	Amplitude of signal current.
b	Band-width of quiescent-circuit response at -3 db.
b_s	Band-width of super-regenerative response (at $-8{\cdot}7$ db. for slope-control; -6 db. for step-control).
b_e	'Effective' band-width of quiescent circuit defined as $\frac{1}{A(f_0)}\int_{-\infty}^{+\infty} A(f)\,df$.
b_{es}	'Effective' band-width of super-regenerative circuit defined as $\frac{1}{S(f_0)}\int_{-\infty}^{+\infty} S(f)\,df$.
b_n	Energy band-width of quiescent circuit.
b_{ns}	Energy band-width of super-regenerative circuit.
B	Band-width of frequency spectrum of output pulses after detection (at $-8{\cdot}7$ db. for slope-control; -6 db. for step-control).
C	Total circuit capacitance.
e	Base of Naperian logarithms.
$f=\omega/2\pi$	Signal frequency.
$f_0=\omega_0/2\pi$	Natural frequency of resonant circuit.
f_q	Quench frequency.
f_m	Modulation frequency.
$F(t)$	Arbitrary function in conductance formulae.
$G(t)$	Circuit conductance function.
G_0	Quiescent circuit conductance.
G_1	Limiting value of negative conductance (step-control).
$H=(f_q/b_s)N_s$	Super-regenerative gain factor on design curves.
I_L, I_C, etc.	Branch currents in equivalent oscillator circuit.
j	$\surd(-1)$.
L	Total circuit inductance.
M	Mutual inductance.
$N_0=\log_e \mu_0$	Slope-factor or step-factor gain in nepers.
$N_s=\log_e \mu_s$	Super-regenerative gain in nepers.
$N_t=N_0+N_s$	Total gain in nepers.
P, P_1	Frequency-dependent terms in basic equation of super-regeneration.
Q	Circuit Q-factor.
$r=V_1/2\sigma$	Ratio of signal to noise at input to detector.
S	Factor, dependent on electrode voltages, appearing in formulae for sinusoidal quench.
$S(f)$	Frequency spectrum.
t	Time variable.
t_1, etc.	Specific instants in conductance cycle.
$T_q=1/f_q$	Quench period.

T_b	Build-up period, slope-control.
T_d	Damping period.
V_s	Signal voltage at input.
V_a	Instantaneous anode voltage.
$\bar{V}_a$	Mean anode voltage.
V_g	Instantaneous grid voltage.
$\bar{V}_g$	Mean grid voltage.
V_{0a}	Anode voltage at which oscillation begins.
V_{0g}	Grid voltage at which oscillation begins.
Y	Circuit admittance.
Z	Circuit impedance.
$\alpha = G_0/4\pi C,\ \beta = G_1/4\pi C$	Used for substitution in expressions for frequency response.
$\delta = (\omega/\omega_0) - 1$	Departure from resonance.
μ_0	Slope or step voltage gain factor ($= e^{N_0}$).
μ_s	Super-regenerative voltage gain factor ($= e^{N_s}$).
σ	Root mean-square deviation of amplitude of output pulse from the mean amplitude.
τ	Width of output pulse at -1 neper.
ϕ	Arbitrary phase angle.
$\Phi = \int_{-\infty}^{+\infty} F\,dt$	Function used in conductance formulae.
ψ	Arbitrary phase angle.
$\omega = 2\pi f$	Radian frequency of signal.
$\omega_0 = 2\pi f_0$	Natural radian frequency of tuned circuit.

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