## 24 DESIGN PHASE OPTICAL TESTS

#### 24.1 INTRODUCTION

- 24.1.1 Uses. Optical testing methods are widely used in all branches of scientific and technical work. The basic techniques, or modifications thereof, enable some of the most sensitive and precise measurements man has ever known. Gage blocks may be measured to better than 0.0000001" with relatively simple interferometric apparatus while velocities of satellites hundreds or thousands of miles away may be measured with the Doppler shift techniques common to older astrophysics problems.
- 24.1.2 Related fields. It will be noted that from time to time reference will be made to work that has been done in the field of microwave antennas. This has been done in the belief that it will be very instructive to become acquainted with design techniques involving wavelengths that are frequently approaching a tenth of the radiating aperture. Further, the use of aberrations, interference, diffraction, and control of aperture illumination are discussed and demonstrated in a way frequently difficult at light optics frequencies. The very recent achievements in light optics where the aberrations are all reduced (save color) to the diffraction-limited stage is many years old in the microwave-antenna field. Microwave antenna designers borrowed heavily from older optical techniques and it is quite possible that a study of their efforts will be highly rewarding to the light-optics designer.

## 24.1.3 Methods and problems pertinent to optics.

- 24.1.3.1 While these methods cover a wide gamut, discussion in this section will be confined to a small sampling of the methods particularly suitable to the design, construction, and evaluation of visual optical systems. A few words regarding the origin of the testing problems which will be encountered will be in order.
- 24.1.3.2 The design of an optical instrument is obviously predicated on a need having been established. Sometimes the nature of this need is such that electrical and mechanical considerations may dictate, to a considerable extent, the physical shape of the optical system. However, even after this has been determined there still remains the problem of translating the customer's purely optical requirements into a form that is significant to the lens designer. Field of view, curvature of field, transmission over a given spectral band, distortion, etc. can be specified rather accurately and unambiguously. Questions, however, as to image quality and what figure of merit is to be used in deciding whether this or that design will most closely give the customer the information he seeks when he uses the instrument, raise problems that have yet to be solved completely. There seems to be more and more evidence of late that to phrase the problem in this way--viz. that the optical instrument be an "information handling system" -- is preferable to the more vague requirement that it be a system that forms a good image. Agreed, the former actually sounds more vague, but current effort indicates the above sentence is probably correct.
- 24.1.3.3 The postulating of a figure of merit implies that one must test proposed designs to see if they meet the assumed theoretical criterion. Once the design is firm, the optician takes over and now he must perform tests to see that his construction faithfully follows the prescription given to him by the lens designer. Here we must point out that there is another testing step necessary. The optician's job may be considered complete and accurate when the radii, edge thicknesses, center thicknesses, spacings, indices, etc. agree with the specifications handed down by the lens designer. The fact that the optician's work is presumably accurate does not, however, serve as a complete check on the usability of the system. It must be remembered that the designer used some theoretical criterion such as amount of energy in a point image, phase front or Seidel aberrations. The next step therefore is to see how well the constructed system lives up to his predictions in one or more of these respects.
- 24.1.3.4 There is little doubt that the ultimate test of any system is a field test under the original conditions imposed in the customer's specifications. A system can conceivably be excellent in the laboratory and yet be so sensitive to vibration that it is useless in the field. Further laboratory testing under simulated field conditions is therefore indicated; installation in field equipment being attempted only after the prototype has been tested thoroughly in the laboratory.
- 24.1.3.5 Here is another point that should be strongly raised. Granted that field tests are the ultimate in one sense, we should not lose sight of the fact that the nature of field tests frequently is such as to cloud the performance of the optical system by the introduction of parameters not basically a part of the problem. The writer clearly recalls airborne cameras yielding several hundred lines per minute resolution in the laboratory and only 20-30 lines per minute in the air. The trouble was definitely not with the camera or optical system but rather with the mechanical mounting in the plane. Some more or less absolute standard of perfection based on the customer's optical requirements is therefore mandatory. Tests in this category are extremely valuable. Resolving power, sine-wave tests, etc. fall into this category.

## 24.1.4 The testing program.

24.1.4.1 A consideration of the principles outlined above indicates that the complete testing program rather naturally falls into the following categories. It should be pointed out that many more types of tests are known in each category, but space permits only this limited sampling.

#### 24.1.4.2 Testing during the design phase.

- (1) Calculation of the Seidel Aberrations
- (2) Calculation of the Spot Diagrams
- (3) Determination of the phase front and perhaps the predicted diffraction by knowing the phase front and amplitude distribution over the aperture.

#### 24.1.4.3 Testing during the manufacturing phase.

- (1) Foucault Test
- (2) Star Test
- (3) Ronchi Test
- (4) Interferometric Tests and/or determination of phase front.
- (5) Measurements of curvature of field, astigmatism, transmission, field of view, front and back focal lengths etc.

#### 24.1.4.4 Testing during the evaluation phase.

- (1) Any or all of the tests in 24.1.4.3 above.
- (2) Measurement of the resolving power.
- (3) Measurement of the sine-wave response.

We will now proceed to discuss each of these tests.

## 24.2 CALCULATION OF THE SEIDEL ABERRATIONS

- 24.2.1 Object-image relationship. From a strictly theoretical point of view, an optical system may be said to be perfect if its response is "collinear" i.e. points are imaged as points, lines are imaged as lines, and planes are imaged as planes. A further qualification is required--namely that the definition just given applies strictly and only to an optical system where the magnification is unity for all image points. While such systems do have significance, most optical systems require either minifications (telescopes, field cameras, etc.) or magnification (microscopes, etc.). We therefore qualify the concept of collinearity by adding that magnification or minification may exist, but should be constant for all points in the image. The above definition, even with its qualifications, applies more to photographic than to visual optical systems because of the reference to a flat focal surface. While curved focal surface systems have been used in photography, they are rare because of the practical problems involved in film handling. Almost all photographic systems require a flat focal surface, i.e. a focal plane. For visual optics we may relax this requirement somewhat. Indeed the ideal system is one whose curvature of field matches that of the eye.
- 24.2.2 The importance of Seidel Aberrations. It has been found possible by Seidel (1) to express the deviation of an actual image produced by a system, from the theoretically perfect system by a series expansion. This series expansion was given previously in Section 8. The monochromatic deviations from the ideal flat focal surface collinearity are called aberrations and include spherical aberration, coma, astigmatism, curvature of field and distortion. To the extent, then, that this series expansion accurately depicts what happens to an image point, the calculation of these Seidel aberrations constitute a powerful first approximation in the design of an optical system. It is equally clear that the calculation of these aberrations may be considered as a theoretical test of such a system. The method of calculating these aberrations, and the detailed significance of each has been previously treated. The subject is raised here again to point out the use of these aberrations in the theoretical tests which may be applied to an optical system. The reader should refer to Sections 8-10 for more details. It should also be pointed out that these aberrations are strictly geometrical and that

<sup>(1)</sup> Seidel: Astronomische Nachrichten, 43, 289-332 (1856).

two different systems may have the same aberrations and yet show quite different images due to the fact that the wave nature of light is completely ignored (except for the variation of index with wavelength).

24.2.3 Seidel Tolerances. The criticism sometimes levied is that it is pointless to design a system on the basis of purely geometrical optics because of the neglect of interference etc. To our knowledge no optical system has been designed, at least in recent years, without reference to the wave nature of light. Frequently this is done by explicitly placing tolerances on the aberrations by reference to the Rayleigh (2) stipulation that the maximum path deviation from a given object point to a given image point be not more than  $\sqrt{4}$ . Discussions of this may be found in Conrady (3) and Martin(4). These optical tolerances are:

For primary marginal spherical,	•
permissible primary LA' = $4NN' \sin^2 U'_m$	(1)
For primary zonal spherical, (assuming LA' = 0)	
permissible LZA' = $6\lambda/N' \sin^2 U'_m$	(2)
For primary Coma (Coma <sub>s</sub> )	
permissible Coma <sub>s</sub> = $\pm \lambda/2N'$ sin $U'_{m}$	(3)
For focal range,	
Focal range = $\lambda/N'$ sin $U'_m$	(4)
For astigmatism, Ast's	
permissible Ast's = $\lambda/4N' \sin^2 U_m$	(5)
For curvature of field,	
permissible X' = focal range = $\lambda/N$ ' $\sin^2 U_m$	(6)

Note:  $U_m$  is the angle between the ray and the axis, N is the index of refraction in image space; and  $\lambda$  is the wavelength of the radiation.

24.2.4 Use of the Seidel Tolerances. One should use these tolerances with exceeding care particularly with high speed systems. This occurs because the focal range allowed by the \$\sqrt{4}\$ path difference criterion is assumed small compared with the actual focal length. Secondly the field angle is assumed sufficiently small so that  $\sin^2(U'_m) = 1/4 \sin^2 U'_m$ . One should further regard these tests as representing a theoretical arbi-

trary standard which may be too tight or too loose in special circumstances. For fast systems (microwave antennas are a good example outside of the field of visual optics), the tolerance on spherical aberration as computed from (1) is too loose-usually by a factor of 4 or more. The tolerance on coma is too loose for many visual systems where the coma may be the most serious aberration and every attempt should be made to reduce it sensibly to zero. The astigmatic tolerance is usually too tight, and a lens may be expected to produce good results even if the astigmatic tolerance is exceeded by a factor of 2.

24. 2. 5 Conclusions. The subject of Seidel aberrations from purely geometrical optics is considered here in conjunction with tolerances imposed by physical optics because they have been the prime standards against which lenses were compared until the relatively recent present. Most lenses still are designed on this basis today although there are some who think that sine-wave response calculations may replace them in future years. In conclusion we may say that the reduction of aberrations to within, or at least close to, the stipulated tolerances is a necessary but not sufficient condition that to assure a lens so constructed will perform well. Actually the reduction of the aberrations to the specified limits results in a wavefront that is sensibly spherical in image space. The true image, however, involves amplitude as well as phase, and the Seidel aberrations give no explicit information regarding amplitude.

## 24.3 THE SPOT DIAGRAM

<sup>(2)</sup> Lord Rayleigh, Collected Papers, vol. 1, pp. 415-459.

<sup>(3)</sup> Conrady, Applied Optics and Optical Design, pp. 136, 395, 434 et seq., Dover, (1957).

<sup>(4)</sup> Martin, Technical Optics, vol. 1, p. 139, Pitman, (1948). also Jacobs, Fundamentals of Optical Engineering, 443, McGraw Hill, (1943).

- 24.3.1 Introduction. In the past, the labor involved in doing any but the simplest of ray tracing was such that relatively few rays were traced in the actual lens design process. With the advent of electric desk calculators, it became possible to trace more rays in the same time. As a result tracing rays out of the meridional or tangential fan became more common. It was not until the relatively recent present that the designer was freed of this time limitation by the development of the high-speed, electronic-computing machinery. It is now possible to trace hundreds of rays in the same time it took to trace just a few some years ago. This has resulted in lenses being designed much more carefully than ever before. The aberrations determined by tracing rays as just discussed are definitely an approximation that is very good under some circumstances, but the usual Seidel third-order aberrations are frequently misleading: higher order aberrations sometimes being dominant.
- 24.3.2 Aspherics. Another factor brought into being recently is the use of aspheric surfaces. Desk calculators or no, tracing through aspheric surfaces can be a monumental task when done by hand. There is ample evidence, however, that freed from the restriction of purely spherical surfaces, the designer can almost always do a far better job with aspherics than he can with spherical surfaces.
- 24.3.3 Development and limitations. One of the first testing techniques that took full advantage of the power of the large computers was that evolved by Herzberger (5) and later by Hopkins (6) and was called the "spot diagram." In essence the entrance pupil is divided into equal areas, and a ray is traced through the center of each area—the assumption being that the energy represented by each ray is the same. The intersection of these rays with an assumed focal plane was a spot, hence the term "spot diagram." The more compact this spot, the more nearly perfect was the lens judged to be by the standards of geometrical optics. This is discussed in Section 8. We should thus clearly realize that this technique is restricted to non-diffraction limited systems. In this connection we should also realize that while most optical systems today are not diffraction—limited, there is a growing class of high precision systems widely emphasizing aspherics where the only aberration left is color, and where the performance is almost an order of magnitude better than it was ten years ago. For such systems, the spot diagram can serve only as a rough first approximation. The vast majority of visual and photographic optical systems are aberration—limited rather than diffraction—limited so the spot diagram is still a powerful tool.
- 24.3.4 Techniques. There are basically two techniques for getting a spot diagram. In one the required number of rays is actually traced, and the intersection points with the assumed focal surface are plotted. In the other a relatively small number of rays is plotted, and the intersection coordinates of the others are obtained by an interpolation and extrapolation process developed by Herzberger. It should be noted here that the interpolation process does more than just give the intersection points. Via the series expansion required for the interpolation it also gives a set of terms not unlike those of Seidel. The difference is major, however, in that the Seidel aberrations work particularly well near the axis while the "Herzberger aberrations" fit well over the entire aperture. Space does not permit us to go more deeply into this use of spot diagrams, but the reader is encouraged to refer to Herzberger's articles on this subject (5), (7), (8).
- 24.3.5 Examples. Those interested in this subject are also urged to obtain National Bureau of Standards Report No. 5640 entitled "Numerical Analysis of a 6" f/3.5 Aerial Camera Lens (006BC035 15)". This report by Stavroudis and Sutton shows clearly the extent to which the spot diagram testing is currently employed. Not only are the spot diagram shown for various assumed focal plane positions and angles of obliquity, but also the values of vignetting, distortion, chromatic aberration, energy distribution, and resolving power are derived for this lens directly from the spot diagrams. It is interesting to note the excellent correction that seems to have been achieved in this lens. For full aperture the diameter of the Airy disk is 4.0 microns. If we inspect the following table, Table 24.1, taken from Stavoroudis report, we see that 80% of the total points fell within a circle on axis whose diameter was 3.93 microns. For an aberrationless system theory indicates there will be 83% of the total energy within the Airy disk. The close agreement between theory and spot diagram prediction indicates the excellence of the design, at least for on axis work. In another series of experiments Stavroudis and his colleagues at the National Bureau of Standards calculated the spot diagram of a completed lens. The comparison of the spot diagrams and corresponding actual photographs for two given positions is shown in Table 24.1.

## 24.4 PHASE FRONT CALCULATIONS

24.4.1 The spherical wavefront. It has been pointed out that the Seidel Aberrations, when they are fully corrected, result in a spherical wavefront converging on the image point. Modern computing machinery has enabled the designer to calculate directly the wavefront and thus determine not only the phase errors over the aperture but where the focal point should be placed.

<sup>(5)</sup> Herzberger, J. Opt. Sec. Am 37, 485 (1947).

<sup>(6)</sup> Hopkins, J. Opt. Sec. Am 44, No. 9, 692-698 (1954).

<sup>(7)</sup> Strong, Concepts of Classical Optics, Appendix L by Herzberger, p. 537, Freeman (1958).

<sup>(8)</sup> Herzberger, Optical Image Evaluation, National Bureau of Standards Circular No. 526, U.S. Gov't. Printing Office (1954).

% Total	0 <sup>Ο</sup>	' 7 <sup>0</sup>	11 <sup>0</sup>	14 <sup>0</sup>
points	μ	μ	μ	μ
10 20 30 40 50 60 70 80 90	0.674 1.22 1.69 1.97 2.43 3.05 3.71 3.93 6.08 12.1	3.66 6.54 14.0 24.7 37.8 53.3 71.7 97.8 132.	4.88 12.2 25.6 43.7 66.2 89.3 118. 149. 187.	4.02 13.2 27.2 45.7 67.6 92.2 119. 148. 189. 311.

Focal length = 5.972460Plane of best focus at -0.042 mm.

Table 24.1- Energy Distribution 006 BC01515.

The table gives the diameters of the smallest circles containing specified percentages of the total number of points in each of the four spot diagrams at the plane of best focus. The common center of the circles for a given spot diagram was taken where the density of the points appeared greatest.

The diameters are listed in microns to three significant figures. Note that the diameter of the Airy disk for a perfect lens as a full aperture of f/3.5 is  $4.0\,\mu$  . Dr. R. N. Wolfe of Eastman Kodak Co. Research Laboratories made a similar series of experiments in 1947 in conjunction with some of Herzberger's early work in this field (9). The subject has been extensively investigated as regards automatic data reduction by Goetz and Woodland (10) at IBM. Miyamote (11), Keim and Kapany (12) as well as many others have studied this very interesting optical test.

24.4.2 Technique. There are many ray tracing programs that will give this information. The one developed by Feder (13) is offered here. Again the techniques of using this method of testing are varied but the following one is typical. See Figure 24.2. Three or more rays are traced from plane PP through the entrance pupil, the optical system into image space. The entrance pupil is EE. Frequently among the rays of interest are the upper rim ray (U), principal ray (Pr), and lower rim ray (L). A point B' on the principal ray in image space is picked arbitrarily and, from the ray tracing data, the optical path length BB' is determined. From the ray tracing data for rays U and L as well as those originating at other points (frequently zonal) such as D and F, optical path lengths equal to BB' are laid off along the rays. The termination points C', D', F' and K' are then marked and the curve passing through them constitutes the equiphase front in the plane of the paper. The deviations from a perfect circle (or sphere in three dimensions) are clear and corrections may be made as necessary.

24.4.3 Applications and limitations. This phase front technique has long been used in the design of microwave antennas because of the optical simplicity (generally speaking) of such systems. It is particularly useful in optical design as the phase front may be determined experimentally by long established techniques. This gives the designer an immediate check on how well the optician has fulfilled the prescription given to him. It should be pointed out that the diffraction pattern may now be determined, providing the amplitude distribution over the front is known. In some cases it is simpler to use basically the same technique but actually determine the phase variation over the exit pupil. The amplitude distribution over the exit pupil is determined and the diffraction pattern calculated as before. The possibility of varying the amplitude over the aper-

<sup>(9)</sup> Herzberger, J. Opt. Sec. of Am. 37, 485 (1947).

<sup>(10)</sup> Goetz and Woodland, J. Opt. Soc. of Am. 48, 965 (1958).

<sup>(11)</sup> Miyamoto, J. Opt. Soc. of Am. 48, 57, (1958); 48, 567 (1958), and 49, 35 (1959).

<sup>(12)</sup> Keim and Kapany, J. Opt. Soc. of Am. 48, 351 (1958).

<sup>(13)</sup> Feder, J. Opt. Soc. of Am. 41, 630 (1951).

ture by control of aperture shape, variation of transmission, or illumination with radius has been known for some years. A few of the efforts in this direction are the work of Conder and Jacquinot (14) in spectroscopy, the work of Osterberg and Wilkins (15) with microscope objectives, and the work of Silver (16) on tapered illumination of microwave antennas.

 $20\,^{\circ}$  Off-axis at Gaussian Focus

10° Off-axis, 0.3mm in from Gaussian focus.

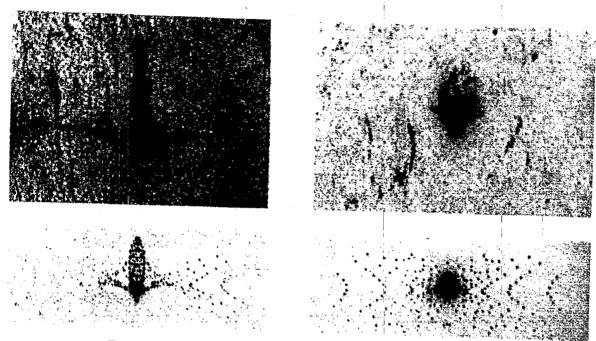


Figure 24.1- Comparison of spot diagram and actual photograph.

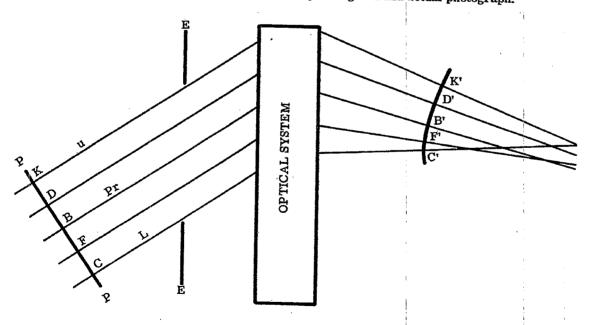


Figure 24.2- Determination of constant phase front from ray tracing data.

Conder and Jacquinot, "Methode pour l'observation des radiations de faible intensite au voisinage d'une raie brillante" Compte Rendus de l'Académie des Sciences (Paris), 208, 1639, (1939). Osterberg and Wilkins, "The Resolving Power of a Coated Objective," J. Opt. Soc. of Am. 39, 553 (1949).

<sup>(16)</sup> Silver, "Microwave Antenna Theory and Design" 187, McGraw Hill (1949).

#### 25 PRODUCTION PHASE OPTICAL TESTS

#### 25.1 INTRODUCTION

- 25.1.1 General. Intrinsic in the design of most optical systems is the calculation of the Seidel aberrations. In this section we will outline procedures for measuring these aberrations experimentally during either the production or the evaluation phase. In this connection it should be noted that over the years different laboratories have developed their own techniques for making these measurements. Frequently the difference between techniques is not so much a matter of difference in basic principle, as it is in the equipment that a particular laboratory happens to have on hand. While there are, then, many, many different ways to make each measurement, we will limit ourselves to one example of each. The interested reader may consult the references for additional information.
- 25.1.2 Theory vs practice. Before leaving this introduction to the measurement of Seidel aberrations, a few words of caution are in order. Aberrations may be completely isolated only in theory. The actual image embodies, simultaneously, all aberrations pertaining to it. This, of necessity, complicates the measurement, and particularly complicates the detailed checking of the theoretical predictions as to the values of the individual aberrations. It should be pointed out also that the accuracy with which the aberrations need to be measured is a function of the importance of the particular aberration to the job at hand. The experiments to be described generally assume a white light source. Chromatic effects are determined by use of the appropriate filters.

#### 25.2 FOCAL LENGTH

- 25. 2. 1 Importance of focal length. Certainly one of the fundamental constants of any optical system that is of prime importance in the evaluation of the significance of all Seidel aberrations is focal length. Not only is the value of the focal length of importance, but also a precise statement of the point from which the focal length is to be measured is mandatory. Some years ago an aerial camera lens was designed and simultaneously the camera body was fabricated, presumably for the same focal length systems. When the lens was installed in the body, a photographic check showed hardly any semblance of an image. To say that it was "out of focus" was charitable. The error was tracked down ultimately to the fact that focal length meant measurement from the secondary principal point to the lens designer and meant from the rear surface to the machinist. Through human error, both lens and body were allowed to be fabricated on this erroneous basis. Since the secondary principal point lay several inches inside the lens, the horribly blurred image was not surprising. Recently a similar situation developed in a missile-tracking system where the optical designer measured the focal point with respect to the front surface and the machinists assumed that it was measured from the rear surface (the optics involved a thick mirror). Since the system was quite fast with short focal length, the one inch central thickness of the thick mirror played havoc with the performance of the system when finally assembled.
- 25. 2. 2 Measurement of focal length. While there are many methods for measuring the focal length of an optical system (1) (2) (3) (4), one of the most accurate for lenses of medium focal length employs the nodal slide. A photograph of one in use at the National Bureau of Standards is shown in Figure 25. 1. The essential part of the nodal slide is the provision for moving the lens system longitudinally with respect to a vertical axis of rotation. This vertical axis is mounted so that it may be positioned longitudinally with respect to a collimator of appropriate size. Usually the object for the collimator is a very small point source set at the focal point of the collimator.
- 25. 2. 3 Test setup. The equipment is set up as shown schematically in Figure 25. 2. In use the magnifier or microscope is set up approximately at the focal point. The lens under test is then moved backward and forward along the nodal slide until rotation of the nodal slide through a small angle, B, produces no sidewise shift in the image. The focal length is then the distance between the axis of rotation of the nodal slide and the appropriate focal point of the magnifier or microscope. There are many variations on this technique, some employing auto collimation, some focusing the image on a card, etc. Negative optics may also be tested in this manner by the addition of a positive lens of known characteristics. Knowing the position of the secondary nodal point (coincides with the principal point if the index of refraction of image space is air), the focal length may be specified with respect to the vertex of the rear surface (this distance is known as the "back focal length") or to any other convenient part of the lens.

<sup>(1)</sup> Cheshire, Trans. Optical for (London) 22, 29 (1920-1921).

<sup>(2)</sup> Kurtz, Jour. Opt. Sci. of Am. and Rev. of Sci. Instr. 7,103 (1923).

<sup>(3)</sup> Searle, Experimental Optics, Exp. 37, 185 Cambridge Univ. Press (1925).

<sup>(4)</sup> Wagner, Experimental Optics, Exp. 67, 136, Wiley (1929).

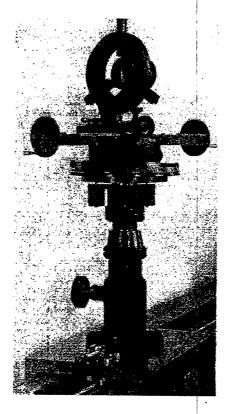


Figure 25. 1- Nodal slide developed at the U. S. National Bureau of Standards.

 $L_c = collimating system$  $L_x = system under test$ 

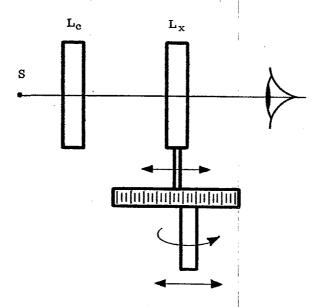


Figure 25.2- Measurement of focal length by use of visual nodal slide.

#### 25.3 LONGITUDINAL SPHERICAL ABERRATION

25.3.1 On-axis performance. Of considerable importance in almost all optical systems is the on-axis performance. Since the principal use of many visual optical systems is tracking in one form or another, systems are ultimately pointed directly at the target. The nature of the image on-axis is thus important. A factor also of much significance is the degree to which the system may be "opened up" and still maintain a good image. This latter requirement involves spherical aberration.

25.3.2 <u>Hartmann test.</u> While there are many techniques for doing this <sup>(5)</sup> <sup>(6)</sup>, the simplest is perhaps the Hartmann test <sup>(7)</sup> <sup>(8)</sup> <sup>(9)</sup>. It may be done either photographically or visually and can be made reasonably sensitive. It is probably not as accurate as a newer method developed by Washer, but is chosen here for its directness and simplicity.

25.3.3 Test procedure. Blocking off all but holes 1 and 8 in the Hartman Disk shown in Figure 25.3 will give the marginal focus. Holes 2 and 7 should be located at the zonal positions, and their intersection will give the zonal focus. The paraxial focus may be determined with holes 4 and 5, or by direct inspection of the lens stopped down to a very small circular aperture. From the data just obtained the longitudinal spherical aberration may be determined. Filters may be used to get the aberration for different colors if so desired.

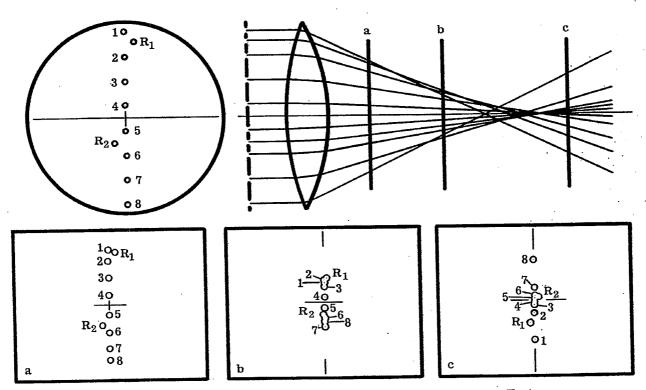


Figure 25.3-Measurement of spherical aberration by the Hartman Test.

(After Strong's, Concepts of Classical Optics, W.H. Freeman and Co. 1958)

<sup>(5)</sup> Washer, Jour. of Res. of Nat'l Bureau of St'nds 61, No. 1, 31, (July 1958).

<sup>(6)</sup> Monk, Light-Principles and Experiments 349, McGraw-Hill (1937).

<sup>(7)</sup> Strong, Concepts of Classical Optics, 354 Freeman (1958).

<sup>8)</sup> Hartmann, Zeit. f. Inst. XX IV, 1 (1904); and subsequent papers in 1904.

<sup>(9)</sup> Bureau of Standards Scientific Papers No. 311 and 494.

#### 25.4 COMA

- 25.4.1 Asymmetrical flare. Asymmetrical flare produced by coma is one of the most important aberrations to eliminate. The reason for this is that most of the other aberrations produce an image degradation that is more or less symmetrical with respect to the principal ray. For example this means that even though astignatism may be present, the system may be pointed with a high degree of accuracy by centering the point of greatest density of the image on the cross hairs, etc. When the image is degraded asymmetrically, this same procedure can produce a pointing, or boresight, error.
- 25.4.2 <u>Collimator check for coma.</u> Coma, being an off-axis aberration, is somewhat difficult to separate from astigmatism. Usually in testing optical systems the optician will simply use a collimator to illuminate his lens at successive angles off axis. The focal plane image is then studied, and, if the flare is more than that allowed in the specifications, the system is reworked.
- 25. 4. 3 Hartmann disk. Coma may be demonstrated to a fairly successful degree by use of the Hartmann disk placed before the lens with the lens illuminated by off-axis parallel light and image space then studied, as indicated in the measurement of spherical aberration. Another simple method for measurement of coma using the Hartmann method is described in Hardy and Perrin (10), and refers to a method described previously in the National Bureau of Standards Scientific Papers No's. 311 and 494.

# 25.5 ASTIGMATISM AND CURVATURE OF FIELD

25.5.1 Measurement of astigmatism. Astigmatism may be measured accurately by a series of Foucault Tests with the knife edges at right angles in the basic manner described in the section devoted to the Foucault test. It may also be measured quite simply by the arrangement illustrated in Figure 25.4. In practice, the lens under test,  $L_x$ , is rotated and the traveling microscope,  $M_x$ , is adjusted until the image of the reticle,  $R_x$ , is found. The microscope is then adjusted; first until the vertical lines are in best focus; then until the horizontal lines are in best focus.  $L_x$  is then rotated to successive angles up to the maximum field angle, and the positions of best focus as just described measured at each angle. A plot of the positions of best focus for the vertical lines will give the sagittal (secondary) focal surface, and the corresponding plot of the positions of best focus for the horizontal lines will give the tangential (meridional or primary) focal surface. The reason why the tangential plane images the horizontal lines best is shown clearly in Jenkins and White (11). The microscope must of course be movable laterally as well as longitudinally to examine the astigmatism at various points on the focal plane. If in addition to making determinations of the position of best focus of the horizontal and vertical lines, one also notes the position of best overall focus of the central point in the grid system, the curvature of field may be determined.

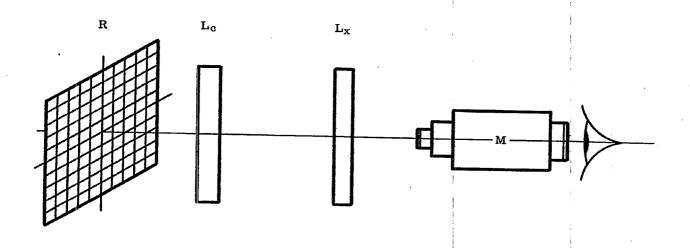


Figure 25.4- Measurement of astigmatism with a grid reticle.

- (10) Hardy and Perrin, The Principles of Optics, 382, McGraw-Hill, (1932).
- (11) Jenkins and White, Fundamentals of Optics, (2nd edition), 139, McGraw-Hill, (1950).

- 25.5.2 <u>Determination of curvature of field</u>. Generally, curvature of field may be determined with respect to the flat focal plane of a camera by placing a flat glass plate with a grease pencil mark across a giameter in the position occupied by the film. The grease pencil mark is towards the camera lens. One then notes the position of the traveling microscope when focused on the grease pencil mark. The camera being set up similar to that in Figure 25.4, the microscope is then focused on a star image (or central point of the reticle). The difference between the two readings is a measure of the curvature of field. Curvature of field measurements are very important in visual systems as the curvature of field of the object must match that of the eveplece or else considerable image degradation ensue.
- 25.5.3 Consistency of test procedure. One should note clearly in all of these testing methods that if a system is to be used visually, then ideally it should be tested visually. Photographic testing does have its advantages, however, as it furnishes a record.

#### 25.6 DISTORTION

- 25.6.1 <u>Importance of distortion study.</u> In optical systems designed essentially for visual observation and study of objects on-axis, the aberration known as distortion is really not too important. There are many systems, however, where, while the target may be centered in the eyepiece, measurements must be made over the entire field of view. An example of such a system is a rangefinder.
- 25.6.2 <u>A rapid check for distortion</u>: Distortion may be measured photographically very simply by replacing the microscope in Figure 25.4 by a good quality camera, known to be well corrected over the field of the optical system under test. The grid reticle is then photographed and the distortion, whether pin cushion, barrel, or irregular, is immediately obvious when D is set = 0.
- 25.6.3 An accurate distortion measurement for small optical systems. An excellent method of making this measurement for small optical systems with the basic nodal slide has been outlined by Washer, Tayman, and Darling (12). The procedure is as follows:
  - (a) The optical system under test is placed on the nodal slide shown in Figure 25.2.
  - (b) A measuring microscope is adjusted with respect to the lens until a focus is found.
  - (c) The lens system is then moved in the usual way along the nodal slide until small rotations (the microscope having been kept in focus by longitudinal movement) show no lateral movement of the image.
  - (d) Assuming that the focal length, f, is now measured or known, it is clear that if the microscope were moved off-axis yet remaining in this focal plane that the distance to the lens would now be f sec  $\beta$ .
  - (e) If now, instead of moving the microscope, the lens is rotated in the nodal slide by an angle  $\beta$  and moved longitudinally a distance (f sec  $\beta$  f) or f (sec  $\beta$  1) toward the collimator, the microscope should again see the image clearly. Actually the image will probably not be on-axis and the microscope will have to be moved laterally a small distance to pick it up again.
  - (f) The reading of the micrometer measuring this lateral shift is noted as R.
  - (g) The lens is now rotated through an angle  $-\beta$  and the microscope, when repositioned, gives a reading L.
  - (h) The distortion. D, at the angle  $\beta$ , is then given by the simple expression,

$$D_{\beta} = \frac{(R-L)}{2} \sec \beta \tag{1}$$

# 25.7 AUXILIARY OPTICAL MEASUREMENTS

25.7.1 Introduction. In the fabrication and testing of optical instruments, it is frequently necessary to make measurements that are made considerably less frequently in regular machine shops. One of these measurements is the radius of curvature of spherical and aspherical surfaces: another is the measurement of the index refraction.

## 25.7.2 Radii of curvature.

25.7.2.1 The radius of curvature of an optical surface whose diameter is on the order of 1 - 3" may be done very conveniently with a spherometer (13). This device takes many forms—one of which is shown in Figure 11.9.

<sup>(12)</sup> Washer, Tayman, and Darling, Journal of Res. of N.B.S., 61, No. 6, 509 (1958).

<sup>(13)</sup> op. cit., (10), pg. 366.

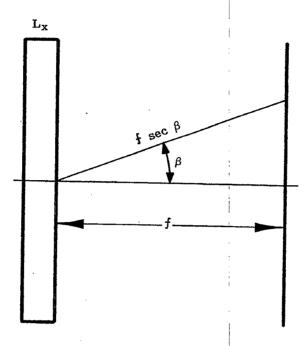


Figure 25.5- Basic diagram for measurement of distortion by nodal slide.

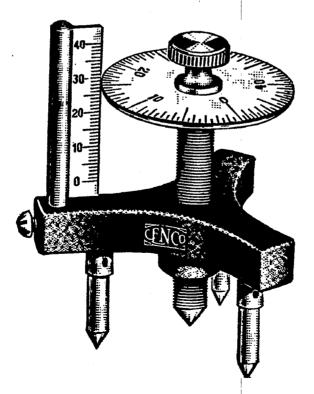


Figure 25.6- Elementary spherometer.

25.7.2.2 In use, the central spindle is screwed up and the three legs placed on the surface in question. The spindle is screwed down until it just touches the surface. The spherometer is then placed on a flat surface and the distance, S, the spindle must be advanced to meet the flat is noted. The procedure is reversed for a concave surface. The radius of curvature of the surface is then obtained from the following equation,

$$\mathbf{r} = \frac{\mathrm{d}^2}{6\mathrm{s}} + \frac{\mathrm{s}}{2} \tag{2}$$

where d = the average distance between the legs.

25.7.2.3 For large surfaces the Foucault method described in paragraph 25.8.2 is used. For very small surfaces, less than about an inch across, a different method is employed. A provision is made for illuminating from the side, the cross hairs or reticle of a Gauss eyepiece, or equivalent, in a microscope or short focal length telescope (the choice depending upon the curvature of the sample to be tested). The microscope is focussed first on the surface of the sample, and the longitudinal position of the microscope recorded. The microscope is then racked back until there is no parallax between the illuminated cross hairs and the image from the surface. The cross hairs are then at the center of curvature. This position of the microscope is also recorded. The difference between the two positions is the radius of curvature. A telescope would be used in exactly the same way for greater radii of curvature. A similar technique can be used for positive surfaces.

## 25.7.3 Index of refraction.

25.7.3.1 Where it is possible to grind and polish a small sample of the material, the spectrometer furnishes a very fundamental method for measuring the index of refraction. The theory and method are outlined in Hardy and Perrin (14) and Sawyer (15). With a good spectrometer the values of the index so determined are good to  $\pm$  .00003. A high precision spectrometer is shown in Figure 25.7.

25.7.3.2 For many samples it is not possible to get the sample in the form required for the spectrometer and for these the refractometer is frequently well suited. One of the many refractometers is the Pulfrich (16). This method, based on refraction at the critical angle, will give values correct to  $\pm$  2 parts in the fifth decimal place, and is shown schematically in Figure 25.8.

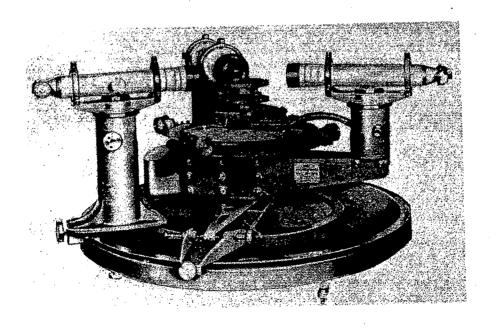


Figure 25.7 - The Guild-Watts precision spectrometer.

(14) op. cit., (10), 549

(16) op. cit., (10), 350.

<sup>(15)</sup> Sawyer, Experimental Spectroscope, 55, Prentice Hall, (1944).

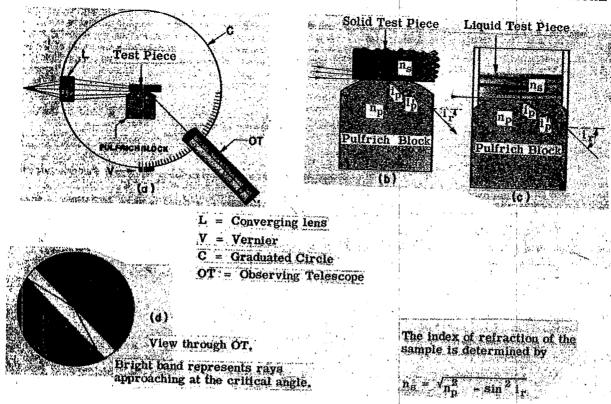


Figure 25.8 - Schematic of the Pulfrich refractometer.

# 25.8 OPTICAL DEVICES, TESTING SYSTEMS AND PROCEDURES

# 25.8.1 Interferometry principles.

25.8.1.1 The most common and simplest method for testing flatness of polished surfaces of glass or other transparent material utilizes interference fringes that are formed between the tested surface S<sub>1</sub> and an optically flat surface S<sub>2</sub> as illustrated in Figures 25.9 and 25.10. The preferred source of light is an unfiltered, tubular, Cooper-Hewitt lamp, L, which is provided with a diffusing reflector, R, and a diffusing glass plate, G. The advantage of the arrangement of Figure 25.9 is that it permits the interference fringes to be viewed at normal incidence. The positions of the light source and the eye may be interchanged. Figure 25.10 illustrates the most common arrangement where viewing at normal incidence is sacrificed to gain greater freedom as regards working space. The light emitted by the Cooper-Hewitt mercury lamp is preponderantly green. It can be rendered quite monochromatic at 5461 A, whenever desired, by means of readily available optical filters that can be held near the eye. The interference fringes are usually viewed in unfiltered light. Contrast in the fringes is improved by placing black felt or paper beneath the optical flat in the manner indicated.

25.8.1.2 It should be noted that the interferometer surfaces  $S_1$  and  $S_2$ , Figures 25.9 and 25.10, are in close contact. Excess dust and other dirt must be removed in order to reduce the thickness d of the airing interference fringes belong to a select class of fringes known as Fizeau or as Newton's fringes. The principles underlying these fringes have been discussed in paragraphs 16.12.1.2 and 16.13.1.5. These fringes are characterized by the following important properties and propositions.

- (a) The interference fringes appear in good contrast when one focuses upon the air-film between the interferometer surfaces, S<sub>1</sub> and S<sub>2</sub>, provided that the reflectances of these surfaces are approximately alike. It is often said that the fringes appear to be localized in the film.
- (b) Because the separation, d, of the interferometer surfaces is small, the location laterally of the fringes between the surfaces does not depend markedly upon the angle of incidence, provided that one views the fringes approximately along the normal to the surfaces  $S_1$  and  $S_2$ .

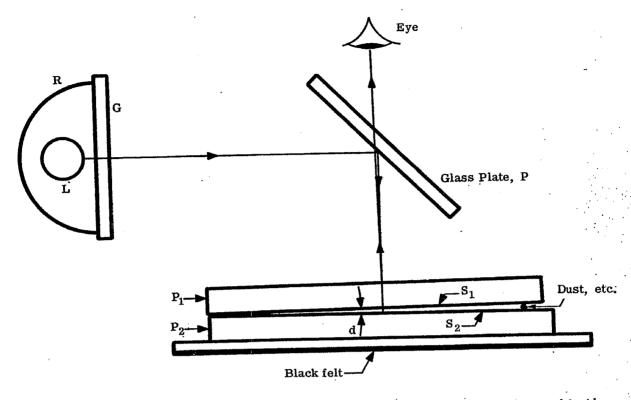


Figure 25.9 - Simple interferometer for viewing Fizeau fringes or Newton's fringes at normal incidence.

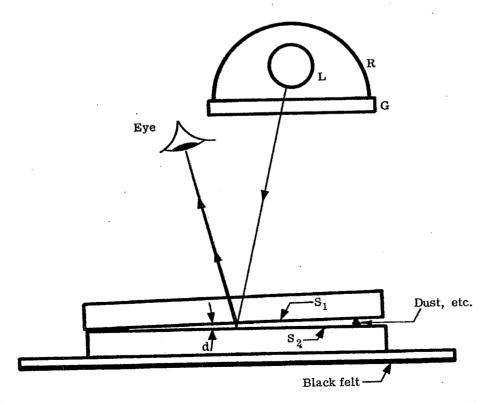


Figure 25.10- Most commonly used method for illuminating the interferometer. The fringes formed between surfaces  $S_1$  and  $S_2$  are observed at near - normal incidence.

- (c) Each fringe marks the locus (lateral) of points for which the separation, d, is a particular constant. This constant is different for each fringe.
- (d) When either surface is moved or distorted by the application of force or heat, each fringe moves in such a direction as to maintain the constant separation, d, associated with that fringe.
- (e) Upon passing from one fringe to the next fringe of equal brightness or darkness, the separation, d, changes by one half wavelength.
- (f) Upon passing from a bright fringe to the next dark fringe, the separation, d, changes by one fourth wavelength. It is assumed tacitly in
   (e) and (f) that the surfaces do not possess discontinuous jumps.
- (g) When surfaces S<sub>1</sub> and S<sub>2</sub> are of nonabsorbing materials such as glass, dark fringes occur at separations, d, for which

$$d = \nu \frac{\lambda}{2}$$
;  $\nu = 0, 1, 2, 3, 4$ , etc. (3)

and bright fringes occur at separations, d, for which

$$d = \mu \frac{\lambda}{4}$$
;  $\mu = 1, 3, 5, 7$ , etc. (4)

wherein  $\lambda$  denotes wavelength.

Of these propositions and properties, (c) and (d) are of the greatest importance to the maker of optical flats. These two propositions or rules enable him to interpret the fringes for high and low areas on the surface under test. The optical worker recognizes fringes as contour lines on a contour map of his surface. Movement of the fringes upon application of pressure serves to distinguish the up-hill direction. Propositions (e) through (g) reveal the heights between contour lines. It is emphasized that propositions (e) and (f) are more general (and hence more often correct) than (g).

- 25.8.1.3 As one tests for flatness of a surface, the fringes become straighter as the surface becomes flatter. The effect of reducing the angle of the air-wedge between surfaces  $S_1$  and  $S_2$  by the removal or crushing of dust particles, is to widen the fringes and to increase their curvature except when  $S_1$  is optically flat. Let, h denote the fringe width, i.e. the distance from one bright fringe to the next. For optical flats of high quality, the degree of flatness can be specified by requiring that any fringe shift from straightness shall not exceed a stated fraction of the fringe width, h, over a stated diameter or other dimension of the tested surface. As an example of the sensitivity of the method, a fringe shift h/5 corresponds to a change of separation, d, by the amount  $10^{10}$ . Fringe shifts smaller than h/10 become difficult to detect and to measure in this type of interferometer.
- 25.8.1.4 In a second, and often preferred test for optical flats of high quality, the surfaces  $S_1$  and  $S_2$  are placed so closely in contact or are rendered so nearly parallel that a single fringe spreads over surface  $S_1$ . This broad fringe is examined for uniformity of color with an unfiltered source of light. It is customary to specify without further qualification that the surface shall be "polished to a uniform color".
- 25.8.1.5 The following procedure applies to that great class of test cases in which the departure of more than one fringe from flatness is tolerated. If the test surface is convex, only one area will contact the reference flat and this area will be surrounded by a number of alternately dark and bright Newton's fringes. The specification of flatness may be stated as the number of allowable Newton's fringes per inch, or other unit. If the test surface is concave, a ring-shaped area will contact the reference flat. The number of concentric fringes within this area can be counted and compared with the maximum tolerable number of Newton's fringes per inch or other unit. In practice, the surface S<sub>1</sub> is likely to display one or more convex or concave areas. Close examination of the fringes will distinguish between these convex and concave areas. If pressure is applied to a convex area in such a manner as to reduce the separation, d, between surfaces a given fringe about the area of closest contact must move outward from its center in order to maintain the locus of points for which d is a given constant.
- 25.8.1.6 No essential modification of the method of Figures 25.9 or 25.10 is needed for testing spherical surfaces. The reference flat,  $S_2$ , is replaced by a concave or convex reference surface whose radius is equal to the desired radius of the completed test surface. Suppose that surface  $S_1$  has a smaller radius than surface  $S_2$  as illustrated in Figure 25.11. Concentric Newton's fringes will appear around point O. The maximum allowed number of Newton's fringes per inch along the radial direction from O may be stated as the permissible departure of surfaces  $S_1$  from the "test glass" having the surface of reference  $S_2$ . For surfaces,  $S_1$ , of high optical quality, the radius of surface  $S_1$  will be made to match that of  $S_2$ . At the match point, it will

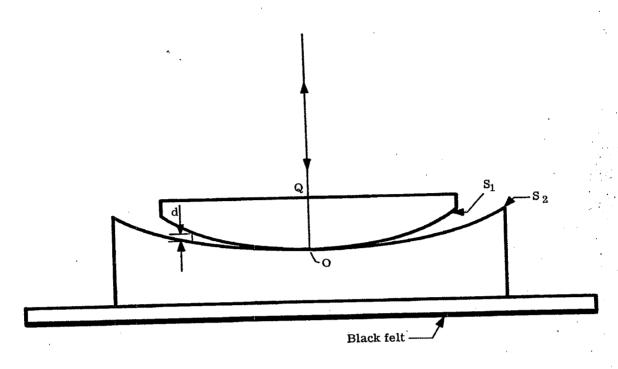


Figure 25.11- Arrangement of the interferometer surfaces  $S_1$  and  $S_2$  for obtaining Newton's fringes.

be possible, as in paragraph 25.8.1.4, to place surfaces  $S_1$  and  $S_2$  into sufficiently close contact so that a single fringe spreads over surface  $S_1$ . For work of highest quality, it is customary to specify that this single fringe shall be made uniform in color.

- 25.8.1.7 The method of the sagitta (see paragraphs 16.13.1.1 and 16.13.1.6) enables one to make a good estimate of the radius of surface  $S_1$  when this radius departs only slightly from that of the test glass. Consequently, it is not always necessary to provide a test glass whose radius is equal to that of the completed surface  $S_1$ .
- 25.8.1.8 When elliptical fringes appear around point O, Figure 25.11, surface  $S_1$  is not spherical. The minor and major axes of the elliptical fringes may be measured, and the ratio of the minor axis to the major axis computed. This ratio is a measure of the ellipticity of surface  $S_1$  and is often utilized as a specification of the maximum tolerable ellipticity. When departures of many fringes from the test glass can be tolerated, another measure of ellipticity is to count the number of fringes along some convenient length in the direction of the major and minor axes of the elliptical fringes and to utilize the ratio of these fringe counts in specifying the tolerable ellipticity.
- 25.8.1.9 An extreme amount of irregularity in the shape of the fringes is an indication that the tested surface has been improperly polished or molded. "Orange peel" and other defects of polished surface produce irregularities in the observed pattern of fringes.
- 25.8.1.10 Contrast in the fringes deteriorates as the reflectances  $a_1$  and  $a_2$  of surfaces  $S_1$  and  $S_2$  become more unlike. The light beams reflected from these two surfaces can interfere to produce systems of fringes having zero intensity in the dark fringes (and hence displaying maximum contrast) only when the amplitudes,  $a_1$  and  $a_2$ , of the two, coherent, interfering beams are alike. The distribution of intensity in the fringe system when  $a_1$  and  $a_2$  are unlike can be ascertained from paragraphs 16.1.1.3, 16.1.1.5, 16.8.1.1, 16.8.1.2, and 16.9.1.4. In spite of reduced contrast in the fringes, the interferometers of Figures 25.9 and 25.10 are often applied to testing polished surfaces of metals. With metals and other opaque substances, surface  $S_2$  of Figures 25.9 and 25.10 must be that of the opaque material.
- 25.8.1.11 The reflectance of the "test glass" can be increased by the deposition of a high reflecting coating or by increasing the refractive index of the test glass. In this way, contrast in the fringe system will be improved in testing high reflecting surfaces. Metallic surfaces and coatings produce phase changes on reflection

that differ from zero (as in the reflection from a glass-to-air interface) or that differ from  $\lambda/2$  (as in the reflection from an air-to-glass interface). Consequently, Equations (3) and (4) require modification. However, the effect of the modified phase changes on reflection is only to shift the location of the fringes. As a result, the interpretation of sections 25.8.1.3 to 25.8.1.9 remains unchanged when applied to coated or metallic surfaces. For testing surfaces of high quality, any coating applied to the test glass must be extremely uniform in thickness and composition because phase changes on reflection can vary appreciably with thickness and composition of the coating.

- 25.8.1.12 As the reflectance of surfaces  $S_1$  and  $S_2$  is increased from that of polished glass, the nature of the interference fringes formed by the interferometers of Figures 25.9 and 25.10 alters gradually until, finally, these fringes are classified as multiple beam interference fringes. Inter-reflections between surfaces  $S_1$  and  $S_2$  serve to sharpen the fringes formed by the reflected or the transmitted light beams in the manner discussed in paragraph 16.17. With suitable changes in the technique of observation, these sharp (narrow) fringes can be used to detect and measure surface irregularities as small as 10 Angstroms in height. Polished surfaces are found to be rough terrains whose hills and valleys vary in height and depth from 10 to 120 Angstroms. These small irregularities are not visible in the "double beam" interferometer of Figure 25.10 when surfaces  $S_1$  and  $S_2$  are of polished glass.
- 25.8.1.13 The interferometer method of Figures 25.9 25.11 is essentially a "contact" method. Experience and care are required in order to avoid undue scratching of one surface by the other. Life of the test glass is shortened by wear and scratching. A number of convenient interferometer techniques can be applied to testing flat surfaces without placing two surfaces in contact. However, existing interferometer methods for avoiding contact between two spherical surfaces are either so inconvenient to manipulate or so difficult to interpret that the contact method remains the standard method of the optical shop.

## 25.8.2 The Fizeau Interferoscope.

- The Fizeau interferoscope, Figure 25.12, is a double beam interferometer that permits one relatively flat surface,  $S_1$ , to be tested against another flat surface,  $S_2$ , without placing these two surfaces in contact. The increased "working distance", d, is made possible without undue loss of contrast in the fringes by restricting the effective size of the light source to a pinhole, H, and by illuminating the pinhole with monochromatic light. Since improved monochromaticity and smaller pinholes entail loss of light, the ultimate working distance, d, is restricted by the required level of illumination. Distances of d greater than 2cm must be considered "large" and should be avoided in designing and planning the interferoscope. Fizeau interferoscopes have been varied in design to meet the needs of various users. The use of beamsplitters as illustrated in Figure 16.2 is to be avoided in order to conserve light. The instrument illustrated in Figure 25.12 is an example of one of the more flexible types of interferoscopes. When this instrument is to be used for testing surface S1 against surface S2, the two sets of leveling screws, L1 and L2, are adjusted in the order mentioned so that the light beams reflected from S1 and S2 are refocused by the collimator as images of the pinhole, H, at the aperture, A. When the pinhole images formed at A are brought almost into unison by further relative adjustments on screws L1 and L2, straight fringes will appear on the observer's retina as he looks through the aperture, A, provided that the test surface, S1, is optically flat. Interpretation of the interference fringes remains the same as with the simpler interferometers of Figures 25.9 and 25.10. Except for the more convenient and elegant manner in which the relative inclinations of surfaces \$1 and \$2 can be adjusted with the aid of the leveling screws, the procedures and methods of sections 25.8.1.3 - 25.8.1.5 apply again.
- 25.8.2.2 Many optical workers use the Fizeau interferoscope exclusively for ascertaining the degree of parallelism of the surfaces of a plane parallel plate. One surface,  $S_1$ , of plate,  $P_1$ , is first made optically flat. With interferoscopes of the type illustrated in Figure 25.12, test plate,  $P_2$ , is removed from table,  $T_2$ . The leveling screws  $L_1$  are adjusted so that the two beams reflected from surfaces  $S_1$  and  $S_{11}$  are focused within aperture A as images of pinhole, H. If both surfaces of plate  $P_1$  are optically flat but are not quite parallel, the fringes are parallel to the line of intersection of surfaces  $S_1$  and  $S_{11}$ . The fringes curve as surface  $S_{11}$  departs from flatness. As  $S_{11}$  is made optically flat and brought into parallelism with  $S_1$ , a single fringe of uniform intensity spreads over the field of view determined by the area of plate  $P_1$ . Equations (3) and (4) must be modified to include the refractive index n of plate  $P_1$ . Thus dark fringes occur when

$$nd = \nu \frac{\lambda}{2}$$
;  $\nu = 0, 1, 2, 3, 4, \text{ etc.}$ ; (5)

and bright fringes occur when

$$nd = \mu \frac{\lambda}{4}$$
;  $\mu = 1, 3, 5, 7, etc.$  (6)

<sup>\*</sup> See paragraphs 16.4 and 16.5 for the effect of monochromaticity, pinhole size and separation, d, on fringe contrast. Other principles underlying the use and interpretation of the Fizeau interferoscope are discussed in 16.2 and 16.2.2.

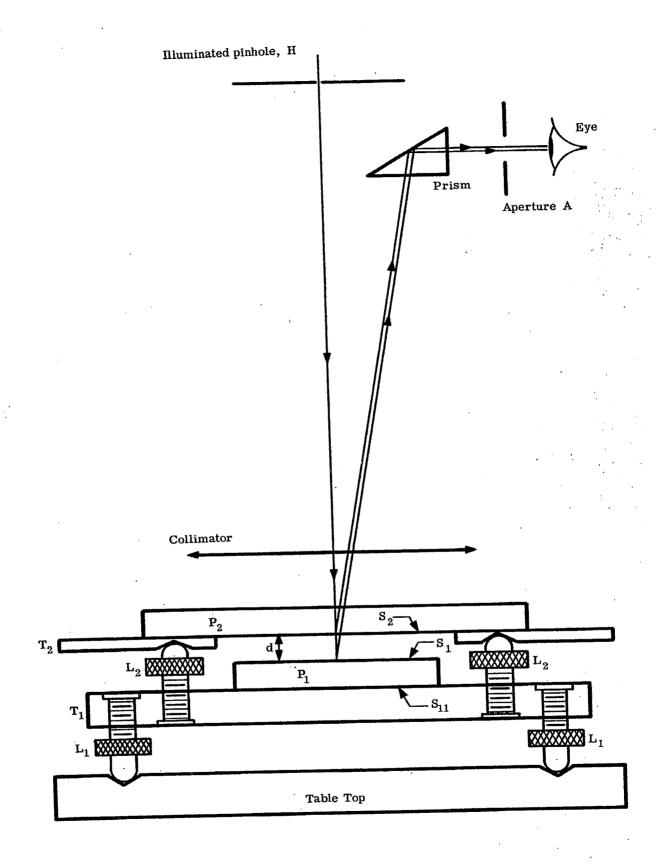


Figure 25.12 - A Fizeau interferoscope.

In other words, dark fringes occur when the optical path nd of the plate is equal to any integral number of half wavelengths, and bright fringes occur when the optical path is an odd number of quarter wavelengths. These conclusions can be expected intuitively when one considers that the phase change on reflection will be  $\lambda/2$  at surface  $S_1$ , and 0 at surface  $S_{11}$ , so that the difference in the phase changes on reflection is  $\lambda/2$ . The beam reflected from  $S_{11}$  passes through plate  $P_1$  twice, and thus is increased in phase by twice the optical path or 2(nd). Because the phase change on reflection is  $\lambda/2$  at the air-to-glass interface,  $S_1$ , the two interfering beams proceed toward the observer with a phase difference,  $\Delta$ , given by

$$\Delta = 2nd - \lambda/2. \tag{7}$$

If now, nd is given by Equation (5),  $\Delta = (\nu - 1/2) \lambda$  so that destructive interference takes place. But if nd is given by Equation (6),  $\Delta = (\mu - 1) \lambda/2$ . Since  $\mu - 1$  must be an even number,  $(\mu - 1)/2$  is an integer and  $\Delta$  is an integral number of wavelengths. Hence we verify that constructive interference takes place when the optical path obeys Equation (6).

25.8.2.3 As an example of the sensitivity of the Fizeau interferoscope in testing for parallelism of the two surfaces of a plate, let us suppose that the diameter of the plate is 5cm, that its refractive index is 1.5, that the wavelength  $\lambda$  is 0.5461 x 10<sup>-3</sup> mm and that the optically flat surfaces define a wedge whose optical path differs by  $\lambda/2$  at the extreme ends of the wedge. Suppose that a bright fringe appears at the thin edge of the wedge. A bright fringe must appear at the thick end of the wedge since the optical path is greater by  $\lambda/2$  at the thicker end of the wedge. This conclusion follows at once from Equation (6); for if nd is increased by  $\lambda/2$ ,  $\mu$  is increased to the next odd number,  $\mu + 2$ , the spectral order number of the next bright fringe, Equation (5) will be satisfied at the center of the plate so that a dark fringe occurs here. The field of the plate will appear very nonuniform. It presents one dark and two bright fringes. Despite this nonuniformity, the angle,  $\alpha$ , between the surfaces of the plate is less than one second of arc. Since nd changes by  $\lambda/2$  across the plate, the thickness of the plate changes by  $\lambda/2$ n. Therefore

 $\alpha = \frac{\lambda/2n}{\text{diameter}} = \frac{0.5461 \times 10^{-3}}{3 \times 50} = 3.64 \times 10^{-6}$  radians or 0.75 seconds of arc. If the variation of intensity across the plate is reduced to 0.1 fringe,  $\alpha$  will be reduced to 0.075 seconds.

#### 25.8.3 A Modified Michelson Interferometer.

A flexible instrument, with the aid of which any surface (whether glass or metallic) can be tested 25.8.3.1 for flatness against an optical flat without contact, is illustrated in Figure 25. 13 as a specialized form of Michelson's interferometer. If  $S_1$  is a surface of polished glass, surface  $S_2$  is chosen as polished glass. If surface S<sub>1</sub> is metallic or high reflecting, an optical flat P<sub>2</sub> having surface reflectance approximating that of S1 will be provided. The user can afford to supply several optical flats since these flats will not have to be replaced because of wear and scratches. The housing, H, is compact and rigid. It is designed to support the beamsplitter and the optical flat, P2, with minimum vibration. The line OB is pointed in the vertical direction so that the test plate, P1, is simply laid upon an auxiliary, stable support, Q. The arms OB and OA of the interferometer will be designed so that these arms are nominally of equal length and so that these arms are easily adjusted for equal lengths. Adjusting screws L can be utilized both for equalizing the lengths of the arms and for tilting surface S2 with respect to S1 to control the fringe width. The mechanism for tilting plate P2 must be designed with great care because it is this mechanism that determines the operator's convenience in making quick and certain adjustments of the fringe widths as well as in checking arms OB and OA occasionally for equality by finding the "white light position". Supports Q should be ground to equal thicknesses in order to avoid hunting for the white light position each time Q is replaced by another support. An auxiliary, tiltable mirror M is provided for deflecting the light beam toward the observer.

25.8.3.2 Michelson's interferometer does not differ in principle from the interferometers of Figures 25.9 and 25.10. Consequently, the conclusions of paragraph 25.8.1.2 remain valid and the methods of paragraphs 25.8.1.3 - 25.8.1.5 apply again. Figure 25.14 illustrates why the Michelson interferometer ochaves as the interferometers of Figures 25.9 or 25.10.

#### 25.8.4 4 The Twyman-Green Interferometer.

25.8.4.1 The Twyman-Green interferometer is similar to the Fizeau interferoscope as regards basic principles\* and interpretation. In both of these interferometers, the allowable optical path difference between the two interfering waves is increased by reducing the effective size of the source to that of a pinhole and by increasing the monochromaticity of the source of light. If, for example, mercury arcs are utilized, they should be operated at reduced pressure and followed by a high quality filter for 5461 Angstroms. The Twyman-Green interferometer has many points of mechanical similarity with Michelson's interferometer. However, Michelson's interferometer is invariably intended for use with broad sources of light.

<sup>\*</sup> The principles underlying the Twyman-Green interferometer are discussed in paragraph 16.3.

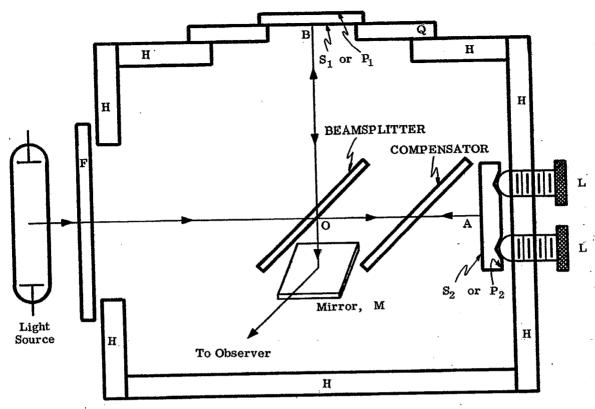


Figure 25.13- A modified Michelson interferometer.

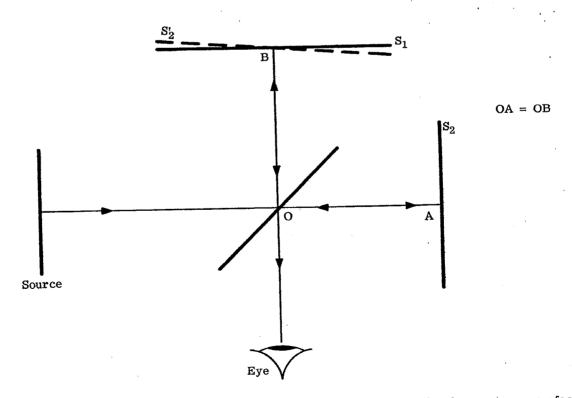


Figure 25.14- Michelson's interferometer as a method of Fizeau fringes. The observer sees surface  $S_2$  as though it were located at  $S_2$ , consequently, the fringes are formed as by reflection from two surfaces,  $S_1$  and  $S_2$ , that are in close contact.

- 25.8.4.2 Emphasis in the design of the Twyman-Green interferometer is placed upon taking advantage of the permissibly large optical path difference, d, between the two arms of the interferometer in measuring the variations of optical paths through plates or prisms. The instrument is not intended for regular use in checking flatness of surfaces. For maintaining best contrast in the fringes, the optical paths OA and OB, Figure 25.15, should be kept approximately equal. Accordingly, end mirror,  $M_2$ , is mounted on a slide that permits  $M_2$  to be moved without appreciable wabble along the line AO. An iris diaphragm whose opening can be reduced to 0.75 mm, or less, in diameter is ordinarily used as pinhole, H. By means of this adjustable iris, the observer can choose his own compromise between brightness and contrast of the fringes. End mirrors,  $M_1$ , and  $M_2$ , are adjusted by means of suitable mechanisms (not shown) involving screws,  $L_1$  and  $L_2$ , until pinhole images of H, formed after reflection at the end-mirrors, appear within aperture, A, at the rear focal plane of the telescope. If these pinhole images are not too far apart within A, fringes will be seen when the eye is placed behind A. One obtains the desired fringe width by further adjustments of  $L_1$  and  $L_2$ . Failure to obtain good fringes as the pair of pinhole images is brought into unison indicates that the optical path difference between arms OA and OB is
- 25.8.4.3 Figure 25.15 illustrates the arrangement for testing plates, P. Suppose that one knows that the surfaces of the plate are optically flat and that he wishes to check the uniformity of the optical paths through the plate. These optical paths can differ due to nonparallelism of the surfaces or due to nonuniformity in the refractive index, n. In one procedure the end mirrors,  $M_1$  and  $M_2$ , are adjusted so that one fringe spreads over the field of view before plate, P, is inserted. The effects of introducing plate, P, are then observed. If the broad fringe is left practically undisturbed, the optical paths through the plate are sensibly uniform. The appearance of straight fringes indicates that the surfaces of the plate are not parallel. Irregularities in the fringes indicate that the refractive index is not constant. Let  $\Delta t$  and  $\Delta n$  denote variations in the thickness, t, and refractive index, n, of the plate, respectively, and let  $\Delta d$  denote the corresponding variation in the optical path difference, d, between the two arms of the interferometer. If  $\Delta n$  is negligible,

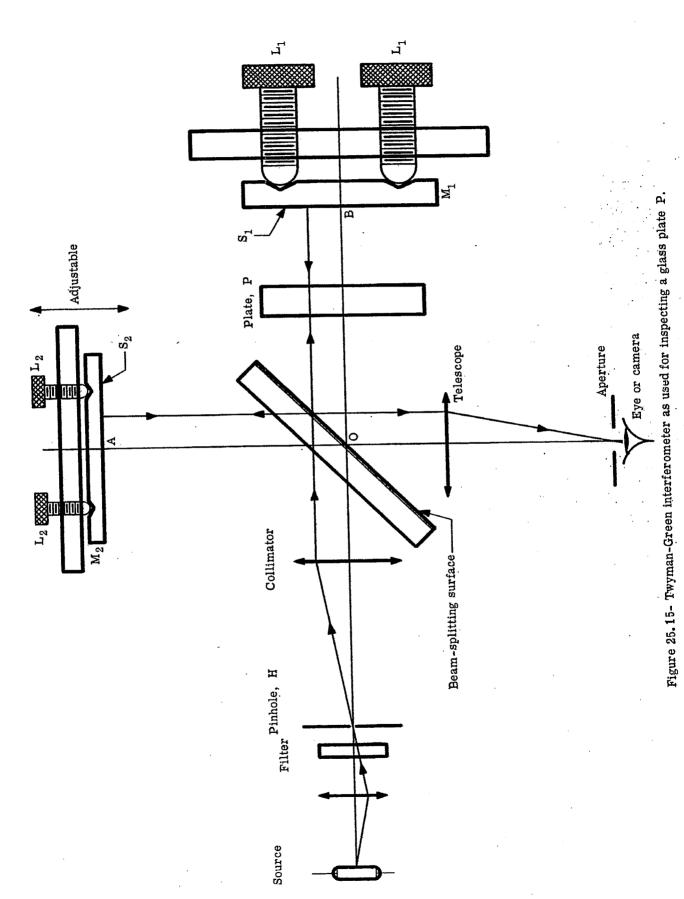
$$\Delta d = 2(n-1) \frac{\Delta t}{\lambda}$$
 wavelength numbers. (8)

If  $\Delta t$  is negligible,

$$\Delta d = 2 \Delta n \frac{t}{\lambda}$$
 wavelength numbers. (8a)

The factor, 2, enters because light waves traverse the plate twice. The following principles should be kept in mind.

- (a) Each fringe is associated with a particular value of Ad.
- (b) When one of the arms of the interferometer is altered in length by pressing upon the plate that supports the elements of the interferometer, each fringe moves such that Δd remains constant. With respect to the problem of interpreting the fringe system for the direction of the wedge that produces the straight fringes, one has only to shorten or to lengthen one arm of the interferometer and to note the corresponding movement of the fringes.
- (c) In passing from one fringe to the next fringe of equal darkness or brightness,  $\Delta d$  changes by unity.
- (d) In passing from a bright fringe to the next dark fringe,  $\Delta d$  changes by 1/2.
- 25.8.4.4 The base plate of the Twyman-Green interferometer is grooved or otherwise constructed so as to permit the end mirror, M1, to be swung into orientations for testing prisms, etc. The configuration for testing right angled prisms is illustrated in Figure 25.16. Mirror, M1, is adjusted for the desired fringe width. If all surfaces of the prism are known to be optically flat, the observed fringes reveal the degree of uniformity of the refractive index of the prism. More frequently, one will not have explored independently the degree of flatness of the surfaces of the prism. In such cases the observed fringes reveal the combined effects due to departures from surface flatness and due to inhomogeneities in refractive index. This method does not appear to have been modified to yield information about the angles of the prism or about deviation by the prism.
- 25.8.4.5 The following expedient is used, as the occasion demands, for distinguishing between effects due to inhomogeneity of refractive index and due to inadequate flatness of surface. The method is particularly effective in testing plates. As illustrated in Figure 25.17, plate, P, of Figure 25.15 is "immersed" between two optically homogeneous plates, E and F, that have outer surfaces of high degree of flatness. These plates are preferably, but not necessarily, plane parallel plates. For best results, the refractive indices of the immersion oil and of plates E and F should match the refractive index of the test plate, P. It can be useful to



25-17

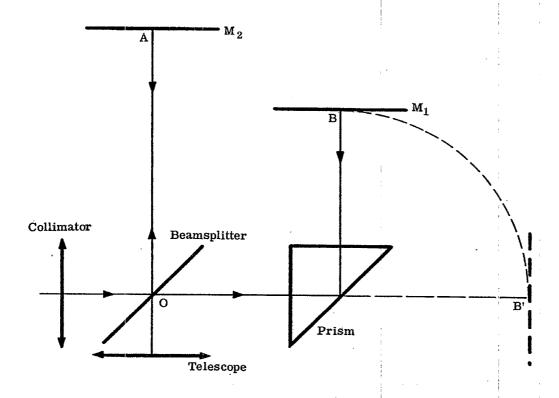


Figure 25.16- Modification of the location of end mirror  $\,\mathrm{M}_1\,$  for testing prisms in the Twyman-Green interferometer.

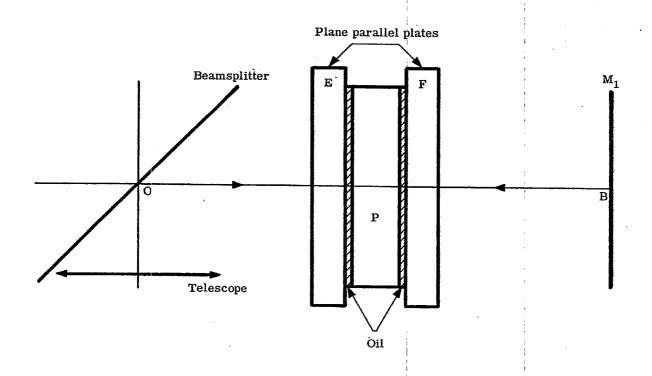


Figure 25.17- Method for obviating surface flatness of the test plate P.

"immerse" the prism of Figure 25.16 between two plane parallel optical flats. However, it is not possible to avoid lack of flatness at the reflecting surface of the prism.

25.8.4.6 Twyman-Green interferometers are provided, as illustrated in Figure 25.18, with an accessory fixture for observing spherical aberration of objectives throughout a wide range in focal lengths. Suppose that neither the collimator nor the test objective L possesses spherical aberration. Then rays from the axial point within pinhole, H, will be converged upon the axial point, C, at the rear focal plane of lens, L. If a spherical, convex mirror, M<sub>1</sub>, is placed as indicated with its center of curvature at point C, all rays, ef, will be reflected back upon themselves and will emerge from lens, L, as normals to a plane wave that is propagated toward the beamsplitter. This plane wave interferes with the plane wave reflected from M<sub>2</sub> in the other arm of the interferometer to produce upon the observer's retina a family of straight fringes whose fringe width depends upon the angular adjustment of the end-mirror, M<sub>2</sub>. In particular, a single fringe can be spread over the field of view. If the test objective has spherical aberration, all rays, ef, cannot be reflected back upon themselves. Consequently, the wave emerging from objective, L, will not be plane and will interfere with the plane wave from the other arm of the interferometer to produce interference fringes that display axial symmetry provided that the elements of objective, L, have axial symmetry and are well centered.

25.8.4.7 With objectives having long focal lengths, it is preferable to locate the convex mirror  $M_{\rm L}$  near the objective. This means that spherical reflectors having a series of radii should be provided. With objectives having short focal lengths, such as microscope objectives, the available working distance will not permit a convex reflector,  $M_{\rm L}$ , but rather a concave reflector,  $M_{\rm L}$ , centered about point C must now be used.

25.8.4.8 In actual practice, the exact location of the center of reflector, M<sub>1</sub>, with respect to the axial point near the rear focal plane of lens, L, is problematical and becomes often a matter of choice. The spherical aberration can be less with respect to a focal plane that falls on one side or the other of the paraxial focal plane. Secondly, some consideration will show that the observed spherical aberration is not necessarily that of the objective alone. When the test objective has spherical aberration, the observed aberration is, in fact, the aberration of the combination consisting of the test objective, L, and the spherical mirror, M<sub>1</sub>. Interpretation of the fringe displacements for the spherical aberration of the objective alone is not without objections of fundamental nature. But inspite of this difficulty, the Twyman-Green interferometer method is one of the better methods for indicating actual or relative amounts of spherical aberration in objectives of high optical quality.

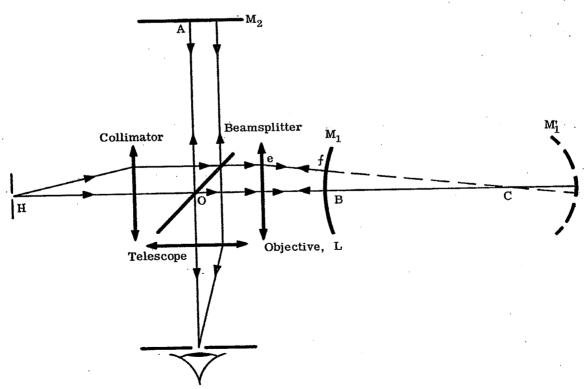


Figure 25.18- Adaptation of the Twyman-Green interferometer to the observation of spherical aberration of lenses, L.

## 25.9 THE RONCHI TEST

25.9.1 <u>Introduction</u>. One of the fairly common tests used currently by optical workers is the Ronchi Test (17). This technique actually falls into a class known as the "shadow-fringe method". The geometrical aspects of the technique are outlined in Figure 25.19.

## 25.9.2 Theory.

25.9.2.1 The essence of the theory is as follows. Let us suppose as a start that a lens, L, produces a perfect star image at the paraxial focus, O, Figure 25.19. (The edge ray shown will not pass through B but through O.) Now if a plane grating of 4-8 lines/mm (Jentsch method 18 and 19) or 10-20 lines/mm (Ronchi method) is positioned at an arbitrary distance, g, from the focal point, O, as indicated in Figure 25.19, a shadow of it would be formed on a screen placed in a plane of projection arbitrarily distant p from O. Since we are, for the present, considering a perfect lens, rays from all parts of the lens aperture pass through O and this point is the single center of projection of the grating onto the plane of projection. Hence the shadow image of the constant magnification over its area of

$$\frac{\sqrt{\eta + \xi}}{\sqrt{y + x}} = \frac{p}{g} \tag{9}$$

This ideal aberrationless case is usually not found and there is some spherical aberration, S, (shown for the edge ray in the figure) with the intersection points of other rays filling the space between O and B (simple undercorrection). Thus there is no longer a single point which can be considered as the center of projection for the entire grating. In this case the shadow image of the grating cast on the plane of projection will have a varying size scale over its area because centers of projection for points on the entire area of the grating lie between B and O. The magnification for any point on the grating becomes a function of the spherical aberration, S, of

$$\frac{\sqrt{\eta + \xi}}{\sqrt{y + x}} = \frac{p - S}{g - S} \tag{10}$$

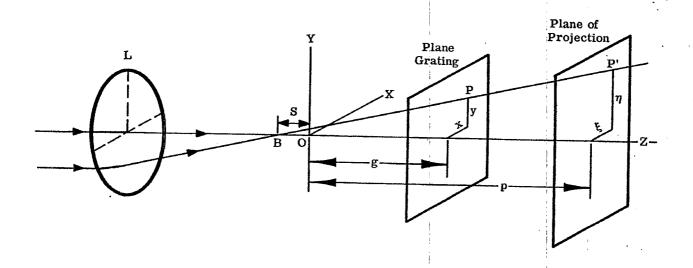


Figure 25.19-Geometrical theory of the Ronchi Test. (After Martin's, Technical Optics, Vol. II, Pitman Pub. Co. 1950)

- (17) Ronchi, Ann. d. R. Scula Normale Supriore di Pisa, Vol. XV (1923)
- (18) Jentsch, Physikal Zeitschr, XXIX, 66, (1928)
- (19) Martin, Technical Optics, Vol. II, 289, Pitman, 1950

If the grating is composed of straight lines its shadow image will show the lines as curved, and from the geometry of the figure, it can be shown that the curvature indicates the amount of spherical aberration present.

25.9.2.2 The complete theory must take into account not only the geometrical aspects outlined briefly here, but also the fact that interference may well be significant if the grating is as fine as those used by Ronchi or if more definitive interpretations are required. F. Toraldo Di Francia (20) has treated this subject in quite some detail, and the interested reader is urged to consult his paper.

# 25. 9.3 Ann Arbor Tester.

- 25.9.3.1 There has appeared a commercial unit known as the Ann Arbor Tester based on this principal and manufactured by the Ann Arbor Optical Co. The tester is the device on the right of Figure 25.20.
- 25. 9. 3. 2 The following Figures 25. 21 and 25. 22 are taken from the instruction booklet furnished with the instrument. Figure 25. 21 [4(a)-(g)] shows the testing of an eight-inch focal length spherical mirror as a 175 line/inch grating is moved through focus on axis. Figure 25. 21 [(5(a)-(d)]] shows the patterns obtained with the Optical Tester and a thirty-inch focal length paraboloid cut 7° off axis. The pattern in [5(a)] shows the tester on the optical axis while [5(b)] shows the tester off axis; [5(c)] and [5(d)] were taken in the same positions but the parabolic mirror was rotated. The experimental arrangement is shown in Figure 25. 22.
- 25.9.4 <u>Jentsch's grid method.</u> A similar testing technique using the coarser gratings of Jentsch is shown in Figure 25.23 taken from Martin (21) and showing the presence of spherical aberration. One optical shop checks all its work with this technique finding it a very sensitive and simple method for examing mirrors and lenses. The shadow fringe method has much in its favor as an experienced worker gains a feeling quickly as to the nature of the defects of the system under test.
- 25.9.5 <u>Summary</u>. In the last analysis, however, all optical tests depend upon evaluation, and the experience of the optical worker himself is a vital factor. It is largely a matter of what the workers in a particular laboratory have previously used. One of the largest laboratories in the United States does practically no Ronchi-type testing as they have accumulated other equipment and know-how over the years that gives them the information they need.

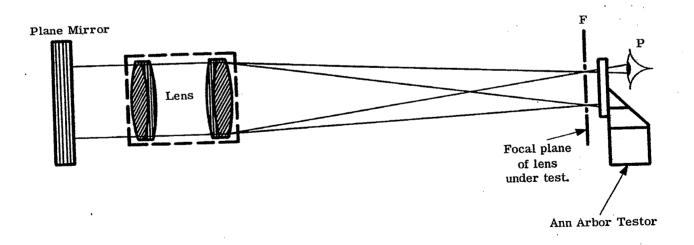


Figure 25. 20 - The Ann Arbor Tester.

<sup>(20)</sup> di Francia, Optical Image Evaluation, (NBS Circular 526), 161, U. S. Gov't. Printing Office, (1954).
(21) Martin, Technical Optics, vol. 1, p289, Pitman, (1948). also Jacobs, Fundamentals of Optical Engineering, McGraw Hill, (1943).







(b) -0.031"

(c) -0.006"



(d) 0.000"



(e) 0.008"



(f) 0.025"



(g) 0.051"

Figure 4



(a)



(b)



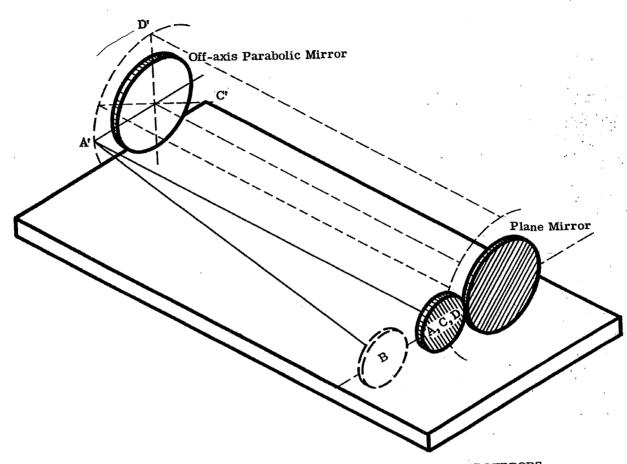
(c)



(d)

Figure 5

Figure 25.21- Patterns seen with the Ann Arbor tester for various optical systems.



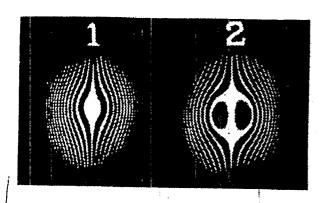
DETERMINING THE OPTICAL AXIS OF OFF-AXIS PARABOLIC MIRRORS

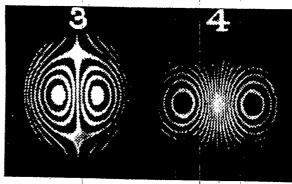
Often off-axis parabolic mirrors are received from the fabricator without markings indicating the side of the mirror which is toward the optical axis. Also, even with such markings, only the plane containing the axis is defined, and it is still necessary to locate the axis.

The position of the optical axis is easily determined by observing the pattern obtained with the Optical Tester. Only when the Tester is on the optical axis will the fringes be equally spaced and parallel to each other and to the lines in the grating when using an arrangement as shown in Figure 25. 20.

The resulting patterns for the Tester in the correct and other positions with respect to the axis can be seen from the patterns pictured in Figure 25.21. Figure 5(a) in 25.21 shows the pattern obtained with the Tester on the optical axis of a 30" focal length parabolic mirror cut 7° off-axis. This position is illustrated above, with the Tester at A and the edge of the parabolic mirror closest to the optical axis at A'. Figure 5(b) in 25.21 shows the pattern when the Tester, parabolic mirror, and plane mirror are still in the correct plane, but the grating is located outside the optical axis (B in the Figure above).

Figure 25. 22- Determining the optical axis of off-axis parabolic mirrors with the Ann Arbor Tester.





1 and 2 are shadow fringes outside the caustic.

3 and 4 are shadow fringes inside the caustic.

The appearances follow in the progression 1, 2, 3, 4.

Figure 25.23-Jentsch's grid method. (From Martin's, Tech. Optics, Vol. II, Pitman Pub. Co. 1950)

## 25.10 FOUCAULT TEST

#### 25.10.11 Introduction.

25.10.1.1. Having been given a lens or mirror surface prescription and having ground and polished the surfaces by hand or machine methods, the question arises as to what areas need to be "figured", i.e. be repolished to achieve perfectly the prescription. While helpful, the viewing of the image in toto is of less value than might be supposed, in that it represents the summation of the contributions from all parts of the surface. A technique which allows for inspection of the surfaces themselves is obviously required. One of the simplest and yet most delicate of all such techniques of surface testing was developed by Foucault (22) in 1859. The method requires, in its simplest form, merely a pinhole, a knife edge, the lens or mirror, and the eye of the observer. The system is shown diagrammatically in Figure 25.24 for a mirror.

25.10.1.2 In essence, the pinhole provides a small source of light which illuminates the surface of the mirror but which is so shielded that it sends no light directly into the eye. If we assume that the spherical surface is perfect and that the longitudinal aberration is negligible, then all rays striking the mirror will be focussed at some point, F. If the pinhole is located at the center of the curvature of the system, then F also will be at

25.10.1.3 Assuming that the eye is placed close enough to the image so as to view the mirror in the Maxwellian sense (i.e. the eye receives all the rays coming to the focus), then the mirror surface will be evenly illuminated. (Actually even a perfect surface will show some deviations which will be discussed later.) If now a knife edge K is advanced in the direction indicated, the mirror surface will appear to go from completely bright to completely dark as the knife passes through F. Again for reasons to be discussed later this does not quite happen. If the knife edge is displaced toward the mirror from F then as it cuts into the beam the lower side of the mirror is darkened gradually and not until the beam is completely occulted will the eye see no light. If the knife edge is displaced from F away from the mirror, the reverse occurs. Clearly then the point where the intensity varies most rapidly with knife edge movement from bright to dark is the focus. If the knife edge always remains in the plane of the pinhole, then there will be only one place where the mirror darkens uniformly and rapidly with lateral displacement of the knife edge. This point is of course the center of curvature.

25.10.1.4 The extreme delicacy of this measurement will become more obvious if we study Figure 11.28 where a highly exaggerated error is present. Suppose there is an error of slope, angle,  $\theta$ . The rays hitting (22) Foucault, Ann de L'Obs. de Paris, V, 197 (1859).

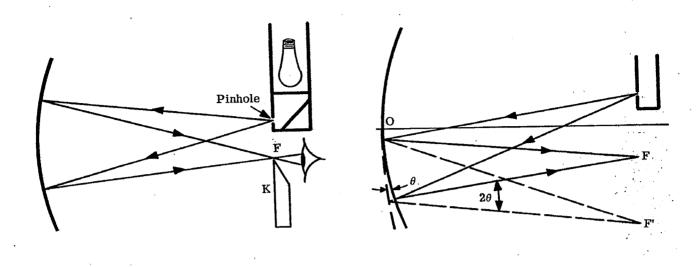


Figure 25.24 - Experimental arrangement for a Foucault Test of a mirror.

Figure 25.25- Schematic demonstration of sensitivity of Foucault testing.

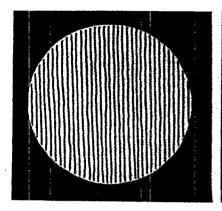
this area will then be deflected from the correct focus by an angle of  $2\theta$  and the focal point for this area will be moved laterally a distance approximately equal to FO x  $2\theta$ . If the surface has a radius of curvature of ten and a slope angle error of  $10^{-5}$  radian, then the deflection is  $10^{-4}$  inches or .025 mm. An error this size on a lens of such small curvature would be barely detectable but on a larger focal length system it would be clearly visible. The test is actually so sensitive that slope angle errors of  $10^{-6}$  radians are easily seen on the long focal length mirrors used in some telescopes.

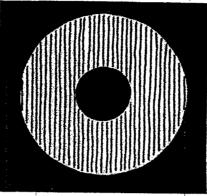
25.10.1.5 Errors of this type usually appear as zones on the surface rather than isolated areas. Unfortunately while Foucault tests are very common, they are almost always done visually and few photographs are taken. The photographs of some drawings from Strong (23) are shown in Figure 25.26. The artistry of Roger Hayward, illustrator for Strong, clearly shows the variations in mirror illumination produced under the Foucault Test.

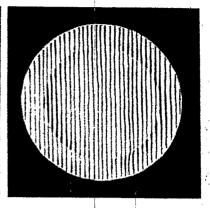
## 25.10.2 Detailed discussion of Foucault Test for spherical surfaces.

25.10.2.1 The preceding discussion has been highly qualitative and obviously over-simplified in some respects. To begin with the focus is never a pure geometrical point as we have demonstrated earlier. Secondly, the surface of even a "perfect" mirror does not appear all bright or all dark. It has been known for a long time that at the edge of a circular mirror there appears a very bright ring even when the knife edge has apparently cut through all of the rays. Banerji (24) has also observed that the surface for a real system with finite focal area does not grow continuously darker as the knife edge advances but rather the entire surface presents large variations in illumination. The peak illuminations get smaller and smaller until finally the whole surface is dark. Lord Rayleigh (25) attempted to explain the first of these two effects and was relatively successful. It remained for Zernike (26), Gascoigue (27), and recently Linfoot, in his articles and more recently in his book (28), to carry the interpretation of the Foucault patterns into the realm of the quantitative. Linfoot shows that the patterns may be determined analytically for an aberration-free system by assuming that electromagnetic waves originate at the surface being tested and combine according to the usual interference principles. In the event that the system is not aberration free, one must assume that the electromagnetic waves start at the pinhole and are reflected from the surface in the usual way.

- (23) loc. cit., (7), 296,297
- (24) Banerji, Astrophysical Journal 48, 50, (1918)
- (25) Rayleigh, Phil. Mag. 33, 161, (1917)
- (26) Zernike, Physica 1, 689 (1934)
- (27) Gascoigue, M. N. 104, 326, (1945)
- (28) Linfoot, Recent Advances in Optics, 128 at req. (1955) Oxford



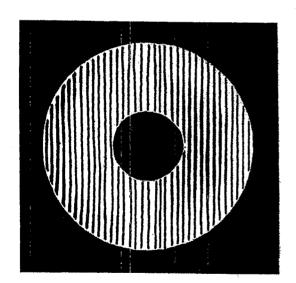




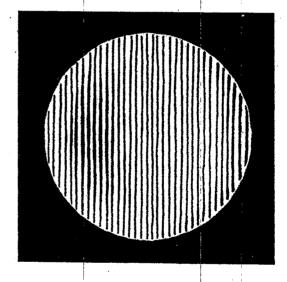
center of curvature.

(a) Spherical mirror tested at the (b) Parabolic mirror tested with a flat testing mirror.

(c) Spherical mirror with a raised annular ridge as tested at the center of curvature.



(d) Spherical mirror tested with a flat testing mirror.



(e) Parabolic mirror tested at the mean center of curvature.

Figure 25.26-Foucault test appearances. (From Strong's, Procedures in Experimental Physics, Prentice-Hall Inc., 1938)

25.10.2.2 Linfoot concludes that the <u>variation of illumination</u>, D(x', y'), of the knife edge is given, to a good approximation, by the equation

$$D(x', y') = \frac{1}{2\pi} \int_0^\infty du \int_{-\infty}^{+\infty} e^{(-iux' - ivy')} W(u, v) dv$$
 (11)

Where

$$u = \frac{2\pi x}{\lambda s}$$

$$v = \frac{2\pi y}{\lambda s}$$

$$i = \sqrt{-1}$$

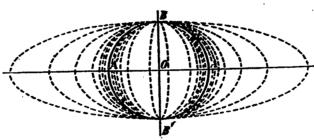
s = distance from the vertex of the surface to the knife edge.

W(u, v) = 
$$\frac{1}{2\pi}$$
  $\int$  E(x, y)  $e^{(iux + ivy)}$  dx dy

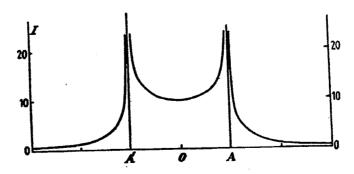
E(x, y) = illumination over the surface

Note carefully that (x, y) represent coordinates of a point on the surface under test; (x, y) represent the corresponding coordinates in the plane of the knife edge; and (x', y') represent the corresponding coordinates in the image of the  $x_1$ ,  $y_1$  plane.

25. 10. 2. 3 An application of this equation to a true mirror with knife edge central gives Figure 25. 27 (from Linfoot). Here the brilliant zone around the edge of the mirror is clearly predicted.



Isophotal lines for a true mirror under the Foucault test, with the knife edge central.



Intensity distribution along the horizontal diameter (after Linfoot).

Figure 25. 27 - Isophotal lines and intensity distribution for a mirror under the Foucault test (From Linfoot's, Recent Advances in Optics, Oxford Univ. Press, 1955)

25.10.2.4 If the knife edge is kept in the central plane but moved a distance, C, laterally we see the oscillations of Benerji (29). In this diagram  $C' = \frac{2\pi C}{\lambda S}$ , where C' is the distance moved in the plane of the image of the knife edge.

25. 10. 2.5 A practical problem frequently occurs in using the Foucault Test for surfaces of low reflection. The small pinhole that must be used results in very low light intensities, and the low light intensities make detection of the Foucault shadows difficult. To improve the situation a slit may be used. The slit must not only be narrow, but also of a length such that the aberrations of the system under study are effectively constant over this length. The details for interpreting the pattern resulting from this type of source are given in Linfoot (30).

## 25.10.3 The Foucault Test applied to non-spherical mirrors.

25.10.3.1 The Foucault Test can be used for paraboloidal as well as spherical mirrors. To simplify the interpretation, an additional flat is required as shown in Figure 25, 28. For paraboloidal mirrors one may use several modifications of the basic Foucault test. One employs a flat mirror with a hole in the center. The arrangement is equivalent to that shown, but the observer looks along the axis of the surface being tested.

25.10.3.2 Another modification is the technique developed by Gaviola (31). This method is more sensitive than the basic test and is particularly useful as a guide in very close control of zonal errors. The experimental arrangement is shown in Figure 25.29. The Gaviola technique depends on the fact that for off-axis areas of a paraboloid the positions of best focus do not lie on the center line of the paraboloid but rather lie on a caustic which originates at the center of curvature. The method is essentially as follows. First the paraxial focal point is determined by the regular knife edge method. From this datum the equation of the caustic for the non-aberrated paraboloid is calculated. Next one calculates where the center of curvature  $(\xi i, n_i)$  should be for a given facet or area. A knife edge is then used to determine where the center of curvature actually is -all of the paraboloid except the facet in question being covered up. The deviations  $\Delta \xi$ ,  $\Delta n_i$  of the actual center of curvature from the ideal center of curvature for various facets are used to map the true surface of the mirror. Symmetry about the center line is assumed.

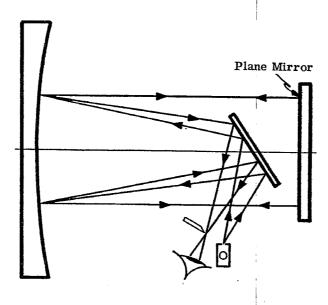


Figure 25.28- Foucault Test set-up for paraboloidal mirrors.

<sup>(29)</sup> loc. cit., (24)

<sup>(30)</sup> loc. cit., (28), 146

<sup>(31)</sup> Gaviola, JOSA 26, 163 (1936); also Strong, loc. cit., 23, p. 298

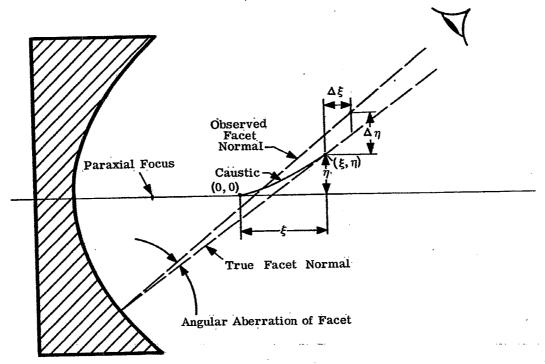


Figure 25. 29-The Gaviola technique of Foucault Testing.

(After Strong's, Concepts of Classical Optics, W. H. Freeman and Co. 1958)

25. 10. 4 The Foucault Test applied to lenses. The previous discussion may have left the impression that the Foucault test is really applicable only to mirrors. This is not so. Basically any type of system may be tested with the same advantages accruing. Several examples are given in Strong (32). The essential technique is the same in each instance with modifications made as dictated by the system under test. In each case, it is the possibility of inspecting the zonal contributions more or less individually rather than seeing them in their integrated form which makes this test so important. Recently one of the leading precision optical makers commented that due to the increasing use of aspheric surfaces, virtually all of his lenses and mirrors were tested by this method.

#### 25.11 THE STAR TEST

25.11.1 Introduction. It has been pointed out before that the choice of optical testing technique is frequently strongly related to the individual and his particular experience. It is generally acknowledged that H. Dennis Taylor carried the star test to its present heights and his disparaging comments on the knife-edge, or Foucault, test are interesting to read (33).

#### Technique. 25.11.2

25.11.2.1 The star test, as practiced by Taylor actually used a star as source of parallel light. The most frequently used star was Polaris although for checking achromatism, he also used the bluish star, Vega, and the reddish star Orionis. Today with the press of work, etc., it is seldom practical to depend on the visibility of the night sky, and artificial stars are used. One of the best "stars" is made by piercing a needle point, well honed, to varying degrees into layers of very thin tinfoil backed up by something like a very hard plastic or ebonite. Several trials should result in a very small perfectly round hole. The resulting aperture is then illuminated by a Pointolite lamp or equivalent and placed at the focus of a well corrected collimator as shown in Figure 25.30. Without much doubt, one of the best collections of star test photographs appears in Taylor's book and repeated also in Twyman's (34). It is reproduced in Figure 25.31. For convenience reference will be made using the figure numbers that appear in the photograph from Taylor, following in parentheses our figure number e.g. Figure 25.31(10a) is Taylor's Figure 10a, etc.

Strong, Procedures in Experimental Physics 70-72 Prentice Hall (1953) (32)

<sup>(33)</sup> Taylor, The Adjustment and Testing of Telescope Objectives, 50, Grubb, Parsons and Co. (1946)

<sup>(34)</sup> Twyman, Prism and Lens Making, 369, Hilger (1957)

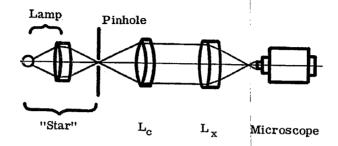


Figure 25.30 - The experimental arrangement for a star test.

- 25.11.2.2 The star test technique consists of examining the image of the star with a fairly high power magnifier or telescope. It is pointed out clearly by Taylor that a careful examination of the observer's own eye is mandatory if a correct analysis of the system under test is to be obtained. The tests such as rotating the objective are simple to make. If the astigmatism rotates with it, the fault is with the objective, if not, the fault is with the objective, if not, the fault
- 25.11.2.3 In most instances more information is to be gained by examining the image out of focus and watching what happens as it goes through focus than trying to evaluate the system by an examination of only the infocus image. The perfect figure will expand concentrically in an even fashion as the image passes through focus, with the intensity varying regularly in the ring structure. A perfect lens is shown in Figure 25.31 (17) while Figure 25.31 (18) shows the variation as a well-corrected lens passes through focus. It will be instructive to consider the principal star tests in the manner of Taylor, and this will now be done with frequent reference to Figure 25.31.

#### 25.11.3 Squaring-on.

- 25.11.3.1 A telescope objective is considered "square-on" when the optical axis of the objective passes directly through the center of the axis or stated another way—when the optical axis of the objective and the eye-piece coincide. Should the objective be cocked with respect to the eyepiece, the appearance of the image will depend upon the residual aberrations in the objective (assuming the eyepiece is effectively perfect). Usually the aberrations at focus that distort the image will be coma and astigmatism. Such a system is shown in
- 25.11.3.2 Reference should be made to Taylor for the actual process of squaring-on the objective, but the principle is clear from the photographs, viz. that the incorrectly adjusted objective will result in even the best focus being non-symmetric. Further, as the eyepiece is racked through focus, the image does not expand concentrically about the best focus image, but does so about a point to one side of the best focus image.

## 25.11.4 Achromatism.

25.11.4.1 It is a fundamental principle of instrumentation that we are interested in the performance of the whole instrumentation system and not just a part of it. Thus with visual optics and the known defects of the eye, we must design systems that take these defects into consideration. Occasionally we can put the defects to good use, but at all times we must be conscious of their effect upon the rest of the system. Even a perfect reflector

will show a colored star test visually because of the achromatism of the eye. This may be checked easily and due account taken of it in judging systems whether they be reflective, refractive, or a combination thereof.

25.11.4.2 The defects of the eye are of course involved in the choice of eyepiece with which to judge the objective. Usually a fairly high power objective with a magnifying power of 50 - 100 times is suitable for use with a well-corrected eyepiece. Lower power objectives involve more of the eye aperture and consequently are affected more by the achromatism of the eye.

### 25.11.5 Astigmatism.

- 25.11.5.1 The nature of astigmatism has been discussed elsewhere in this manual so its details will not be reworked. It suffices one to say that the aberration known as astigmatism results in a star being focussed into two "lines" that are at right angles to each other, and displaced by an amount that depends upon the angle of view. This aberration is very easily detected with the star test. As the eyepiece is racked through focus one might see the image vary as in Figure 25.31 (12 d, d", d'). For corresponding positions inside and outside of focus one might see images as in Figures 25.31 (13) and (14).
- 25.11.5.2 A study of the photographs in Figure 25.31 and of the aberrational theory indicates that the startest is indeed a marvelously simple, and yet accurate, method for testing for astigmatism. It must be understood that seldom will a system have just one aberration and that particularly as one goes off axis, it becomes increasingly difficult to stipulate exactly the cause of the image degradation. It is here that experience plays such a vital role.
- 25.11.5.3 Assuming in the present case that only astigmatism is involved, one will usually find that the position of best focus will show a roughly circular image the disk of least confusion half way between the two astigmatic focal lines and of a diameter approximately equal to one-half the length of either focal line. While astigmatism is not as bad an aberration as some, if one is interested in "pointing" because of its symmetry, it may increase the spot diameter several hundred percent. This clearly decreases the resolution possible with the system. Once again we call the attention of the reader to the fact that the requirements of a good pointing system are not as stringent as those for a system of high resolution. This fact is too often overlooked.
- 25.11.5.4 As previously indicated, the defects of the eye must be taken into account and a truly stigmatic system may appear to the eye to be astigmatic. It is not only possible to separate the astigmatism of objective, eyepiece, and eye, but careful design can result in a system that shows no astigmatism to the eye, yet each component of the system, objective, eyepiece, and eye, each have demonstrable amounts of this aberration. Again the tests of the eye should be made initially with a low power eyepiece. More detailed drawings of star effects showing astigmatism, after Zernike and Nienhuis (35) are shown in Figure 25.32.

# 25.11.6 Zonal and marginal spherical aberration.

- 25.11.6.1 Perhaps the best way to get a feel for how the star test demonstrates zonal and marginal spherical aberration is to refer to Figure 25.33 where three possible extremes are depicted: (a) no spherical (b) marginal spherical and (c) zonal spherical. In each instance the lens under test (the element could of course be a mirror, etc.) is directed toward a distant star or equivalent as previously explained. In case (a) there is no spherical aberration at all and geometrically all rays come to point focus. Actually of course interference spreads the point into the familiar interference pattern and such a lens would show a perfect figure such as Figure 25.31 (17) at focus. Either side of, but not far from focus, such a lens might produce images such as Figure 25.31 (22) and (23). A glance at Figure 25.33 (a) demonstrates why this is so.
- 25.11.6.2 If the lens has marginal spherical aberration but little or no zonal, then we might see within focus an image similar to that in Figure 25.31 (15). Note carefully that there is no very bright center as contrasted with Figure 25.31 (22). Note also the way the intensity of the rings varies with transverse distance. Figure 25.31 (15) shows positive marginal spherical, i.e. the edge rays come to focus between the paraxial focus and the lens. Figure 25.31 (15a) shows negative marginal spherical aberration. The reason for Figure 25.31 (15) is again clear by reference to Figure 25.33 (b). Within focus there is a greater concentration of light for the marginal than for the central rays.
- 25.11.6.3 Where the marginal spherical aberration has been corrected, there may be residual zonal. This manifests itself, as might be expected, by an image of the form of that shown in Figure 25.31 (20) (inside focus) and 25.31 (20a) (outside focus). That there should be this high concentration in the third and fourth rings inside focus and in the 2 and 3 and 5th rings is reasonable providing the zonal error is as shown in Figure 25.33 (3c).
- 25.11.6.4 When checking for zonal or marginal spherical, the best technique is to check through focus and not just at focus. Further, the inspection should be made far enough from focus so that several rings appear as this is a more sensitive test. Generally speaking, the rules for interpreting zonal spherical aberration are

<sup>(35)</sup> loc. cit., (26), 96, Plate V

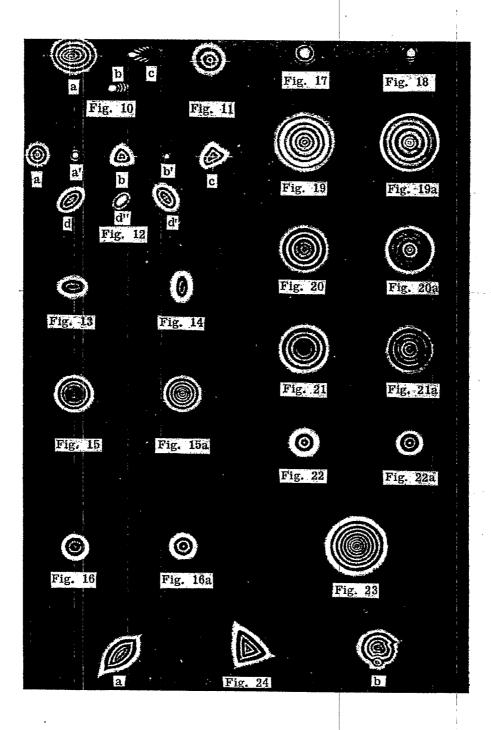


Figure 25.31-Star testing. (From Taylor's, The Adjustment and Testing of Telescopes Objectives, Grubb, Parsons and Co. 1946)

- Fig. 10a.- Eccentric appearance of interference rings, due to the objective being out of adjustment.
  - c.- The focussed image of a star, when the maladjustment is about as much as in the last case.
  - b.- The focussed image, as visible when the objective is moderately out of square.
- Fig. 11. A section of the cone of rays taken closer to the focus, exhibiting a more moderate degree of maladjustment.
- Fig. 12. a, b, c, d, and d' are out-of-focus sections, as will be seen when the objective is correctly "squared on," and quite irrespective of other faults.
  - a', b' and d" are appearances of the focussed image corresponding respectively to a, b and d. d, d and d" are also examples of astigmatism.
- Fig. 13. A section taken a very little way within focus, under a high power, exhibiting the fault of astigmatism. Fig. 14. - The corresponding appearance to Fig. 13, as shown by a section taken at the same distance beyond focus.
- Fig. 15. Section within focus, showing result of positive spherical aberration.
- Fig. 15a.- The corresponding section, taken at the same distance beyond focus.
- Fig. 16. A section taken closer to focus under a high power, exhibiting a slight residual spherical aberration; the central rings rather weak.
- Fig. 16a.- The corresponding appearance at the same distance beyond focus; the central rings relatively strong.
- Fig. 17. The spurious disc or image of a star yielded by a perfect objective, and viewed under a very high magnifying
- Fig. 18. The spurious disc sometimes yielded by a large objective when resting upon three points, without intermediate supports being supplied to counteract the flexure due to the weight of the lenses.
- Figs. 19 and 19a. An example of marked zonal aberration, being sections of the cone of rays taken inside and outside of focus respectively.
- Figs. 20 and 20a. Another example of zonal aberration.
- Figs. 21 and 21a. Example of the general figure of an objective being tolerably good, but there is a region in the centre having a focus somewhat beyond the main focus.
- Figs. 22 and 22a. Two sections of the cone of rays yielded by a perfect objective, taken very near to and on opposite sides
- Fig. 23. A section of the cone of rays yielded by a perfect objective, taken at about 1/4-inch on either side of focus, and of focus. and viewed under a high power. viewed under a moderately high magnifying power.
- Figs. 24 and 24a. Examples of violent mechanical strain, due to imperfect mounting or bad annealing.
- Fig. 24b. Example of the effects due to the presence of veins in the material of the objective.

Index to Figure 25.31

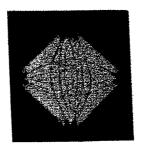
(From Taylor's, The Adjustment and Testing of Teles. Objectives, Grubb, Parsons and Co., 1946)



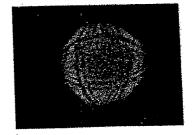
± 1.4x



± 2.7x



± 3.5 \cdot



± 6.5λ

Figure 25.32-Star tests showing astigmatism of varying degrees. (From Linfoot's, Recent Advances in Optics, Oxford Univ. Press, 1955)

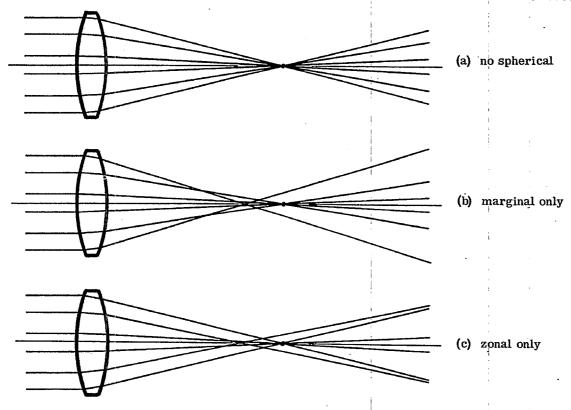


Figure 25.33 - Various types of spherical aberration.

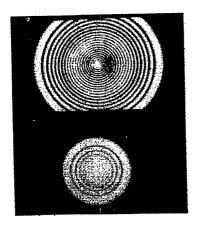
fairly straightforward with a bright zone or ring inside focus corresponding to a zone that focusses short. A bright zone or ring outside focus corresponds to a zone that focusses long. This corresponds of course to positive and negative zonal aberration. Again, Nienhuis (36) gives somewhat more detailed data, but not quite as much general information as Taylor. Figure 25.34 shows star images with varying degrees of primary spherical aberration at various focal positions.

25.11.6.5 It is clear that the star test furnishes a sensitive measure of the integrated effect of the whole lens. There is some question as to whether it gives as much information about a specific part of the lens or mirror as might a Foucault Test.

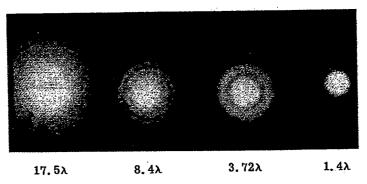
25.11.7 Coma. A good photograph of the effect of coma appears in Kingslake (37) and is reproduced in Figure 25.35.

<sup>(36)</sup> loc. cit., (26) 48, Plate II; also, Thesis, Groningen (1948)

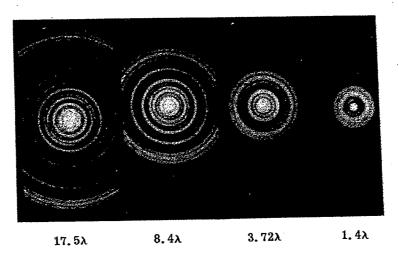
<sup>(37)</sup> loc. cit., (28) 84, Plate IV



(a) Images in presence of primary spherical aberration of amount  $16\lambda$ , at marginal focus and at circle of least confusion.

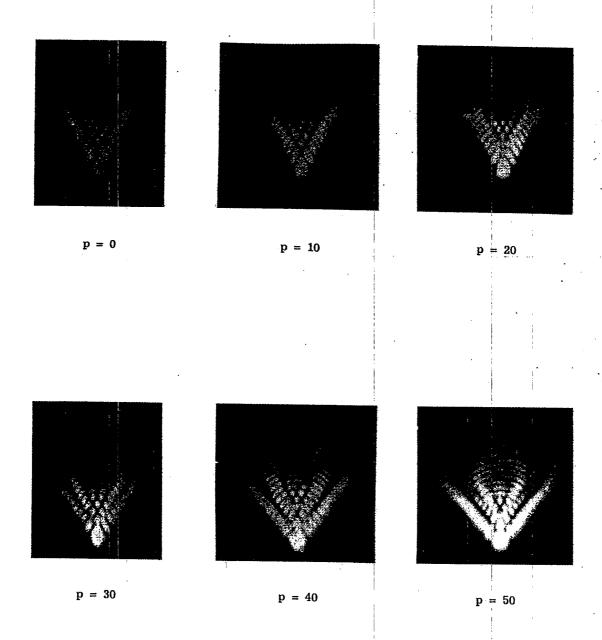


(b) Images in plane of paraxial focus, in presence of primary spherical aberration.



(c) Images in plane of least confusion, in presence of primary spherical aberration scale three times that of (b).

Figure 25.34-Star images showing various amounts of primary spherical aberration. (From Linfoot's, Recent Advances in Optics, Oxford Univ. Press, 1955)



Primary coma -  $\phi$  is  $2\lambda$  (r  $^3$  - 2/3 r) cos x at focal settings corresponding approximately to the given values of p.

Figure 25.35-Star images exhibiting coma. (From Linfoot's, Recent Advances in Optics, Oxford Univ. Press, 1955)

## 26 EVALUATION PHASE OPTICAL TESTS

## 26.1 RESOLVING POWER TESTS

## 26.1.1 Introduction.

- 26.1.1.1 The reason for the popularity of this general method stems from the feeling that, artistic considerations aside, the function of an optical system is to give information as to the detail in an object which is usually quite some distance away. Short of looking at the actual detail of the type on which the instrument under test is to be used, it has seemed reasonable to use some sort of artificial but definite target. Since many targets of military significance have sharp edges, targets with sharp edges seem to make sense. The nature of optical system performance is such that the edges should occur in at least two orientations and these preferably at right angles to each other. This deceptively simple process culminates then in a statement as to how many lines per millimeter can be resolved on the film of a camera, or as seen by the eye in a visual device. Actually it makes more sense to talk about a limit of resolution in terms of lines per unit solid angle, etc.
- 26.1.1.2 It will pay us to look somewhat more closely as to why this apparently straightforward process is called "deceptively simple". To begin, we have the fundamental question of what kind of target are we going to choose as a representative sampling of the in-use object. The USAF has been using the target in Figure 26.1 for years, while the National Research Council of Canada (1) has been using annuluses on a dark background as shown in Figure 26.2 along with a sector target proposed by Nutling. The U.S. National Bureau of Standards until recently used a line target as shown in Figure 26.3. This target and its applications were discussed in the reference cited. Recently NBS has adopted a new target and this is shown in Figure 26.4.
- 26.1.1.3 In addition to these, other groups have chosen targets made up of letters or numbers or combinations of special symbols or objects (2). To get informative as to the response of the optical system, at all angles, a target consisting of alternate black and white sectors has been used. (3) Apparently even the choice of the form of the target has been far from unanimous!
- 26.1.1.4 Let us look deeper. Even putting aside the question of form there is a considerable controversy over the contrast to be used between the dark and light portions. At least until the new NBS low contrast target came out, the British and Canadians were maintaining stoutly that the USAF high contrast targets were unrealistic as most of the objects photographed from an aircraft exhibited low contrast on the majority of days when photo-reconnaissance could be performed. We need not labor this point further except now we realize that not only the form but also the contrast is the subject of controversy.
- 26.1.1.5 With all this controversy the fact still stands that the system does have merit. Pestrecov (4) gave an excellent survey of the methods to date, and the serious student is referred to his work as regards relative merit of each. The particular claim for this technique is that it does give a single number that may be used to compare the performance of different lenses. The big question obviously is "granted it does give a figure by which to compare lenses but so do other techniques such as f- numbers, T- numbers, etc.", but does this really enable one to evaluate how a lens will perform in the field or does it merely tell how it would perform when photographing the very uninteresting lines and spaces on the test target? Unfortunately the answer to this question is not an unqualified "yes it does serve to state positively that this lens will be better than that in the field."
- 26.1.1.6 Having discussed these general ideas, let us now look at how the resolving power charts are actually used. As can be gathered from the above, different laboratories have their own techniques so we will sample three of the more common methods.

#### 26.1.2 The NBS method.

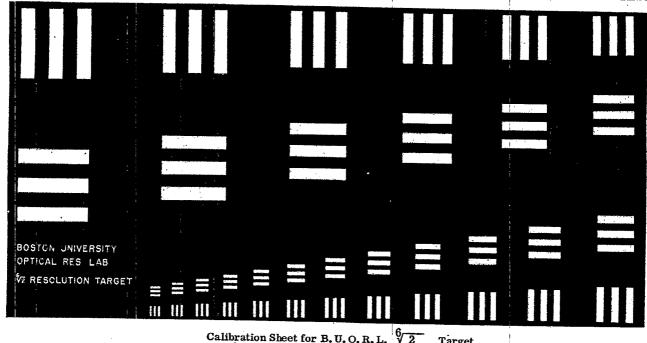
26.1.2.1 Figure 26.4 shows the high and low contrast NBS charts. The dimensions of these patterns are given in the table below the charts. The contrast of the black on white is 1.4 while that of the black on grey is 0.20. The numbers on the chart "14, 20, 28", etc. refer to the number of lines/mm when this chart is used at a minification of 25X. The numbers refer to both the horizontal and vertical patterns whose linear extention

<sup>(1)</sup> Howlett, L. E., Photographic Resolving Power, Canadian Journal of Research, Vol. 24, Sec. A, No. 4, 15-40 (1946)

<sup>(2)</sup> MacDonald, NBS Circular 526, 51

<sup>(3)</sup> Jewell, A Chart Method of Testing Photographic Lenses, JOSA Vols. 2-3, Nos. 3-6, 52, (1919)

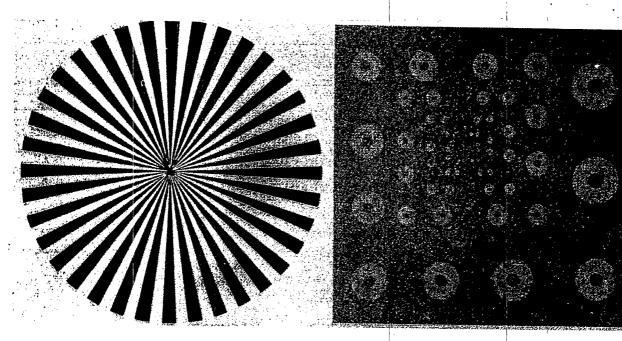
<sup>(4)</sup> Pestrecov, Photographic Resolution of Lenses, Photogrammetric Engineering, Vol. 13, (1947)



Calibration Sheet for B. U. O. R. L. Target Unit 6 7 8 9 10 12 13 14 Conversion 34.2 38.3 43.0 48.3 54.2 60.8 68.3 76.7 86.0 96.6 108 122 137 153 Figure

These are resolution values for a B. U.  $\sqrt[6]{2}$  target of 1mm. width. To determine resolution for each unit in lines/mm for any size target, divide each figure listed above by the width of the target measured from the extreme edge of unit 1 (the largest) to the extreme edge of unit 6.

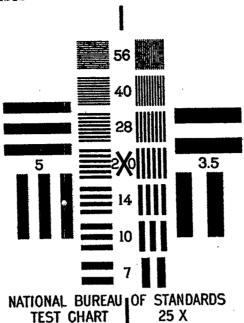
Figure 26.1 - The USAF resolving power target.



Sector target introduced by P. G. Nutting.

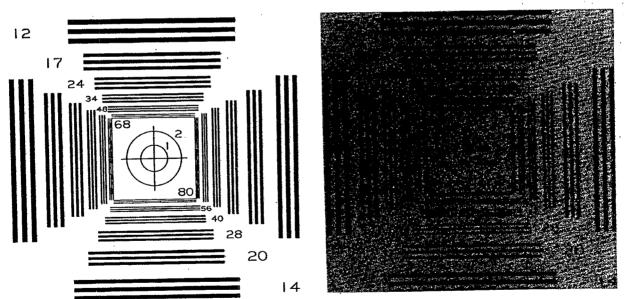
Canadian annulus target of 1.6:1 contrast ratio. The resolution values of the adjacent annuluses are in the  $6\sqrt{2}$  ratio.

Figure 26.2 -The Nutting and Annuli resolving power targets.
(From Pestrecov's, Photographic Resolution of Lenses, Photogrammetric Engineering, Vol. 13, 1947)



This chart formed part of NBS Circular 428. The ratio of the line spacings in successive patterns of this chart is equal to  $\sqrt{2}$ . When the chart is photographed at the standard distance of 26f, the values of resolving power that can be measured with this chart range from 3.5 to 56 lines/mm.

Figure 26.3 - Old NBS resolving power target.
(A Test of Lens Resolution for the Photographer, NBS Circ. 428)



High-contrast N. B. S. resolution test chart Low-contras

Low-contrast N. B. S. resolution test chart.

High-contrast N.	D.D. IC	SOTUL	DIA COOL	Ollege c						· · · · · ·		
0	80	56	40	28	20	14	68	48	34	24	17	12
Pattern Number Width of single black		.233	. 312	.446	.625	.893	0.184	.260	. 368	.521	.735	1.042
or white line												
Width of 3-line	0.781	1.116	1.562	2.232	3.125	4.464	0.919	1.302	1.838	2.604	3.676	5.208
pattern		<u> </u>						<del> </del>		<del>                                     </del>	<del></del>	
Width of space	0.	781 1.	116 1.	562 2.2	32 3.	125	0.	582 .8	25 1.	164 1.	649 2	.328
between patterns		L				00 1	18.0	19.6	21.9	25.1	29.6	36.0
Length of lines	18.0	19.6	21.9	25.1	29.6	36.1	10.0	11000	1 22.0			

Figure 26.4 - The new N.B.S. resolving power targets.

(Charts for Testing the Resolving Power of Photographic Lenses, F. E. Washer and I. C. Gardner, NBS Circ. 533(1953))

would run into the number. The chart used in this manner should be 26 focal lengths in front of the lens. The charts of course may be used both off as well as on axis. A common arrangement is to make a rack holding a series of the charts arranged in roughly the form of a square so that a photographic lens may be tested at all angles simultaneously. If the lens is to be tested visually, then it may be somewhat more desirable to reposition the test chart to the various angles of interest.

26.1.2.2 The observer after setting up the chart at the requisite distance determines which group is just distinguishable as three distinct lines and reports the corresponding number of lines/mm as the maximum resolving power of the lens at the given angle etc. Note that the measurement made in this way gives little or no information as to the response of the system to targets at fewer lines/mm.

26.1.2.3 Table 26.1 taken from NBS 533 shows the variation of resolving power of several hand held cameras. In this connection it is interesting to note the effect of using the high and low contrast targets. Inasmuch as we judge lines to be separated on the basis of contrast, it is important to note particularly Figure 26.5. The high contrast targets clearly may well be a more revealing as to what the actual resolution limits of the lens are. Further, the increased slope of the high contrast curve makes far more accurate measurements. Again we must warn that if the lens is to be actually used on low contrast targets, then we had better check it

Lens	EFL.	F-number		Re		ıg pow ılar se					neter	
	mm			Т	anger	tial		Π		Radi	al	
			00	5 <sup>0</sup>	10°	15 <sup>0</sup>	20 <sup>0</sup>	00	50	10°	15 <sup>0</sup>	20°
		2	68	56	56	48	28	68	56	56	48	40
1		2.8	68	68	68	68	56	68	68	68	68	56
1		· 4	80	68	56	56	56	80	68	68	68	68
1		5.6	80	80	68	68	80	80	80	80	80	80
A	50	8	80	68	68	68	68	80	68	68	68	56
1	•	11	80	80	80	80	68	80	80	80	80	80
		16	56		56	56	48	56	56	48	48	48
		22	56	56	48	48	48	56	56	48	48	40
		4.5	56	34	20	14	24	56	40	40	48	48
		5.6	56	28	17	20	34	56	40	40	56	56
, B	50	8	56	28	24	34	48	56	56	48	80	80
		11	56	34	34	34	56	56	56	56	80	80
		16	₹56	56	56	48	48	56	56	56	68	68
		2	68	68	34	17		68	68	48	34	
C	85	5.6	68	68	48	20		68	68	68	56	
		11	68	68	<b>4</b> 8	24		68	68	68	80	
		4.5	34	34	28	28	28	34	2.4	80		
i i		5.6	40	34	28	28	28	40	34 40	28 28	20	28
D	101	8	40	40	40	34	34	40	48	48	14 24	28 28
		11	40	48	48	40	40	40	48	48	34	34
		16	(34)	48	48	40	40	34	48	48.	40	40
			Tax 2 -									
1		4.5	28	28	24	12	7	28	34	34	28	20
1 1	·	5.6	28	28	20	12	7	28	28	24	28	20
E	101	8	34	28	24	17	14	34	34	34	28	28
L	101	11	28	28	28	20	12	28	40	40	28	28
1 1		16	(34)	34	28	17	12	34	40	40	28	20
]		22 32	34	28	28	17	5	34	40	40	34	24
	Í	32	. <b>34</b>	28	24	17	12	34	34	34	34	28
F			5,6	5.6	5.6	5.6	4.8	5.6	5,6	5.6	4.8	4.8

Table 26.1 - Resolving power at various apertures of several lenses of the type used on small hand-held cameras.

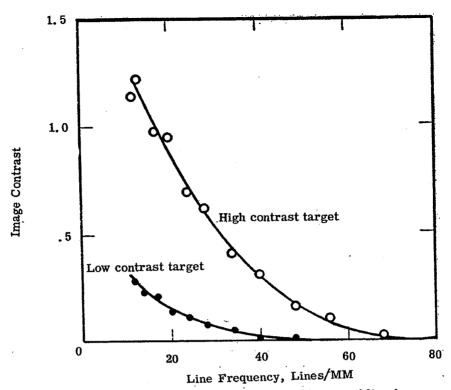


Figure 26.5-Variation of contrast in the image as a funtion of line frequency. (Charts for Testing the Resolving Power of Photographic Lenses, F. E. Washer and I. C. Gardner, NBS Circ. 533(1953))

on low contrast targets. This also is shown clearly by Figure 26.5: if we want to resolve 50/mm at low contrast, then the lens examined is not suitable. If we want to resolve the same number of lines/mm at high contrast, then the lens might well be satisfactory. This is a crucial point in considering the usefulness of resolving power targets as evaluation tools.

26.1.2.4 Looking again at the question of visual optics such as binoculars, telescopes, periscopes, etc. we realize, as previously pointed out that here the most important characteristic is not lines/mm but rather lines/unit solid angle. We can also state this by saying that we are interested in the angular rather than the linear resolving power of the system. Tables 12 and 13 from NBS 533 enable the user to determine from the chart group just resolvable, the corresponding maximum angular resolving power for either the circles or the lines around them.

26.1.2.5 Care must be exercised in judging the resolving power of a visual system to be certain that the resolving power of the eye is taken into consideration. This means that the lines under study must all subtend an angle greater than that just resolvable by the eye—usually about 60 seconds of arc. This means then that the product of the resolving power of the target and the magnification of the system must be greater than, say 60 seconds of arc, if we are to obtain a true test of the resolving power of the system.

26.1.2.6 In this same connection, the resolving power of a sequence of optical systems is analogous to the effective bandwidth, or the effective rise time of a number of sequential amplifiers; the overall resolving power, Re, in terms of the resolving power of the individual components  $R_1$ ,  $R_2$  etc., is given approximately by

$$\frac{1}{Re} = \sqrt{\frac{1}{R_1^2} + \frac{1}{R_2^2} + \frac{1}{R_3^2} + \dots}$$
 (1)

26.1.2.7 The NBS chart, when used in this standard manner, will cover a range of 14 to 80 lines/mm. For systems having higher or lower resolving powers the targets may be moved closer or further away. In some instances it may be convenient, where systems capable of resolving several hundred lines/mm are repeatedly encountered, to avoid the long working distances involved in the method above and reduce the targets photographically. Should this be done, great care must be exercised to see that the resolving power of the film and copying camera are such as to not degrade the targets.

26.1.2.8 While the NBS charts were developed primarily for lens studies, they may also be used as a basis for compliance with certain government specifications, for example

Federal Specification:

GGG-G-501b

Goggles, eyecup, protective, impact-resisting (chippers',

grinders' etc.).

GGG-G-511a

Goggles, eyecup, protective (welders).

GG-T-621

Transits, 1-minute; and transit tripods.

Military Specification:

MIL-O-13830 Ord

Optical components for fire control instruments; general specification governing the manufacture assembly, and inspection of.

Commercial Standard:

CS159-49

Sun glass lenses made of ground and polished plate glass thereafter thermally curved.

#### 26.1.3 The U.S.A.F. resolution target.

26.1.3.1 Originally suggested at the Bureau of Standards and carried to its present status by the U.S.A.F. Photographic Laboratory at Wright-Patterson Air Base, the U.S.A.F. target was designed primarily to evaluate the performance of aerial camera lenses. While the use of this target is controversial, it is probably the most widely used of all at the moment. The following comments of A. Katz (5), then of Wright Field, are much to the point. They were made during a discussion following a paper by R. E. Hopkins.

"In connection with the points raised by Dr. Pestrecov and in earlier papers, I notice that a number of people have been gleefully trying to kick the three-line resolution target to death. I want to point out again—and I have done this in other meetings—that it has served its purpose well. This purpose, simply stated, is to serially grade lenses in a manner that will correlate with their photograph—making rank. I have yet to be shown that our use of the three-line target in the judging of lenses to be used for aerial photography has led to any error, let alone consistent error."

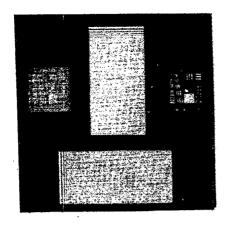
"Now we have lots of data, most of which is not neat and packaged. The exigencies arising with the working conditions in the Air Force are such as to effectively preclude the careful running of planned experiments. We substitute large numbers of airplane flights and tests, and after a number of years we come to pretty definite conclusions—by statistical osmosis, if you will. We know by now that when we get a lens that performs well in the laboratory (on the much maligned three-line high-contrast target) it will take high-quality photographs in the air on good days as well as bad days. The converse is also true. Laboratory test enable us to predict the quality of actual aerial photographs. I can't expect much more of a laboratory test. Let us not forget that it is only within the last 10 years that lens performance began to be specified in terms of resolution requirement over the field and that manufacturers began to use these tests, and it is only within the last couple of years that photointerpreters have begun to hear of lines per millimeter as a measure of performance."

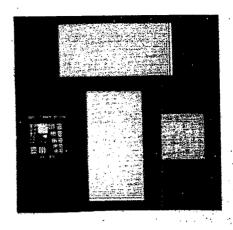
26.1.3.2 The type of tests (6) to which Mr. Katz is referring are well demonstrated in a portion of a series of through-focus trials shown in Figure 26.6 on a 40-inch f/5 Baker telephoto aerial camera looking through a window of poor quality. The target was the standard high contrast U.S.A.F. target plus a low contrast version of same plus a two variable frequency high contrast targets first introduced by Washer and Rosberry (7). The target was distant from the camera some 35 focal lengths to minimize the effects of spherical aberration. The term "window" here refers to the glass covering the hole in the skin of the aircraft through which the aerial camera sees the ground. More or less comparable focal positions are shown side by side for ease of

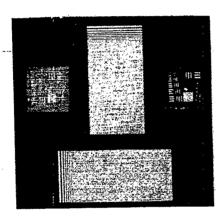
<sup>(5)</sup> NBS Circular No. 526, 200

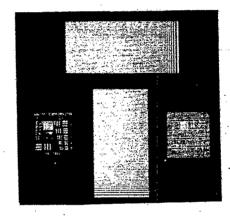
<sup>(6)</sup> These tests were run by Mr. William C. Britton while at the Boston University Physical Research Laboratories and under a U.S.A.F. contract. Mr. Britton is now with Itek Corp.

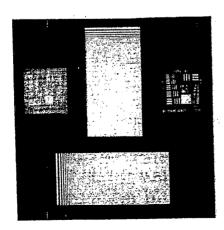
<sup>(7)</sup> Washer and Rosberry, JOSA vol. 41, No. 9, 597, (1951)



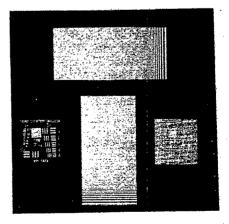








window 45° Obliquity



No Window

Figure 26.6 - Resolution target testing for Baker 40" f/5 telephoto aerial camera.

comparison. The focal settings was changed by .005" between successive exposures. The variation of the resolution limit with the high and low contrast targets is clear. The effect of off-axis aberrations is also clear.

26.1.3.3 Composite target tests such as these demonstrate the difficulty of deciding on which target, if any, to settle on to the exclusion of all others. In fact it is pretty generally the opinion of the "conservatives" that no one target gives all the information that is needed to fully evaluate a lens. Were a given optical system always to be used on exactly the same type target, that would give a one to one correlation between laboratory testing and field performance. It, thus, is the very versatility of optical systems that gives rise to our difficulty.

#### 26.1.4 The Kinetic Definition Chart.

- 26.1.4.1 There was developed during World War II (8) and subsequently improved upon (9 and 10) a routine system for checking the resolving power of visual optical systems. The system is essentially a resolving-power target approach, but incorporates many features not formed in the spur-of-the-moment setups commonly found in laboratories. The targets employed, as well as plan and side view schematics are shown in Figure 26.7.
- 26.1.4.2 The apparatus derives the word "kinetic" from the motorization of some of its parts, but the term is misleading nonetheless. A glance at the charts will show that they are of constant line spacing but of various contrasts and situated in four positions. The ratio of lines/spaces is 1:1, and is essentially the chart first advanced by Foucault (11) in 1858. The variation in line spacing required to determine the resolving power of a system is effected by the optical reduction unit. This unit consists of four highly corrected microscope objectives of focal length 4, 8, 16, and 32 mm. By varying the distance from the target to the reduction unit by the adjustable space gauge shown in Figure 26.7, the lines/inch may be changed from coarse to fine.
- 26.1.4.3 There are several interesting aspects to the KDC Apparatus. One of these is the "artificial sky" which not only simulates (by varying its illumination) the sky against which many objects must be seen, but also the stray light found in most optical systems. This apparatus thus takes into account not only the low control of the object itself, but also the surround so important in retinal response. Incorporated into the KDC Apparatus is a standard telescope with an aperture that is variable. This very carefully constructed telescope is of superior quality and allows the observer, in effect, to set up a standard against which the test instrument is compared. Once again we see a recognition of the need for removing as far as possible the limitations of the particular observer's eye from the testing procedure. Here this is done by inclusion of an auxiliary telescope of such magnification that the limit of resolving power is determined by the instrument under test rather than the eye. The rest of the system is rather straightforward and all designed to give maximum ease of assessment to the observer.
- 26.1.4.4 The final report on the NBS chart or the USAF chart is the resolving power limit of the system. In this technique the final report is called the K.D.C. efficiency and is defined as follows:

$$KDC efficiency = \frac{\alpha_e}{\alpha_i Mi} \times 100$$
 (2)

where

 $\alpha_i$  = minimum angle resolvable using the instrument under test.

 $\alpha_e$  = minimum angle resolvable with the eye alone.

Mi = magnification of the instrument under test.

Clearly then, this definition is not a statement of the resolving power of the instrument alone, but rather it is a comparison of the effective improvement the instrument affords over the eye alone.

26.1.4.5 The factors directly preportional to  $\alpha_e$  and  $\alpha_i$  are conveniently determined directly from the KDC apparatus as follows. With the auxiliary telescope in place (if it will be required with the instrument under test as previously explained) the observer adjusts the target-to-turret spacing until the target is just resolved and the K.D.C. scale (lower left of drawing, just above the reversing switch) is read. The pointer on this scale is coupled to the target holder. The instrument under test is then inserted in its proper place and the K.D.C. scale again read. The K.D.C. efficiency is now obtained from the equation:

(8) NDRC Report (classified)

(9) Coleman and Harding, JOSA 37, 263, (1947)

(10) NBS, 526, 95, (1954)

(11) Foucault, Ann. de L'observation de Paris, 5, 197, (1859)

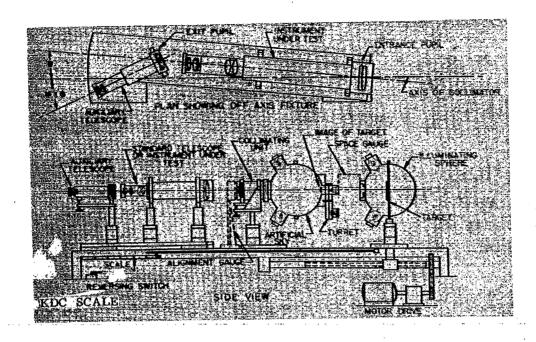
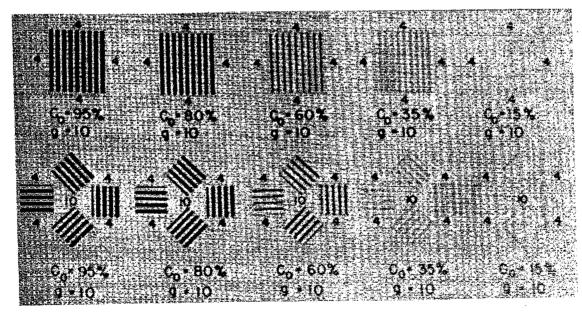


Figure 26.7 (a) - The KDC apparatus schematic.



Modified Foucault resolution targets.

$$C_0 = \frac{B-b}{B} = \frac{\text{Reflectivity of white band} - \text{Reflectivity of dark band}}{\text{Reflectivity of white band}}$$

 $C_{o}$  = inherent target contrast: g = number of white bands per inch.

Figure 26.7 (b) - The KDC apparatus targets.

Figure 26.7 - The KDC apparatus and target charts.

K.D.C. efficiency = 
$$\frac{Xi}{MiXe}$$
 x 100

(3)

where

Xi = K. D. C. scale reading with the instrument under test (and of course the eye)

Xe = K.D.C. scale reading with the eye alone

Mi = the magnification of the instrument under test.

If it is desired to compare a production instrument with the standard telescope, the K.D.C. reading taken with the standard telescope replaces Xe in the above equation.

26.1.4.6 There are many other uses of the K.D.C. apparatus but certainly its versatility and ease of manipulation recommend it when a large amount of work of this type must be done.

#### 26.2 GENERAL DISCUSSION OF SINE WAVE TESTING

#### 26.2.1 Introduction.

- 26. 2. 1. 1 At about the time the controversy as to just what type of resolution target should be used was reaching its zenith, a paper given by Schade (12), an electrical engineer, brought to bear on the problem of optical system evaluation, the full resources of a completely different field viz., communication theory. While others such as Selwyn (13) and Duffieux (14) had preceded Schade in their investigations into this general area, there is little doubt in most minds that Schade (15) was responsible for focusing the attention of the optical world on the optical possibilities of this method.
- 26.2.1.2 It will be of interest to look briefly at Schade's original problem. Schade was studying the problem of optimizing the response of a television system starting with the optical pick-up in the studio through the electronic and electromagnetic systems to the final presentation on a kinescope in the home. His background here as an electrical engineer had taught him that one may study the response of an ordinary amplifier two ways (a) by feeding a single transient pulse to the amplifier and noting its response or (b) using sine waves of different frequencies and noting the phase shift and/or amplitude change as the sine wave signal passed through the amplifier. Fourier analysis shows that all the information contained in (b) is actually implicit in (a) but the transient is harder to use experimentally.
- 26. 2. 1. 3 With the knowledge that this testing technique was a proven method, Schade in effect asked "why can't I do the same sort of thing for the optical part of the system? If I can do this, then I should be able to use the theories already developed for optimizing cascaded amplifiers." The question then arose as to what there was about an optical system that corresponded to the electrical sine waves. After the idea was conceived that the variation in intensity with angle as seen by the lens did indeed constitute a frequency, albeit a spatial Fourier frequency" and not the frequency associated with  $v = f\lambda$ , the way was clear. There did remain then (and still does now) much theoretical work to do but at least the direction was indicated. The problem of translating the Fourier spatial frequencies into the temporal frequencies used in electronic amplifiers was easily solved by scanning techniques already under study in the sister field of flying spot scanner television.

#### 26.2.2 Basic theory.

26.2.2.1 Inasmuch as this manual is not intended to develop all the pertinent theory but rather to acquaint the reader with possible methods, most of the details of the mathematical treatment will be omitted. The reader, however is invited to study closely the many excellent articles in this field. Some of these are in the following

<sup>(12)</sup> Schade, A New System of Measuring and Specifying Image Definition: Symposium on Optical Image Evaluation, NBS, Oct., 1951. Proceedings published in NBS circuler 526, (1954).

<sup>(13)</sup> Selwyn, Theoretical Estimation of Combined Effects of Film and Lens on Resolution; RAE Report N. H. 698, April, (1940).

<sup>(14)</sup> Duffieux, L'integrale de Fourier et ses Applications à L'optique, Besançon, Faculté des Sciences, (1946).

<sup>(15)</sup> Schade, Electro-Optical Characteristics of Television Systems, RCA Rev., 9:5-37, 245-286, 490-530, 653-686; (1948).

references: (16) through (23)

26.2.2.2 As indicated above and by Schade and Duffieux, an optical system may be considered as a two dimensional electrical filter. Further in electrical work we normally think in terms of amplitudes and at least in normal circuit work do indeed measure our signals by determining their amplitude. In optics, however, we cannot measure amplitude directly but instead measure intensity. A negative amplitude has no physical significance (although it can be interpreted as indicative of a 180° phase shift) for optics while it is a common and significant occurrence in electronics. As an aside we might note, however, that in the detection of electromagnetic radiation we can measure only power directly. The spatial frequencies to which we are referring are thus variations of intensity. This is an important point.

26.2.2.3 Let us assume that the coordinates in an object plane are denoted by  $\xi$  and  $\eta$  and in the image plane by x and y. The intensities in the object and image plane are then indicated by  $O(\xi, \eta)$  and i(x, y) respectively. We should note here that the terminology is not yet standardized and we are here following that of O'Neill (loc. cit. 16, p E-3). An object point  $O(\xi, \eta)$  is then spread out into an image point i(x, y), this "spread function" being denoted by i(x, y). If we now apply this spread function to each point in the object, we can predict the appearance of the image by convolving the spread function with the object distribution according to equation (3).

$$i(x,y) = \int_{-\infty}^{+\infty} \int S(x-\xi, y-\eta) O(\xi, \eta) d\xi d\eta$$
 (4)

Assuming for the moment that this convolution is amenable to the techniques of the Fourier transform, we can do the same thing as (4) in the spatial frequency domain by utilizing equation (5).

$$i(\omega_{x}, \omega_{y}) = \tau(\omega_{x}, \omega_{y}) \circ (\omega_{x}, \omega_{y})$$
 (5)

where  $i(\omega_x, \omega_y)$  and  $O(\omega_x, \omega_y)$  are the image and object expressed in terms of Fourier spatial frequencies and  $\tau(\omega_x, \omega_y)$  is the so called "transfer function" of the system (for details see loc. cit. (16) p 232, et seq.)

26.2.2.4 Note clearly what has happened. We have replaced the convolution integral which is difficult to compute, by a product. The two equations of course say basically the same thing and their interrelationship is clearly seen by the more complete definition of the transfer function (loc. cit. (17), p26).

$$\tau(\omega_{x}, \omega_{y}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int S(x, y) e^{-i(\omega_{x} x + \omega_{y} y)} dx dy$$
 (6)

Clearly we must be able to calculate, or otherwise determine, the spread function in (4) before the i( $\omega$ ,  $\omega$ ) may be calculated theoretically. This is a sizeable task. It turns out, however, to be relatively simple to do it experimentally and this is effectively where the art stands at present. The technique, based on experimental determinations of  $\tau(\omega_x, \omega_y)$  has lead to a new, although still controversial, method of evaluation of optical systems. Synthesis by use of this principle as a design method is still in its infancy.

26.2.2.5 Let us back off again and look at why equation (5) is so important an evaluation tool. The reason rests in part on the fact that the object and image are related in the spatial frequency domain by a multiplicative factor while in the spatial domain they are related by a complex summation. If we have two systems

- (16) Proceedings of Symposium on Communication Theory and Antenna Design AFCRC TR-57-105 (ASTIA Document No. AD117067). While this symposium was aimed primarily at antenna designers, the organization of it was such that not only is the optics covered rather well by O'Neil and Parrent but also the basic mathematics and physical requirements are outlined in detail. One should note particularly the bibliography prepared by Parrent on Page M-1.
- (17) O'Neil, Selected Topics in Optics and Communication Theory Itek Corp. (1958) Note This publication has an exceptionally complete bibliography of work in this field.
- (18) O'Neil, Publications of the Theoretical Optics Section, Itek Corp. (1958)
- (19) Marechal, The Contrast of Optical Images and the Influence of Optical Aberrations, NBS Circular No. 526, p9, (1954)
- (20) Elias, Optics and Communications Theory, JOSA, 43, 229, (1953)
- (21) Hopkins, H. H., The Frequency Response of a Defocussed Optics System, Proc. Ray Soc (London), 321A, 91, (1955)
- (22) Blanc-Lapierre, Upon Some Analogues Between Optics and Information Theory, Symposium on Microwave Optics, McGill University, (1953) - Proceedings published by Antenna Section Air Force Cambridge Research Center.
- (23) Parrent and Drane, The Effect of Defocussing and Third Order Spherical Aberration on the Transfer Function of a Two Dimensional Optical System, Optica Acta, 3: (1956)

one of which clearly shows a better high frequency response than the other, we can be sure that this system will have the higher resolving power. Further the process of obtaining the sine wave response, or  $\tau(\omega_x, \omega_y)$ , will give (or usually does) the response at all frequencies and not only at the maximum resolvable condition as with the resolution target system. It is thus, theoretically, possible having  $\tau(\omega_x, \omega_y)$  to predict the image for any object by the use of equation (5).

26.2.2.6 As might be expected, nothing is ever quite this rosy. Always there is the needle in the haystack or thorn in the rose. The difficulty here lies in the fact that the transformation from equation (4) to equation (5) presupposes that the optical system is perfectly linear and invariant over the object and image fields. Unfortunately this does not hold very well in poor systems. In good systems Linfoot and Fellgelt (24) have shown, however, that over the normal working field the assumptions are reasonably valid. A rather good discussion of the restrictions involved in making the jump from (4) to (5) has been given by Zucker (25) both for the case at hand, optical systems, and also for the allied problem-antennas. Much as it would be interesting to go into here more of the basic theory, the limitations of space require that we get on to the actual experimental techniques of measuring the transfer function and its applications. The interested reader will find the references given, however, replete with pertinent information. There are several methods of determining  $\tau(\omega_X, \omega_Y)$  or the equivalent, of which the following are representative only.

### 26.3 SINE WAVE TESTING WITH SINE WAVE TARGETS

## 26.3.1 The Schade system.

- 26.3.1.1 In Schade's original presentation, he demonstrated a system that, stripped to its basic features, was essentially that shown in Figure 26.8 wherein F represents a continuous film with a series of discrete sine wave targets. Each target was made by varying the intensity of the exciter lamp in a sound track camera sinusoidally with time while the film was moving at a constant rate through the camera. Sections of the film are shown in Figure 26.9 (26). P is a projector that allows the test pattern to be seen at any effective distance from the system under test, S. The light from S is focussed (usually with the aid of an auxiliary microscope) onto a scanning aperture, A. This aperture might be of any shape but usually it is most convenient to use a circle. Behind the aperture is a photomultiplier tube, PM, which feeds into a recorder, R.
- 26.3.1.2 In action then, the film moves through the projector producing a spatial frequency sine wave. The fact that the film is moving means that there will be a sinusoidally varying electrical signal from the photomultiplier tube. The sine wave response is then given simply by the ratio of this ac signal at a spatial frequency, N, to that which the system would give if the frequency were extrapolated to zero. In Schade's terminology  $r_{\widetilde{\psi}} = \widetilde{\psi}_n/\widetilde{\psi}_0$  where  $r_{\widetilde{\psi}}$  is the sine wave response. Typical sine wave response factor curves are shown in Figure 26.10. These response curves were taken from research done in this field by Shack (27) when at the National Bureau of Standards. Figure 26.11 from the NBS Report gives the variation of  $r_{\widetilde{\psi}}$  with focal position for a fixed spatial frequency while Figure 26.12 gives the variation of  $r_{\widetilde{\psi}}$  with focal position for a fixed color. Figures 26.13 and 26.14 show the variation of  $r_{\widetilde{\psi}}$  with spatial frequency for different colors. Note the negative amplitude in these figures. It is due to a 180° phase change. Schack's apparatus was much the same as Schade's but Shack used a scanning slit instead of a scanning pinhole.

## 26.3.2 The Lamberts system.

- 26.3.2.1 Lamberts (28) and Lamberts, Higgins, and Wolfe (29) have studied the sine wave response particularly in connection with their lens evaluation program at Eastman Kodak. The reader will find Lamberts' article particularly interesting as he not only discribes the basic theory very lucidly but also presents a rather novel variation on the fundamental method.
- 26.3.2.2 In the Schade method the scanning aperture is very small and usually circular or square. In the Lamberts system the scanning aperture is a long slit. By the use of the slit it is possible to replace a target whose intensity varies sinusoidally by a target with a variable area as shown in Figure 26.15. This type of target has also been used by Lindberg (30). The scanning slit is indicated by SS in Figure 26.16. It can be shown the light distribution in the image is given by,

$$F(x) = b_0 + b_1 \quad |A^*| \quad \cos (2\pi \gamma x - \phi) \tag{7}$$

Where  $b_0$  and  $b_1$  have the meaning shown in Figure 26.15, and  $b_1/b_0$  is the "normalized amplitude" as discussed in Lamberts' article.  $\gamma$  is of course the spatial frequency and x is the shift of any particular aspect

- (24) Linfoot and Fellgelt, On the Assessment of Optical Images, Trans. Roy, Soc. (London) 247, (1955)
- (25) Zucker, loc. cit. 5, p L-1
- (26) Schade loc. cit. 1, p 233
- (27) Shack, <u>Investigations Into the Correlation Between Photographic and Photoelectric Image Evaluation</u>, NBS Report No. 5483
- (28) Lamberts, JOSA 48, 490 (1958)
- (29) Lamberts, Higgins, and Wolfe, JOSA 48, 487 (1958)
- (30) Lindberg, Optica Acta 1, 80 (1954)

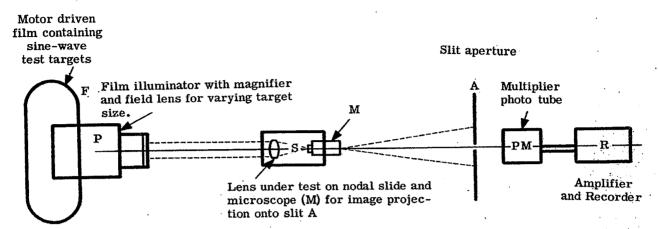


Figure 26.8 - The basic Schade system for determining the sine wave response of an optical system. (Based on O.H. Schade's, Electro-Optical Characteristics of Television Systems, RCA Review, Vol. 9, 1948)

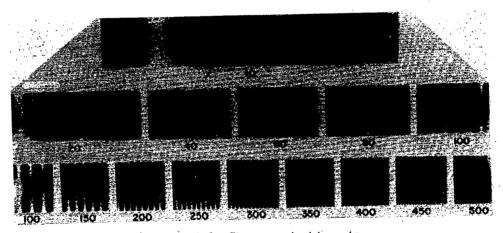


Figure 26.9 - Sine wave test targets.

(Based on O. H. Schade's, Electro-Optical Characteristics of Television Systems, RCA Review, Vol. 9, 1948)

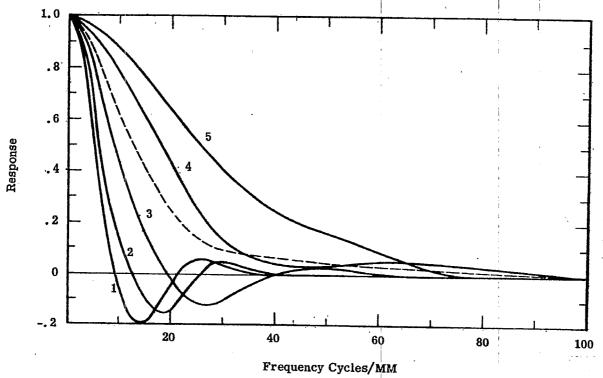


Figure 26.10 - Sine wave response factor vs line number (frequency) of lens A, .4 mm inside focus. In this and in following figures, curves numbered 1, 2, 3, 4 and 5 were obtained with Wratten filters 29, 25, 90, 16 + 60, and 45 respectively. The dashed curve was obtained with no filter (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

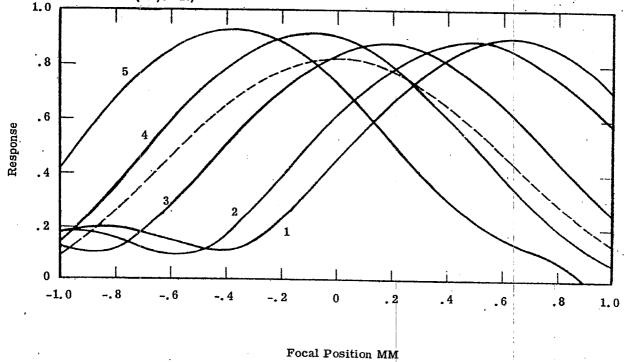


Figure 26.11 - Variation in response with focal position for lens A for different colors at a fixed frequency. The frequency chosen was 8 cycles per mm. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

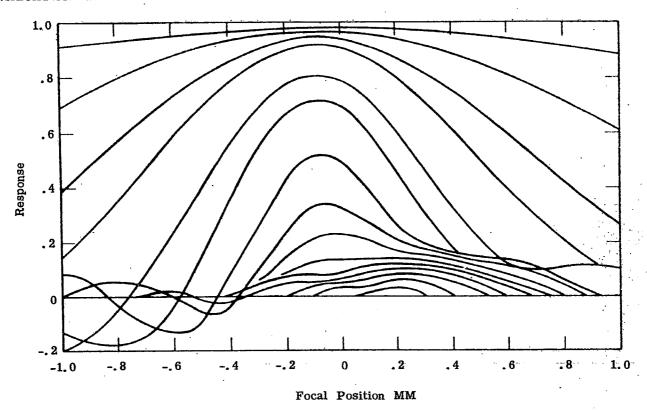


Figure 26.12- Through-focus response curves for lens A with filter 16 + 60. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

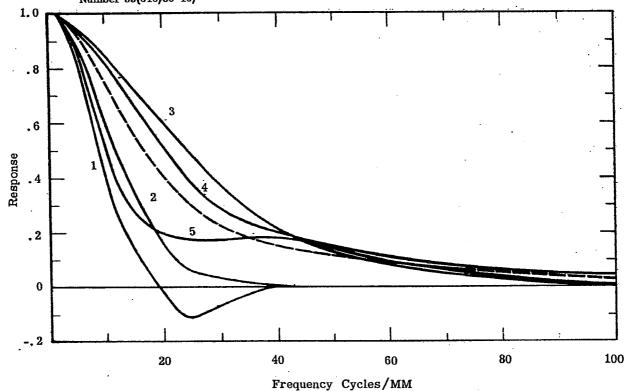


Figure 26.13- Frequency response of lens A at focus for various colors. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

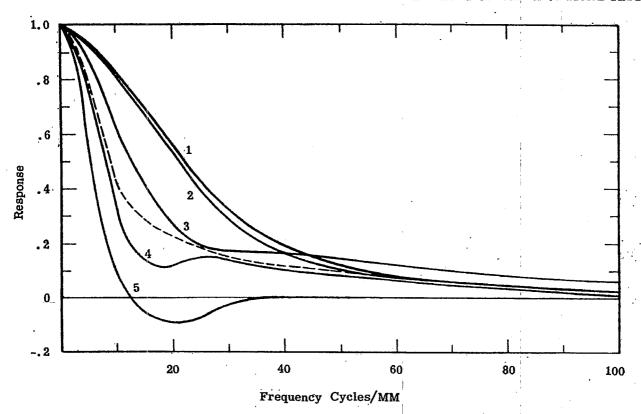


Figure 26.14 - Frequency response of lens A .4 mm outside focus for various colors. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

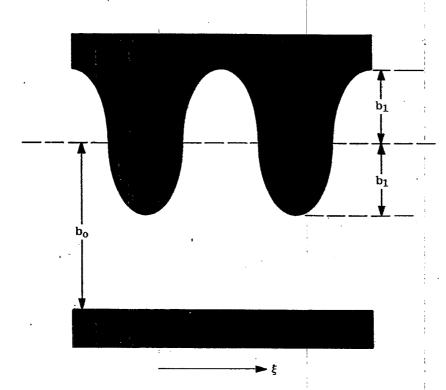


Figure 26.15- The Lambert's test object for measuring sine wave response. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

26-16

of the target.  $\phi$  is the spatial phase angle between object and image and is something not covered specifically in Schade's original work. A\* is the sine wave response previously defined. The lens bench used in the experiments set up to confirm the theory is shown schematically in Figure 26.16. TO is the test object, which for this work was either a slit (used to determine the spread function) or the target shown in Figure 26.9. L is the lens under test, with SS being the scanning slit, and P the photomultiplier and recorder ensemble. T is a tangent bar arranged to tilt the object and lens, L, when studying off-axis response.

- 26.3.2.3 The action of the system is similar to that of the basic method, and the reader may refer to the original article for further details. The reader should pay particular attention to the excellent discussion of the significance of the spatial phase angle, and the symmetric and asymmetric spread functions. Attention is also called to the discussion of the derivation of the spread function from the sine wave response. This is important when one remembers that in the introduction to this section the spread function was defined first, with the sine wave response introduced subsequently as a dependent variable. The fact that the one may be calculated from an experimental determination of the other bears out the statement made earlier about their relationship.
- 26.3.2.4 The significance of phase angle is pointed up in discussions of objects that represent coherent, incoherent, or partially coherent sources. Even with the simple systems checked by Lamberts the phase angle was a strong function of spatial frequency. Figure 26.17 shows both the normalized amplitude in percent (directly relatable to sine wave response via equation 7) and the phase angle as a function of spatial frequency in lines/mm for a certain lens.
- 26.3.2.5 Stephens (31) has recently indicated an interesting way of determining experimentally not only the cosine of the phase angle but also the sine of the phase angle. The advantage is that of increased precision for angles up to 45°.

### 26.3.3 The recording electronic lens bench of Herriott.

- 26.3.3.1 The recording lens bench we are about to describe is a long way from the first exploratory efforts in this field. Actually this lens bench is similar in purpose to the K.D.C. apparatus in that each was designed not so much to do research work as to check out large number of lenses routinely by their respective techniques. The target for this apparatus was first made by W. Herriott (32) and is shown in Figure 26.18. Note carefully that the spatial frequency varies continuously on the actual target with samples taken discontinuously along the length of the film to show the variation in the spatial frequency. The scanning slit is oriented vertically with respect this page. The target is on a 36 in. strip of 35mm film with 50 parallel opague tracks on 0.010 in, centers. The slit is a few microns wide and long enough to span most of the width of the 50 tracks.
- 26.3.3.2 In use the target film is wound around a drum inside of which is the light source and appropriate motors and clutches. Attention is called to the fact that the target does not directly present a sinusoidal variation of intensity to the optical system under test. The scanning slit, however, integrates the image over its length and the result is effectively the same as with the Schade system. The complete schematic layout of the system is shown in Figure 26.19.
- 26.3.3.3 In this method the sine wave response is measured by the contrast rendition which is defined as "image max image min".

  Object max object min

  Defined in this way, the result is independent of the contrast in the object, a point about which there was much discussion in connection with resolving power targets. The contrast rendition is plotted automatically as a function of spatial frequency. A typical recording showing the result of a through focus test is shown in Figure 26.20. (33)

#### 26.4 SINE WAVE TESTING WITH SQUARE WAVE TARGETS

### 26.4.1 General discussion.

- 26.4.1.1 One of the problems involved in sine wave testing is the actual production of the sine wave targets themselves. This has proved to be a major problem, particularly so as the demands of the theorists got tighter and tighter. One method has already been outlined above Other techniques have been developed (34 36) but the fact remains that it is still easier to make a square wave target than a sine wave target. The question has naturally arisen "can we not utilize the known Fourier sine wave content of a square wave to produce the equivalent of a pure multiple frequency sine wave target?" The answer is "yes" with some restrictions. If
- (31) Stephens, Computation of Achromatic Objectives, NBS, (1954)
- (32) Herriott, W., JOSA 37, 472 (1947)
- (33) Herriott, D., JOSA 48, 968 (1958)
- (34) Kapany and Pike, JOSA, 46, 867 (1956)
- (35) Kapany, Eyer, and Shannon, JOSA 47, 103 (1957)
- (36) Kelly, Lynch, and Ross, JOSA 48, 858, (1958)

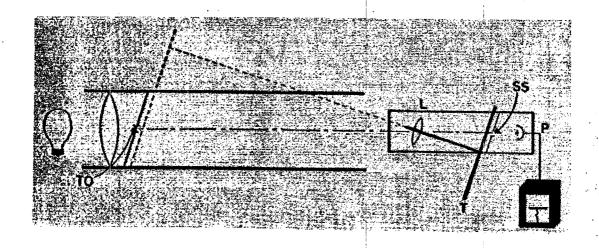
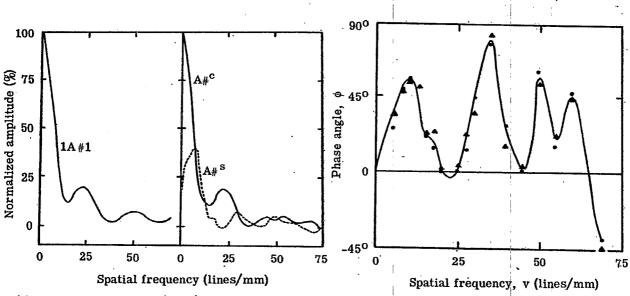


Figure 26.16- Lambert's lens bench for determining sine wave response factors.

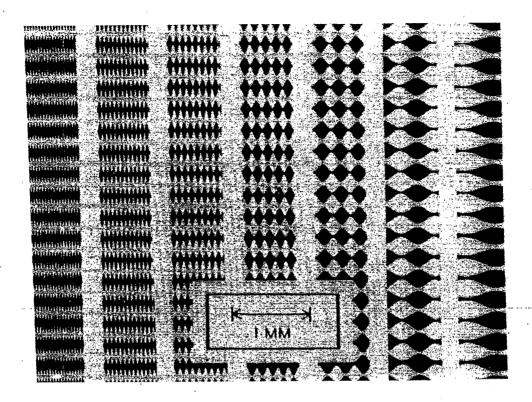


(a) Sine-wave response |A#| (left) and Fourier transfoms of it A#c and A#s (right) for a certain lens. A single sinusoidal test object was used to obtain |A#| and a double test object for A#c; A#s was computed from the other two.

(b) Phase angle as a function of frequency for the lens of fig. (a). The curve represents the mean of the two determinations • and •.

Figure 26.17 - Normalized amplitude and phase angle as a function of spatial frequency.

(From Jour. Optical Soc. America, Lamberts 89, 1958)



Enlarged photographs at intervals along a sinusoidal target on which the frequency change is continuous.

Figure 26.18 - The Herriott continuous spatial frequency target for determining sine wave response. (From Jour. Optical Soc. America, W. Herriott 37; 472, 1947)

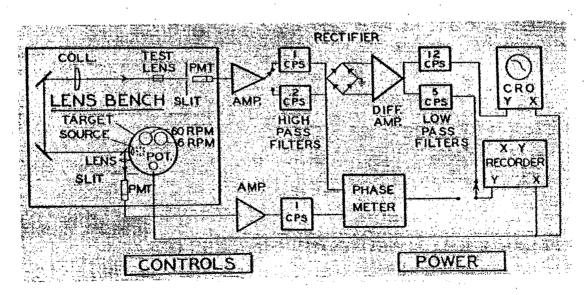


Figure 26.19 - Schematic diagram of the electronic system of the Herriott recording electronic iens bench. (From Jour. Optical Soc. America, W. Herriott 37; 472, 1947)

we suppose the optical system to accurately image a spatial frequency square wave such as a series of alternate dark and bright bars equally spaced, then the image may be represented as a spatial frequency series as,

$$F(x) = B_1 + \Delta B_1 \frac{4}{\pi} \left[ \cos \left( 2\pi n \frac{x}{\delta} \right) - \frac{1}{3} \cos 3 \left( 2\pi n \frac{x}{\delta} \right) + \frac{1}{5} \cos 5 \left( 2\pi n \frac{x}{\delta} \right) \right].$$
 (8)

Where x is the lateral coordinate and is defined as the width of a rectangle with the same area and height as the aperture flux distribution as shown in Figure 26.21 taken from Coltman (37). The square wave response factor is defined then as

$$r(n) = \frac{\Delta B_2/\Delta B_1}{B_2/B_1} \tag{9}$$

and the sine wave response factor is defined as

$$R(n) = \frac{\Delta B_2/\Delta B_1}{B_2/B_1} \tag{10}$$

- 26.4.1.2 It should be noted that there is a variation in the definition of response factors from author to author. This is clear if the reader will go back and check the definition of similar terms by Schade and D. Herriott. The end result in each case is essentially the same and one definition can be converted into another with no basic change in principle.
- 26.4.1.3 Coltman (38) shows that r(n) may be expressed in terms of the sine wave responses R(n) as given in equation (11).

$$r(n) = \frac{4}{\pi} \left[ R(n) - \frac{R(3n)}{3} + \frac{R(5n)}{5} - \frac{R(7n)}{7} \right]$$
 (11)

solving for R(n) by successively subtracting series for  $\frac{r(Kn)}{K}$  we can get,

$$R(n) = \frac{\pi}{4} \left[ r(n) + \frac{r(3n)}{3} - \frac{r(5n)}{5} + \frac{r(7n)}{7} + \cdots \right]$$
 (12)

The reader should see Coltman for the details. Suffice it to say that we have now expressed the sine wave response at a spatial frequency of n, the number of cycles in some unit distance. There are basically two ways of determining R(n). These will now be discussed.

### 26.4.2 The Coltman variable frequency square wave method.

- 26.4.2.1 The Coltman technique is similar in principle to the corresponding technique used in testing electrical amplifiers (39) with variable frequency square waves. Others such as Rosberry have also studied the method. It is usually found to be more trouble than it is worth to test electrical amplifiers this way, since if you have to vary the frequency of square wave, you might just as well vary the frequency of a sine wave and be done with it. In the optical case it is easier to vary the frequency of the square wave because spatial square waves can be made more easily than can spatial sine waves.
- 26.4.2.2 Coltman's method is similar, then, in principle to that discussed in Schade and Herriott's paper except for an analysis (40) that allows him to measure the sine wave response of the system by use of the more easily manufactured square waves. Not only is Coltman's article highly informative but it also gives an excellent discussion of the basis of the method and a specific example in the field of X-ray fluoroscopic work. Here the relative ease of studying systems in cascade by the sine wave method is shown and a discussion as to why sine wave targets are not used is given.

### 26.4.3 The fixed frequency square wave method.

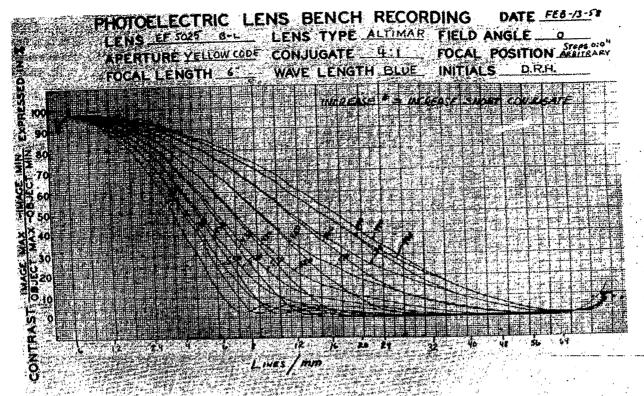
26.4.3.1 Suppose that instead of variable spatial frequency square wave, we used a fixed spatial frequency square wave and get the higher frequency components by wave analysis of the electrical output of the photomultiplier tube. We now assume that the combination of scanning pinhole and associated photomultiplier circuit that transduces the spatial frequency to a temporal frequency spectrum in the image can be directly

<sup>(37)</sup> Coltman, JOSA 44, 468, (1954)

<sup>(38)</sup> ibid.

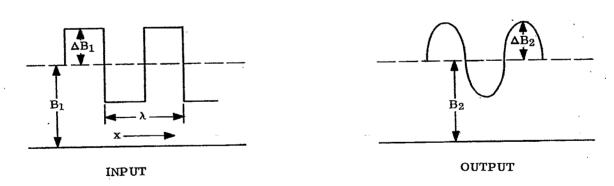
<sup>(39)</sup> Rosberry, A Correlation Investigation Between Photoelectric and Image Analysis, NBS Report No. 5799

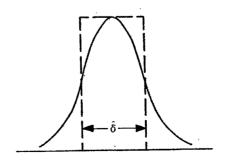
<sup>(40)</sup> Loc. cit.,



## SINE WAVE TARGET SPACINGS

Curves of contrast rendition measured through focus and recorded directly on preprinted paper. Figure 26. 20 - Sample contrast rendition vs spatial frequency recording taken with the Herriott system. (From Jour. Optical Soc. America, D. Herriott 48; 968, 1958)





APERTURE FLUX DISTRIBUTION

Figure 26.21- Quantities used in the Coltman definition of response factors. (From Jour. Optical Soc. America, Coltman 44; 468, 1958)

related to the spatial frequency. Furthermore electrical tunable narrow band temporal frequency filters are standard items and have been for years. Therefore by feeding the output of the transducing element to a temporal frequency filter tuned to the fundamental of the square wave we can determine the sine wave response at that frequency. We then retune the filter to the next harmonic, record the output, etc. Account must be taken of the reduction in amplitude of the harmonics as given by the coefficients in equation (8).

- 26.4.3.2 It might occur to those versed in Fourier analysis that some other wave shape might be chosen such as a triangular wave that has all harmonics and not just the odd harmonics as in the case of the square wave. Such a wave could be used, but again the problem of production is such that it probably would be undesirable. Actually a single square wave target can suffice to cover almost any desired spatial frequency band provided it is used with a minifying or magnifying system whose quality is far superior fo that of the system being tested.
- 26.4.3.3 The difficulty involved with this method of square wave testing is that there is a phase shift associated with the tunable electrical filter. While there are ways to take this into account, they are rather complicated. Furthermore, it was assumed above that the percentage reduction of the amplitude as a function of frequency in the ideal image was known. This is true providing the detecting system is completely linear. For some systems, notably photographic ones, this may well not be so. Hence white we can get the sine wave response at any frequency, it may be difficult to relate it numerically to the response at other frequencies.

## 26. 4. 4 Automatic determination of power of an opthalmic lens by sine wave response.

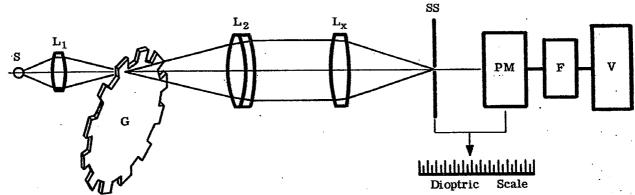
- 26.4.4.1 In 1953 Gunter and Panetta (41) developed a method of applying the sine wave response criteria to the problem of automatically maintaining large aerial cameras in focus. The aerial camera aspects of the technique are of not so much interest to us here as is Gunter's definition of best focus used in connection with their analysis of the problem viz, "best focus is that point in image space where the spatial frequency response is an optimum within the bandwidth of information in which the observer is most interested." This definition is certainly a far cry from that usually found in optics and photography. It stems from the work of Schade rather than from that of the traditional treatments of Conrady etc.
- 26.4.4,2 Shortly thereafter Gunter (42), (43), (44) applied these same principles and this same definition of focus to the automatic determination of the power of opthalmic lenses. This was a research problem to see if the human factor could be removed in the routine inspection of opthalmic lenses. The women who customarily do this work are wont to get tired and their judgment varies. The first target was a square wave made by rotating a square cut gear as shown in Figure 26, 22.
- 26.4.4.3 In the initial study the combination of SS and PM was moved along a lathe bed until the meter showed a maximum, the bandwidth of information having been selected by trial. Specifically this meant that a lens of say 2 diopters as judged by the eye was selected. This lens was placed in the test device and the temporal frequency filter adjusted until the meter output was maximum at a distance of exactly 50 cm. By checking with other standard lenses the variation of focal point as judged by the maximum meter response and the eye were shown to be well within commercial tolerances. A plot of meter response vs. focal position looked essentially the same as Figure 26.12.
- 26.4.4.4 The technique having been proved, the quest was now how to change the system so that a lens could be snapped into place and have the SS-PM unit automatically move so as to maximize the meter response i.e. move to the position of best focus. Important in the final system was a novel "square wave target" suggested by Hayes. The original square cut gear (seen from above) was modified as shown by the dotted lines in Figure 26.23 the hatched part of the gear remaining. The lens under test sees sequentially now edge a, b, a', b', a'', b'' etc. This is the equivalent of a square wave tipped at an angle of  $45^{\circ}$  in so far as  $L_{\rm X}$  is concerned. The light from the source S was focussed midway between a and b so that the edges a and b appeared equally sharp to  $L_{\rm X}$ . The action is simple. Referring to Figure 26.12 we see that near the peak the response curve as a function of focal length is quite symmetrical. An electronic circuit separated the responses from edges a and b and ordered a motor to adjust the position of the SS-PM unit until the responses were equal. The motor then stopped and the power of the lens was read directly from the scale. The system was easily more than sufficiently accurate. Modifications of the system were developed for specific purposes but the basic technique was unchanged.

<sup>(41)</sup> Gunter and Panetta, An Automatic Electronic Focussing Device for Aerial Cameras, Boston University Optical Research Laboratory Technical Note 113, June, 1954

<sup>(42)</sup> Gunter, Whitney, Hayes. U. S. Patent 2897722, Electronic Lensometer.

<sup>(43)</sup> Gunter, Whitney, Hayes. U. S. Patent 2803995, Special Frequency Centering Device.

<sup>(44)</sup> Wing, Whitney, Hayes. U. S. Patent 2792748, Pyramid Centering Device.



S = source of light

 $L_1$  = lens to focus light from S onto the teeth of G.

G = square cut gear rotated at 1800 rpm.

 $L_2$  = a collimating lens.

 $L_x$  = the opthalmic lens under test,

SS = pinhole scanning aperture.

PM = photomultiplier tube and associated circuits

F = a tunable electric filter

V = voltmeter

Figure 26.22- Basic square wave system for studying Opthalmic lens power.

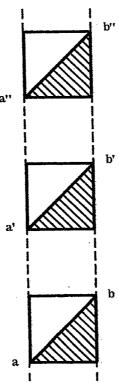


Figure 26.23 - The Hayes target for the American Optical automatic lens power measurement system.

Custodians:

Army - U.S. Army Munitions Command Navy - Bureau of Ships Air Force - Middletown Air Materiel Area Preparing activity:
Army - U.S. Army Munitions Command

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Interp.	Constant	1,012	1,011	1.012	1.013	1, 012	1, 012	1.013	1,013	1,013	1,013	1,013	1,014	1, 014	1,014	1.014	1,015	1,015	1,014	1,016	1,015	1,015	1,016	1,015	1.016	1. 016 1. 016	1.017	1,016	1, 017	1.018	1. O.7	1, 018	1,018	1,018	1.018	1.019	1.019	1.019	1.019	1,020	1.019	1.020	1,020	1.021	1.021	
	Angle	.151579	152591	.153602	. 154614	. 155627	156639	158664	159677	160690	161703	162716	, 163729	.164743	165757	10011	168799	169814	170829	171843	.172859	.173874	.174889	175905	176920	177936	179968	180985	.182001	.183018	184036	186070	187088	,188106	,189124	, 190142	191161	193199	194218	195237	196257	197276	198296	199316	.200337	, 40400 i
	Sine	151	152	153	154	155	156	127	150	160	161	162	163	191	165	166	.9T	160	720	12.	172	173	174	175	176	177	179	180	181	182	183	184	186	187	188	189	190	161	267	187	195	196	197	198	199	2002
	Constant	1 005	1.006	1,005	1,006	1.005	1,005	1,006	1.006	1,005	T. 00%	1.006	1,006	1,007	1.006	1.007	1.007	1.007	1,008	1,007	1.005	1.008	1,007	1,008	1,008	1,008	1,008	1,009	1,009	1,009	1.009	1.009	1.009	1,010	1,009	1.010	1.010	1.010	1.010	1.011	1,011	1,011	1.011	1.011	1,012	1,011
	Angle	101180	100177	103183	.104188	105194	.106199	107204	.108210	109216	110221	111228	119941	114247	.115254	,116260	.117267	.118274	119281	120289	121296	122303	124320	. 125327	, 126335	127343	128351	190968	131377	.132386	, 1333395	134404	135413	137432	.138442	139451	.140461	141471	142481	143491	145512	146523	.147534	.148545	.149556	.150568
	Sing	ATTC.	g :	707 103	104	105	106	101	108	109	110	3 :	112	114	115	116	117	118	119	120	121	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	124	125	126	127	128	129	131	132	133	134	135	137	138	139	140	141	142	143	135	146	147	148	149	150
	Interp.	Constant	1.001	1,001	7.00%	100	1,001	1.002	1,002	1.002	1,001	1,002	1,002	1.002	1,002	1,002	1,003	1,002	1,003	1.002	1,003	1.002	1,003	1 .003	1.003	1.003	1,003	1,003	1.003	1,004	1,003	1,004	1.004	1,003	1.004	1.004	1,004	1,004	1.005	1,004	1,004	000 T	1,005	1.004	1,006	1,005
		Angle	,051021	.052022	. 053023	. 054025	. 056028	. 057029	. 058031	. 059033	. 060035	.061036	.062038	.063040	065045	066047	. 067049	. 068052	. 069054	. 070057	.071059	.072062	,073064	. 074067	076079	077076	6.0870	.079082	. 080085	0801000	083095	. 084098	. 085102	. 086106	. 087109	089117	090121	. 091125	, 092129	.093134	.094138	095142	096147	. 098157	. 099161	.100167
		Sine	051	052	053	† <u>9</u> 0	05 05 05 05 05 05 05 05 05 05 05 05 05 0	000	928	620	090	190	062	063	† 90 90	980	067	890	690	020	071	072	073	074	075	07.0	078	620	080	180	283	984	980	980	180	980	680	160	092	093	094	600	960	260	660	001
•	Interp.	Constant	1.000	1,000	1,000	1.000	1.000	000	36.	900	800	000	1,000	1.000	1.000	1.000	1.000	T. 000	000	200	1,000	1,000	1.001	1,000	1.000	1.000	1.000	1,001	1.000	1.001	1.001	1.000 1.001	1,000	1.001	1,001		1,001	1.001	1,001			1,001	1,001	1.001	1,001	1,001
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		Sine	200	5 8	003	8	98	90	200	800	600	010	110	013	0.14	012	016	017	018	610	020	000	023	024	026	026	020	020	030	031	032	033	035	036	037	038	039	040	045	043	044	045	046	047	048	050

Table 1 - Sine - Angle Conversion Table (Sheet 1 of 5)

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Constant	1,068	1,069	1.070	1.070	1.070	1.071	1.072	1.072	1.071	1.073	1.073	1.074	1.074	1.074	1,075	1.075	1.076	1.076	1.076	1.07.	1.078	1,078	1.079	1,079	1.081	1.080	1.081	1.082	1,082	1,083	1.083	1.084	1.084	1.085	1.085	980 T	1.086	100	1.088	1,088	1.088	1.090	1,090	1.090	1.091	1.021
Angle	. 358638	. 350774	.361843	. 362913	. 363983	. 365053	.366124	. 367196	.368268	, 369339	.370412	.371485	. 372559	. 373633	. 374707	. 375782	.376857	. 377933	30000	381169	382240	. 383318	.384396	.385475	. 386554	. 387635	. 388715	.389796	391959	. 383041	. 394124	. 395207	. 396291	. 397375	, 398460	399545	400631	409804	403891	404979	406067	407155	408245	409335	410425	orett.
Sine	351	202	354	355	, 356	357	358	329	360	361	362	363	364	365	366	367	368	369	87.0 27.0	7)5	3 22	374	375	376	377	378	379	380	385	383	384	382	386	387	80 80	5 0 0 60 0	) 000 000 000	300	368	394	•		397	398	•	, *O0*
Interp. Constant	1.049	1.049	1,050	1,050	1,051	1.051	1.051	1,052	1,052	1,052	1,053	1.053	1,053	1,054	1.054	1.054	1,055	1.056	T. 056	1 056	1.057	1,058	1.057	1: 058	1.058	1.059	1.059	1.060	1:060	1,061	1,061	1.061	1,062	1.063	1.062	1.063	1.064	1 064	1, 065	1,065	1.065	1.067	1.067	1,066	1.067	T. 000
Angle	.305741	. 306790	30888	309938	.310988	. 312039	.313090	.314141	. 315193	. 316245	,317297	.318350	.319403	. 320456	.321510	. 322564	. 323618	. 324673	. 325/29	997841	328897	. 329954	. 331012	, 332069	. 333127	. 334185	. 335244	. 336303	. 338423	. 339483	. 340544	.341605	. 342666	. 343728	. 344791	. 345853	. 346916	- 5% 500 340044	350108	. 351173	. 352238	. 353303	. 354370	. 355437	. 356503	020100
Sine	301	305	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	175	353	324	325	326	327	328	329	330	332	333	334	335	336	337	988	925	340	349	343	344	345	346	348	348	349	مرمه
Interp. Constant	1,033	1.033	1.034	1,034	1,035	1.034	1.035	1,035	1,036	1,036	1.037	1.036	1.037	1.038	1.037	1,038	1.038	1,039	T. 038	1.039	1.040	1,040	1,040	1,041	1.041	1.041	1.041	1,042	1.043	1,043	1.043	1.043	1.044	1.044	1.044	1.040	1.045	1 047	1.046	1.046	1.047	1.047	1.048	1.048	1.047	71.047
Angle	. 253717	.254746	256814	. 257848	.258882	. 259917	.260951	.261986	. 263021	. 264057	. 265093	.266130	.267166	.268203	. 269241	. 270278	. 271316	. 272354	. 273393	274431	976509	. 277549	. 278589	. 27 9629	.280670	. 281711	. 282752	. 283793	. 285877	. 286920	. 287963	. 289006	. 290049	. 291093	. 292137	. 293181	. 294226	296316	. 297363	.298409	. 299455	300502	.301549	, 302597	303645	# cotoo
Sine	251	2020	25.4	255	256	. 257	258	259	260	197	262	263	264	265	266	267	268	269	270	7.2.7	073	274	275	276	277	278	279	280	282	283	284	285	286	287	80 00	607	000	292	293	294	295	296	297	298	300 300	
Interp. Constant	1.021	1.021	1.021 1.022	1,022	1,022	1.022	1.022	1,023	1,023	1,023	1,023	1,024	1.024	1.024	1.024	1.025	1.024	1,025	1.026	1.025	1.026	1.026	1.027	1,026	1.027	1.027	1.028	1.027	1029	1.028	1.029	1.028	1.030	1.029	1.030	1 030	1.03	1.031	1,031	1.031	1,032	1.032	1.032	1.032	1,033	
Angle	. 202378	. 203399	. 204420	206463	. 207485	. 208507	. 209529	. 210551	.211574	. 212597	.213620	. 214643	. 215667	. 216691	.217715	. 218739	. 219764	. 220788	. 221813	. 222333	100077	. 225916	. 226942	. 227969	. 228995	. 230022	.231049	.232077	234132	. 235161	. 236189	. 237218	. 238246	. 239276	240305	049966	242300	.244426	. 245457	.246488	.247519	. 248551	250615	. 25 V6 L5	. 252680	
Sine	I															217	218	219	220	177									:									1								I

Table 1 - Sine - Angle Conversion Table (Sheet 2 of 5)

		Intom			Interp.	5	Angle	Interp.	Sine	Angle	Constant
	ofono	Constant	Sine	Angle	Constant	anve	10101	1.156	551	. 583562	1,199
sine	angre	000	451	.467885	1.121	501	. 524754	1 156	552	.584761	1,200
401	.412607	7.000	452	90069†	1.121	205	018220.	1550	553	.585961	1,200
402	413099	1.092	453	. 470127	1,122	503	000100	1.158	554	.587161	1,202
±03	. 414791	300	454	471249	1.123	504 405	600000	1.159	555	, 588363	1.203
404	415554	1.00	455	.472372	1.123	505	100000	1.160	556	, 589586	1,203
405	416978	1.005	456	. 473495	1,124	206	100001	1.160	557	. 590769	1.205
406	418072	1.005	457	.474619	1.125	200	101100	1,162	558	.591974	1,205
407	419167	1.006	458	, 47 57 44	1,125	208	100700	1.162	559	. 593179	1,207
408	420262	000.	459	476869	1,126	203	. 534040	204	260	. 594386	1,207
409	. 421358	1.096	480	477995	1,127	210	.535185	7.100	561	. 595593	1,209
410	. 422464	1.096	402	47.9122	1.127	511	. 536348	707.7	269	596802	1.209
411	. 423550	1.097	704	480949	1, 128	. 512	.537511	1,165	400	598.011	1,211
412	. 424647	1,098	402	10000	1.128	513	.538676	1,165	200	666003	1.211
413	, 425745	1.099	463	10000	1.130	514	.539841	1.167	2004	20000	1,213
414	. 426844	1.099	46+	000000	130	515	.541008	1.167	200	. 004.00	1 913
415	427943	1.100	465	. 483635	001	516	542175	1.167	266	0.000	1.61.0
418	429043	1,099	466	. 484765	1,100	512	543342	1.169	267	. 602659	910
2 + +	430142	1,101	467	. 485895	1, 131	9 5	544511	1,170	268	. 604074	CT2.T
	491943	1,101	468	,487026	1, 132	3 5	545681	1.170	269	.605289	1.21
410	499944	1,101	469	. 488158	1,133	GTC.	1000th	1,171	570	. 606506	1.217
419	402044	104.1	470	.489291	1.133	020	100010	1 179	571	.607723	1,219
420	. 433440	1100	471	. 490424	1.134	521	220040.	173	572	.608942	1,220
421	434548	707.	47.9	,491558	1,135	522	. 549194	2111	573	,610162	1,220
422	,435650	1.103	473	.492693	1,135	523	. 550367	1.1.0	47.0	.611382	1,222
423	. 436753	1,104 1,104	474	493828	1,136	524	.551540	1.1.0	575	612604	1,223
424	437857		475	494964	1,137	525	. 552715	1,170	576	. 613827	1,224
425	438961	1,105	476	496101	1,137	526	. 553890	1.1(	577	.615051	1,225
426	440066	1 106	477	. 497238	1,138	527	,55506,	1.4.6	578	.616276	1,226
427	441172		478	.498376	1,139	528	. 556244	179	579	617502	1,227
424	44000		479	, 499515	1,140	529	777100.	1 1 7 9	580	.618729	1.228
624	007777		480	. 500655	1,140	530	100000	1,181	581	.619957	1,229
430	445600		481	. 501795	1, 141	122	. 00100.	1, 181	583	.621186	1,230
407	446709		482	,502936	1,142	20 1	589149	.1, 182	583	.622416	1,232
2 6	447817		483	,504078	1.142	200	563324	1.184	584	.623648	1,232
200	448027		484	. 505220	1,143	* 60	564508	1,184	585	.624880	1,234
100	450038		485	, 506363	1.144	90.5	565692	1,185	586	,626114	1.234
967	451148		486	. 507507	1.145	200	566877	1,186	587	, 627348	1,236
437	452260	1,112	487	508652	1,145	98.5	. 568063	1,186	288	628584	1.237
438	,453372		488	,5097.97	1 147	939	569249	1,188	289	. 629821	1.230
439	454485		480	.510943	- 44 t t	540	. 570437	1,189	290	.631059	1.200
440	. 455598		490	000210	1.148	541	. 571626	1,189	591	632298	1.240
441	.456711		491	100000	1 149	542	. 572815	1, 191	289	. 633330	
442	.457826		492	. 01450 00410	150	543	. 574006	1, 191	593	.634780	1.646
443	, 458941		493	. 010004 516684	1, 151	544	. 575197	1,192	594	. 630022	1 945
444	, 460057		#0#	517835	1,151	,545	686949	1, 193	200	0001500	1 245
445	,461173		407	518986	1,152	546	. 577582		200	400004	1.247
4+6	462290		404	590138	1,153	547	. 578776		286	00000	1 949
447	463408		104	521291	1,153	548	. 579971		900	640069	1,249
448	464527		007	522444	1,155	549	1.581167		660	643501	1.251
449	. 465645		P 00 F	593599	1,155	920	. 582364	1,198	000	*000*0	20014
450	. 466765	5 1,120	,,,,,								•

Table 1 - Sine - Angle Conversion Table (Sheet 3 of 5)

i ii	T,		-	4	9	<u> </u>	~						×0			. ~			_									-		-		_		•					<u> </u>			<del>_</del>				····
Interp. Constant	1.516	1.518	1.521	1,524	1.526	1.529	1,532	1,535	1.537	1.540	1.043	1.546	1.548	1 555	1.557	1,559	1,563	1.566	1,569	1.572	1.574	1.578	1.581	1,584	1.587	1,590	1,593	1,597	1.099	1.606	1,610	1,612	1.616	1,619	1.623	7.020	T. 029	1 636	1.640	1.643	1,647	1.650	1.654	1,657	1.661	1.665
Angle	. 849575	851091	.852609	.854130	. 855654	.857180	. 858709	. 860241	.861776	, 863313	. 004503	. 300396	869490	871041	872596	.874153	.875712	.877275	.878841	.880410	.881982	. 883556	. 885134	. 886715	. 888299	. 889886	. 891476	. 893069	. 69*000 896965	. 897868	. 899474	. 901084	. 902696	. 904312	. 905981	901104	01000	912442	914078	915718	917361	919008	920658	922312	923969	. 925630 . 927295
Sine	751	752	753	764	755	997	7.07	708	60%	192	101	707	764	765	766	767	168	769	770	171	772	773	774	775	27.6	777	778	987	781	782	783	784	785	002	2 00	886	280	791	792	793	794	795	436	797	•	800
Interp. Constant	1,403	1.405	1.407	1,409	1.411	1 415	1.410	1 410	1,413	1.421	1.426	1.497	1. 429	1, 432	1, 433	1,436	1,438	1,439	1.442	1.445	1.446	1,449	1,450	1,453	1,456	1.40	1.460	1.464	1,467	1,469	1. 471	1. 474	1.478	1 481	1.483	1,485	1.488	1.491	1.493	1.495	1, 498	1,500	1.503	1,500	1,009	1.513
Angle	.776799	.778202	.779607	.781014	. 782423	785947	78885	788079	789498	790919	792342	793768	. 795195	.796624	. 798056	. 799489	800925	. 802363	. 803802	805244	808689	302130	. 008084	.011034	6120010	815400	816860	818322	.819786	,821253	. 822722	. 024093 805867	. 827143	. 828621	. 830102	.831585	.833070	.834558	.836049	.837542	. 839037	.840535	. 842035	845043	846552	. 848062
Sine	701	702	703	704	2 2	2 2	708	209	210	217	712	713	714	7.15	716	717	718	719	720	721	77.	2 2 2	7.05	796	207	728	729	730	731	732	733	7.85.	736	737	738	739	740	741	742	743	744	746	04.2	748	749	
Interp. Constant	1.318	1,320	1.321	1.323	1.02# 1 995	1.328	1.329	1, 330	1.332	1,333	1,335	1,337	1,338	1.340	1,341	1.343	1,345	1.346	1.048	1.048	1.301	356	1.356	358	1.359	1.362	1.363	1.364	1,367	1.368	1.370	1,373	1.376	1.377	1.378	1,381	1,383	1,384	1.386	1.388	1 300	1 393	1.396	1,397	1.400	1.401
Angle	106807.	.710219	.711539	714189	715507	.716832	.718160	.719489	.720819	.722151	.723484	.724819	.726156	.727494	.728834	. 730175	.731518	794900	795557	738908	738257	739610	. 740965	. 742321	.743679	. 745038	.746400	.747763	.749127	. 750494	753232	. 754604	.755977	. 757353	.758730	.760108	.761489.	. 762872	.764256	767050	.768420	.769812	.771205	.772601	.773998	,775398
Sine	651	652	653	909	656	657	658	. 629	099	199	662	663	664	665	999	299	999	670	671	672	673	674	675	929	677	678	679	089	189	289	684	685	989	687	688	689	. 069	169	260	694	695	969	697	698	669	700
Interp. Constant	1.251	1,253	1.255	1.256	1.258	1.259	1.260	1.262	1,262	1.264	1,265	1,266	1,268	1.269	1.270	1,2/1	1 274	1.275	1.276	1.278	1,279	1.281	1,281	1.283	1,284	1.286	1.287	1.289	1.289	1.231	1,293	1,296	1.296	1.298	1.299	1.301	1.302	1.304	1 306	1.308	1,310	1.310	1.313	1.313	1.315	1.317
Angle	644752	. 646003	. 647256	649766	. 651022	,652280	, 653539	. 654799	. 656061	. 657323	. 658587	. 659852	. 661118	. 662386	. 663655	. 001200	667469	. 668743	.670018	671294	. 672572	.673851	. 675132	.676413	. 677696	.678980	. 680266	. 681553	248289.	685422	. 686715	. 688008	. 689304	. 690600	. 691898	.033187	0044400°	697104				٠			•	. 707584
Sine	109	209	3 6	909	909	607	809	609	610	611	612	613	614	615	979	3 5	619	620	621	622	623	624	625	626	627	628	629	029	150	633	634	635	636	637	929	640	240	642	643	644	645	646	647	648	649	650

Table 1 - Sine - Angle Conversion Table (Sheet,4 of 5)

Sine         Angle         Constant         1.12069           651         1.017887         1.907         901         1.124580           652         1.021707         1.913         902         1.124580           652         1.021707         1.913         903         1.124580           854         1.02707         1.913         904         1.124580           856         1.027821         1.95         904         1.136180           856         1.027821         1.95         906         1.13845           856         1.027821         1.96         907         1.13845           868         1.037821         1.96         901         1.148708           867         1.03723         1.96         912         1.148708           868         1.043161         1.98         912         1.148708           868         1.043161         2.00         91         1.148708           868         1.043161         2.00         91         1.148708           868         1.041178         2.00         91         1.148708           868         1.041160         2.01         91         1.1148134           868		1-4-2			Interp.	S.	Angle	Constant	Sine	Angle	Constant
1,095   682   1,017897   1,917   903   1,124709   2,222   955   1,265705   958   1,265705	al v	interp.	Stne	Angle	Constant	arro	9	0 011	951	1.256454	3, 251
1,772         682         1,01979         1,913         902         1,14570         2,256         693         1,28500         693         1,12870         2,345         693         1,12870         2,345         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,346         693         1,12870         2,446         693         1,12870         2,446         693         1,12870         2,446         693         1,12870         2,446         693         1,12830         694         1,12830         694         1,12830         694         1,12830         694         1,12830         693         1,12830         693         1,12830         693         1,12830         693         1,12830         694         1,12830         693         1,12830         693         1,12830         694         1		Constant	25.1	1, 017887	1.907	106	1, 122069	7.317	252	1,259705	3.284
1,091   685   1,001/107   1,019   908   1,128/102   2,556   656   1,266306   656   1,266306   656   1,001/107   1,019   906   1,113930   2,566   656   1,277000   656   1,277000   656   1,001/107   1,045   666   1,277000   666   1,27700   666   1,277000   666   1,277000   666   1,277000   666   1,277000   666   1,277000   666   1,277000   666   1,277000   666	¥96	1.672	700	1 019794	1, 913	905	1, 124380	2,226	953	1, 262989	3.317
1,686   684   1,02526   1,925   1,92	636	1.676	200	1 021707	1,919	808	1, 126702	2,000	954	1,266306	3,354
1,695   665   1,0024561   1,991   906   1,133748   2,886   966   1,279040   966   1,279040   968   1,279040   969   1,27904	312	1.680	854	1.023626	1,925	904	1,129035	2.340	955	1,269660	3,390
1,000   605   1,000	266	1.683	1 12	1,025551	1.931	902	1, 131380	0000	926	1,273050	3,428
1,000   1,00	675	1,000 1	856	1,027482	1.938	906	1,133730	2000	957	1,276478	3,467
1,000   1,00	202	1.000	857	1,029420	1,943	904	1, 130104		958	1.279945	3,508
1,777   1,000   1,00	1054	7.030	0 00	1,031363	1.950	806	1,138485	90,00	958	1,283453	3,550
1,772         860         1,0828270         1,983         910         1,143292         2,444         961         1,29066           1,772         862         1,087282         1,976         912         1,145194         2,444         962         1,29066           1,772         868         1,04116         1,987         914         1,155096         2,472         965         1,29436           1,720         866         1,04116         1,987         914         1,155096         2,472         965         1,29436           1,722         866         1,04116         1,997         915         1,155096         2,472         966         1,30454           1,723         867         1,04116         1,997         917         1,156096         2,472         966         1,30454           1,744         868         1,04116         1,997         917         1,156096         2,514         966         1,300544           1,745         870         1,057234         2,047         917         1,156094         2,514         966         1,300544           1,745         871         1,067234         2,047         917         1,154190         2,514         966         1,300544	1420	F. 688	0 00	1,033313	1,957	606	1,140878	7,400	960	1,287003	3,593
1,777   861   1,08728   1,969   911   1,445703   2,454   962   1,294235   1,700   861   1,08728   1,969   912   1,445703   2,456   964   1,294235   1,700   862   1,043161   1,969   914   1,145706   2,456   964   1,294235   1,700   865   1,043161   1,969   914   1,155306   2,466   965   1,294235   1,700   965   1,047147   2,007   915   1,155306   2,466   965   1,306444   2,514   965   1,306444   2,514   965   1,306444   2,514   965   1,30644   2,514   965   1,306444	449	1.703	689	1.035270	1,963	910	1,143282	214.2	961	1 290596	3,639
1,772         865         1,0976         912         1,148134         2,444         962         1,2972           1,775         868         1,04178         1,997         914         1,156308         2,475         964         1,304547           1,725         866         1,047176         2,003         917         1,156308         2,475         966         1,304547           1,732         866         1,047177         2,003         917         1,156308         2,475         966         1,304544           1,732         866         1,047177         2,003         917         1,165308         2,475         966         1,304544           1,745         870         1,06120         2,03         917         1,160344         2,514         967         1,31450           1,746         870         1,062202         2,032         917         1,16034         2,514         967         1,31450           1,746         877         1,062202         2,032         922         1,173216         971         1,32537           1,746         877         1,062302         2,032         922         1,173216         2,64         976         1,32537           1,776 <t< td=""><td>162</td><td>1.707</td><td>200</td><td>1 027933</td><td>1,969</td><td>911</td><td>1,145703</td><td>2. 431</td><td>100</td><td>1 004935</td><td>3.686</td></t<>	162	1.707	200	1 027933	1,969	911	1,145703	2. 431	100	1 004935	3.686
1,715         868         1,0426.2         1,0436.4         914         1,156078         2,458         965         1,0275.5           1,720         864         1,043161         1,989         914         1,155036         2,472         965         1,30544           1,723         866         1,04717         2,000         917         1,155036         2,500         967         1,31320           1,724         866         1,04717         2,002         918         1,155036         2,524         966         1,31320           1,724         866         1,06717         2,003         917         1,156037         2,544         967         1,31320           1,724         867         1,067224         2,024         919         1,166081         2,576         970         1,32523           1,744         877         1,065973         2,047         922         1,178216         970         1,32732           1,745         877         1,065486         2,064         922         1,178413         2,691         971         1,37722           1,776         877         1,065486         2,064         922         1,178413         2,644         976         1,37722	888	1.712	198	7.000000	1.976	912	1,148134	2, 444	208	1. 484400	3.736
1,700         868         1,04117         1,050         914         1,158196         2,472         964         1,30129           1,720         864         1,04116         1,997         915         1,158196         2,466         964         1,301294           1,732         866         1,04116         1,997         915         1,169044         2,604         967         1,301280           1,736         867         1,04116         2,018         918         1,118049         2,644         967         1,331380           1,745         877         1,05220         2,022         920         1,186041         2,654         977         1,3323115           1,746         877         1,067234         2,038         922         1,178040         2,640         977         1,323315           1,746         877         1,067320         2,034         922         1,178316         2,640         977         1,323231           1,776         877         1,067320         2,034         922         1,178316         2,640         977         1,325267           1,776         877         1,067320         2,034         922         1,178306         2,640         977         1,325267 <td>1571</td> <td>1,715</td> <td>862</td> <td>T. 038202</td> <td>200</td> <td>913</td> <td>1,150578</td> <td>2,458</td> <td>963</td> <td>1, 297921</td> <td>0 0 0</td>	1571	1,715	862	T. 038202	200	913	1,150578	2,458	963	1, 297921	0 0 0
1,723         864         1,04364         1,070         916         1,187594         2,466         966         1,896444           1,728         966         1,047147         2,003         916         1,187594         2,504         967         1,313180           1,736         867         1,045150         2,010         918         1,1653008         2,524         967         1,32331           1,741         868         1,051378         2,024         910         1,165301         2,524         967         1,32331           1,745         871         1,052302         2,024         922         1,188001         2,559         971         1,32331           1,754         871         1,053202         2,047         922         1,178015         2,641         977         1,32331           1,754         871         1,053202         2,047         922         1,178015         2,641         977         1,32331           1,754         877         1,065346         2,062         924         1,178015         2,641         976         1,32472           1,777         877         877         1,065346         2,102         922         1,118043         2,73         976	3286	1.720	863	1. 041178	960	914	1,153036	2,472	964	1.301657	200
1,728   866	900	1,723	864	1.043161	50A -T	4 5	1 155508	2,486	962	1.305444	3, 540
1,735         686         1,647477         2,003         915         1,166494         2,514         967         1,133180           1,736         6867         1,04150         2,010         915         1,168081         2,524         968         1,331136           1,745         6869         1,065202         2,034         910         1,168081         2,554         970         1,325331           1,745         870         1,065202         2,032         920         1,178216         2,575         977         1,325379           1,754         871         1,055272         2,047         922         1,178216         2,591         977         1,325379           1,755         874         1,065436         2,044         922         1,178106         2,640         976         1,34526           1,776         877         1,065436         2,069         922         1,17810         977         1,335266           1,776         876         1,065436         2,069         928         1,181090         2,640         976         1,345266           1,776         877         1,065436         2,069         928         1,181090         2,640         976         1,345266 <tr< td=""><td></td><td>1 728</td><td>865</td><td>1.045150</td><td>1,997</td><td>CT6</td><td>7000ur .</td><td>. 500</td><td>996</td><td>1,309284</td><td>3,896</td></tr<>		1 728	865	1.045150	1,997	CT6	7000ur .	. 500	996	1,309284	3,896
1.741         686         1.049150         2.010         917         1.165037         2.544         968         1.031160         2.018         917         1.165037         2.544         968         1.031161         2.024         918         1.165037         2.544         969         1.031161         2.024         919         1.165037         2.544         970         1.32151         3.054         970         1.32151         3.054         970         1.32151         3.054         971         1.23237         970         1.32151         3.057         971         1.323368         971         1.323368         971         1.323368         971         1.323368         971         1.323368         971         1.323368         971         1.323368         972         1.775         974         1.324265         972         1.775         974         1.324265         974         1.324265         974         1.324265         974         1.324265         974         1.324265         975         1.7772         975         975         1.7772         975         975         1.7772         975         976         1.324265         976         1.324265         1.344762         1.344762         976         1.324762         1.344762         1.	2 1 2 2	200	866	1.047147	2,003	916	T. TO. 65.	200	7967	1,313180	3, 955
1.756         60.         1.05106         2.523         900         1.05206         2.543         900         1.05201         2.544         900         1.05231         1.05231         1.05231         1.05231         1.05231         1.05231         1.05231         2.047         910         1.166081         2.544         970         1.323319         971         1.323319         971         1.323319         971         1.323319         971         1.323319         972         1.333319         972         1.333319         972         1.333319         972         1.333319         972         1.333319         972         1.333319         972         1.333319         972         1.337319         972         1.337319         972         1.337319         972         1.337319         972         1.337319         972         1.337319         972         1.337319         972         1.337319         972         1.337319         972         1.174413         972         1.34726         972         1.174413         972         1.34726         972         1.174413         972         1.34726         972         1.174413         972         1.34726         972         1.174413         972         1.34726         972         1.174413         972 </td <td>4457</td> <td>1.732</td> <td>500</td> <td>1.049150</td> <td>2,010</td> <td>917</td> <td>1,160494</td> <td>7. 0.14 7. 0.04</td> <td>990</td> <td>1 217135</td> <td>4,016</td>	4457	1.732	500	1.049150	2,010	917	1,160494	7. 0.14 7. 0.04	990	1 217135	4,016
1.741         B009         1.058376         2.04         919         1.166537         2.544         970         1.325201           1.745         877         1.058378         2.02         920         1.166981         2.575         971         1.323879           1.754         877         1.058373         2.047         922         1.17806         2.501         974         1.333898           1.753         877         1.058374         2.064         924         1.17850         974         1.335898           1.776         877         1.068374         2.065         926         1.17850         2.607         974         1.346722           1.777         876         1.067506         2.076         926         1.18876         976         1.346722           1.777         877         1.067506         2.076         928         1.189709         2.679         977         1.366949           1.776         877         1.067662         2.110         929         1.189709         2.635         971         1.366949           1.779         880         1.077761         2.10         928         1.189709         2.635         971         1.346772           1.780	6819	1,736	200	1 051180	2,018	918	1.163008	2,529	900	1 901151	4.080
1,745         869         1,05231         2,032         920         1,168081         2,559         971         1,32531           1,745         871         1,056234         2,039         921         1,168081         2,575         971         1,323589           1,756         871         1,057234         2,039         923         1,17836         2,677         972         1,333582           1,763         874         1,061327         2,047         923         1,17836         2,630         974         1,323587           1,772         875         1,061327         2,069         924         1,178418         2,640         976         1,337882           1,772         875         1,067863         2,069         926         1,186346         2,640         976         1,337892           1,772         877         1,067862         2,069         927         1,186344         976         1,346722           1,776         877         1,067862         2,107         929         1,186344         977         1,36648           1,776         877         1,067872         2,117         920         1,186344         976         1,346722           1,776         877	7925	1,741	202	7.0010	0 0 0	919	1.165537	2,544	ROA	TOTTOO .T	07.
1.744         870         1.055202         2.02         921         1.168081         2.576         971         1.328379           1.754         871         1.052724         2.034         921         1.178316         2.591         972         1.328379           1.758         873         1.052724         2.047         922         1.178306         2.640         974         1.328379           1.767         875         1.063374         2.069         924         1.178418         2.640         975         1.337858           1.772         876         1.067566         2.078         926         1.186374         2.640         977         1.387867           1.772         877         1.067566         2.078         926         1.186374         2.640         977         1.387867           1.772         877         1.067566         2.078         926         1.186374         2.640         977         1.387664           1.772         877         1.18634         2.640         977         1.387648         977         1.387648           1.773         877         1.18634         2.640         977         1.387648           1.775         877         1.18634	9996	1,745	698	1.053175	1000	920	1,168081	2,559	970	1. 325231	0#1.#
1.754         871         1.057234         2.039         22.         1.173216         2.607         972         1.33358           1.758         872         1.063207         2.047         922         1.173216         2.607         973         1.33358           1.763         873         1.063374         2.064         924         1.178466         2.607         974         1.33586           1.776         874         1.06583         2.069         924         1.178418         2.640         976         1.34672           1.776         877         1.06963         2.069         926         1.18634         2.641         976         1.34672           1.776         877         1.06963         2.085         928         1.18634         2.675         977         1.35664           1.781         877         1.073662         2.101         930         1.194413         2.730         978         1.36648           1.791         877         1.077862         2.117         931         1.191743         2.749         978         1.36648           1.800         882         1.077862         2.117         932         1.191743         2.749         977         1.38644 <td>1411</td> <td>1,749</td> <td>870</td> <td>1,055202</td> <td>2,032</td> <td>160</td> <td>1.168081</td> <td>2,575</td> <td>971</td> <td>1, 329379</td> <td>4, 219</td>	1411	1,749	870	1,055202	2,032	160	1.168081	2,575	971	1, 329379	4, 219
1,768         872         1,063273         2,047         923         1,17500         2,607         973         1,34782           1,763         874         1,063274         2,064         924         1,17500         2,623         974         1,34722           1,776         876         1,06536         2,069         926         1,17600         2,668         976         1,35267           1,776         876         1,06536         2,069         926         1,186384         2,675         976         1,35267           1,776         877         1,069683         2,098         929         1,189000         2,675         977         1,365486           1,791         879         1,073761         2,117         930         1,194413         2,730         971         1,365486           1,791         881         1,077772         2,117         931         1,199000         2,749         981         1,365486           1,800         882         1,077772         2,117         931         1,199000         2,749         981         1,36648           1,801         882         1,06069         2,117         931         1,199620         2,749         981         1,36648     <	3160	1,754	871	1,057234	2, 039	170	1 173215	2,591	972	1, 333598	4, 294
1.767         873         1.061320         2.054         924         1.1764136         2.640         976         1.342265           1.767         874         1.063874         2.062         924         1.181036         2.640         976         1.346722           1.776         876         1.063863         2.078         926         1.181036         2.658         977         1.365907           1.776         877         1.063863         2.085         928         1.186376         2.658         977         1.365907           1.776         877         1.063863         2.085         928         1.189099         2.675         977         1.365907           1.781         877         1.073761         2.103         929         1.181702         2.711         979         1.376561           1.781         870         1.077372         2.117         932         1.198992         2.749         981         1.366498           1.800         881         1.077372         2.117         932         1.198992         2.749         981         1.366498           1.800         882         1.082216         2.135         932         1.198992         2.749         981         1.3654	4914	1,758	872	1,059273	2,047	200	1 175906	2.607	973	1,337892	4,373
1,772         874         1,063374         2,062         924         1,11971         975         1,346722           1,772         875         1,065436         2,069         926         1,18876         2,658         976         1,355377           1,772         877         1,0696436         2,085         926         1,18876         2,675         977         1,366499           1,772         877         1,069683         2,085         928         1,18909         2,693         979         1,366499           1,795         877         1,0773761         2,101         930         1,197102         2,749         981         1,36649           1,795         881         1,0773761         2,117         931         1,197143         2,749         981         1,38646           1,806         882         1,080689         2,127         932         1,2899         2,749         981         1,386747           1,806         882         1,080689         2,135         934         1,205460         2,809         981         1,40377           1,806         882         1,080689         2,152         935         1,21994         2,814         1,40377           1,806	6679	1,763	873	1,061320	2.054	923	200011	2.623	974	1,342265	4,457
1, 065436         2, 069         925         1, 136534         2, 658         976         1, 351267           1, 772         876         1, 067505         2, 076         927         1, 186334         2, 676         977         1, 355907           1, 782         877         1, 069568         2, 095         927         1, 186009         2, 675         976         1, 365498           1, 786         878         1, 073761         2, 110         929         1, 189702         2, 711         970         1, 365498           1, 795         880         1, 077862         2, 110         931         1, 187413         2, 749         981         1, 365646           1, 800         882         1, 078062         2, 110         931         1, 187143         2, 749         981         1, 380744           1, 815         882         1, 080069         2, 127         932         1, 200540         2, 749         981         1, 380744           1, 815         884         1, 082216         2, 145         934         1, 200540         2, 809         1, 403877           1, 816         885         1, 096494         2, 162         936         1, 210340         2, 81         1, 403877	8435	1.767	874	1.063374	2.062	426	1 101096	2.640	975	1,346722	4.545
1,762         876         1,067505         2,078         920         1,188034         2,675         977         1,355907           1,782         877         1,069583         2,085         927         1,188039         2,675         977         1,356907           1,785         877         1,071686         2,003         929         1,189009         2,673         977         1,356987           1,791         878         1,077972         2,117         930         1,194743         2,749         981         1,376551           1,805         880         1,077972         2,117         932         1,198982         2,749         982         1,376551           1,805         882         1,030099         2,127         932         1,202661         2,789         982         1,390774           1,816         884         1,034351         2,145         933         1,202661         2,789         982         1,390774           1,826         886         1,086494         2,162         936         1,210990         2,819         982         1,403269           1,826         886         1,086449         2,162         937         1,210990         2,81         1,403269      <	0000	1.772	875	1,065436	2.069	920	7.0007.	2.658	976	1.351267	4.640
1.782         877         1.069583         2.085         926         1.189092         2.693         978         1.360649           1.782         878         1.077668         2.093         928         1.181702         2.711         979         1.36648           1.791         879         1.077662         2.101         930         1.191702         2.711         981         1.370462           1.795         881         1.077862         2.117         931         1.19743         2.749         981         1.370462           1.800         882         1.080089         2.127         932         1.19892         2.769         981         1.375551           1.810         882         1.080089         2.127         933         1.205640         2.809         984         1.380774           1.820         886         1.086494         2.162         986         1.208269         2.811         984         1.31673           1.820         886         1.096897         2.162         986         1.208269         2.827         986         1.403267           1.840         889         1.092977         2.180         986         1.21862         986         1.403267	1074	1:776	876	1,067505	2.078	926	700001	9 675	977	1,355907	4.742
1.781         878         1.071668         2.003         928         1.18700         2.711         979         1.365498           1.795         879         1.073661         2.101         929         1.18102         2.731         980         1.376462           1.796         880         1.077672         2.117         931         1.187443         2.749         981         1.375551           1.800         881         1.077672         2.117         931         1.197443         2.749         981         1.375551           1.800         882         1.080082         2.127         932         1.202640         2.769         982         1.380774           1.810         883         1.084551         2.145         984         1.202640         2.831         982         1.380775           1.820         884         1.084564         2.161         987         1.211090         2.852         986         1.403269           1.820         886         1.096646         2.161         987         1.21861         986         1.403269           1.821         886         1.096646         2.161         987         1.21861         2.814         4140326           1.830	1014	1,782	877	1,069583	2, 085	927	#C00007 *	603	978	1,360649	4,849
1.791         879         1.073761         2.101         929         1.121702         2.749         980         1.370462           1.792         880         1.077872         2.117         930         1.194413         2.749         981         1.370462           1.800         881         1.077872         2.117         931         1.197143         2.749         982         1.370551           1.805         882         1.080099         2.127         932         1.202661         2.789         983         1.38173           1.815         884         1.086494         2.152         934         1.205659         2.831         986         1.39173           1.820         886         1.086494         2.152         936         1.205659         2.831         986         1.40329           1.821         886         1.086494         2.162         937         1.211990         2.852         986         1.40329           1.825         886         1.096494         2.161         937         1.211990         2.852         986         1.40329           1.830         886         1.096977         2.180         939         1.21894         996         1.40329	200	1 785	878	1,071668	2, 093	928	1.189009	2000	979	1,365498	4,964
1.795         880         1,075862         2:110         930         1,194413         2.750         981         1,375551           1.806         881         1,077972         2:117         931         1,1997443         2.749         981         1,375551           1.805         882         1,080089         2:127         932         1,199892         2.769         982         1,380744           1.810         883         1,080494         2:135         933         1,202661         2.789         984         1,391673           1.825         884         1,084351         2:145         936         1,21090         2.852         984         1,391673           1.825         886         1,084494         2:161         937         1,211090         2.856         986         1,409377           1.825         886         1,096494         2:161         937         1,211090         2.856         986         1,415722           1.825         886         1,096944         2:161         937         1,211090         2.866         986         1,415722           1.830         886         1,096947         2:180         940         1,22944         2.997         991         1,44221<	7000	102	879	1,073761	2, 101	929	1.191.02	1000	080	1.370462	5.089
1.800         881         1.077972         2.117         931         1.187143         2.749         982         1.380774           1.800         882         1.060089         2.127         932         1.198992         2.769         983         1.380774           1.810         883         1.08216         2.135         934         1.202661         2.769         984         1.391673           1.815         884         1.084351         2.145         936         1.21090         2.809         984         1.391375           1.820         886         1.086494         2.162         936         1.21090         2.819         986         1.403377           1.821         886         1.0908077         2.180         937         1.213942         2.874         987         1.415722           1.830         888         1.092977         2.180         939         1.213942         2.874         989         1.415722           1.840         890         1.095547         2.180         939         1.225574         2.943         991         1.42257           1.841         890         1.095543         2.188         940         1.225574         2.943         991         1.44221 <td>13.1</td> <td>101.1</td> <td>880</td> <td>1,075862</td> <td>2, 110</td> <td>930</td> <td>1,194413</td> <td>2.730</td> <td></td> <td>1.375551</td> <td>5, 223</td>	13.1	101.1	880	1,075862	2, 110	930	1,194413	2.730		1.375551	5, 223
1,805         882         1,080689         2,127         932         1,198882         2,769         983         1,080444           1,805         883         1,08216         2,135         933         1,202661         2,789         985         1,386144           1,815         884         1,08494         2,145         934         1,205450         2,809         986         1,391673           1,820         886         1,08646         2,161         936         1,213942         2,831         986         1,403269           1,820         886         1,08646         2,161         936         1,213942         2,874         986         1,403269           1,830         888         1,09607         2,180         937         1,213942         2,874         986         1,403269           1,846         889         1,09607         2,180         939         1,219712         2,919         940         1,226574         2,943         990         1,426336           1,846         890         1,095432         2,278         940         1,225631         2,943         991         1,444221           1,867         891         1,100796         2,226         942         1,225631	976	000	88	1.077972	2,117	931	1, 197143	2.749	600	1 380774	5.370
1,810         883         1,202661         2,789         990         1,391673           1,810         884         1,084316         2,145         934         1,205460         2,699         986         1,391673           1,815         886         1,086494         2,152         935         1,208459         2,831         986         1,403269           1,820         886         1,08646         2,161         936         1,211990         2,852         987         1,409377           1,830         887         1,090807         2,180         937         1,211912         2,874         987         1,409377           1,830         889         1,090807         2,180         938         1,211612         2,874         987         1,409377           1,840         889         1,090807         2,186         940         1,222631         2,943         990         1,422336           1,840         899         1,093435         2,207         940         1,225674         2,943         990         1,422336           1,846         892         1,103966         2,226         941         1,225631         2,943         991         1,442221           1,867         894	2060	1.000	688	1,080089	2, 127	932	1.199892	2, 769	000	1 986144	5.529
1,810         884         1,005450         2,143         934         1,205450         2,809         984         1,391075           1,810         884         1,086494         2,162         935         1,208259         2,831         986         1,397375           1,820         886         1,086464         2,161         936         1,211090         2,852         987         1,409377           1,830         887         1,090807         2,170         937         1,21342         2,874         987         1,409377           1,830         889         1,090807         2,170         937         1,213641         2,896         989         1,42236           1,840         899         1,090847         2,188         940         1,222631         2,943         991         1,42236           1,846         890         1,090843         2,207         941         1,23654         2,943         991         1,43221           1,846         891         1,104366         2,218         942         1,23654         2,943         991         1,44221           1,846         892         1,104366         2,226         943         1,231538         3,014         994         1,44221	2703	che T	200	1 080216	2,135	933	1,202661	2,789	200	T-1000 -1	200
1.816         6854         1.00101         2.831         986         1.208269         2.831         986         1.237670           1.820         686         1.086494         2.161         936         1.211090         2.852         986         1.403269           1.825         886         1.086466         2.161         937         1.213942         2.874         987         1.403269           1.830         887         1.090807         2.180         938         1.21816         2.896         988         1.415722           1.835         889         1.095167         2.188         939         1.219712         2.919         980         1.42535           1.840         890         1.09750         2.188         940         1.225674         2.967         991         1.436532           1.861         892         1.101750         2.218         941         1.225674         2.967         991         1.44221           1.862         893         1.101750         2.218         942         1.231533         3.018         994         1.46128           1.867         894         1.106194         2.226         943         1.24421         991         1.46121	4508	1.810	200	1,000 1	2, 143	934	1,205450	2,809	984	T. SETONS	200
1. 820         686         1. 080646         2.161         936         1.211090         2. 852         986         1. 400269           1. 825         886         1. 086646         2. 161         937         1.213942         2. 874         987         1. 409377           1. 830         887         1. 0908077         2. 180         938         1. 216816         2. 896         988         1. 415722           1. 840         889         1. 095157         2. 188         940         1. 222631         2. 919         989         1. 422386           1. 846         890         1. 097345         2. 198         940         1. 225674         2. 967         991         1. 422365           1. 861         892         1. 101750         2. 218         942         1. 228674         2. 992         992         1. 444221           1. 865         893         1. 106194         2. 226         943         1. 234561         3. 018         994         1. 461198           1. 867         894         1. 106194         2. 236         944         1. 234561         3. 044         995         1. 461198           1. 878         896         1. 1106194         2. 236         946         1. 240666	86318	1.815	* 000	100500	9.152	935	1,208259	2.831	988	1, 387370	# GO . 0
1. 825         686         1. 090070         2.170         937         1. 213942         2. 874         987         1. 409377           1. 830         887         1. 090877         2. 180         938         1. 216816         2. 896         988         1. 415722           1. 840         889         1. 092077         2. 188         939         1. 219712         2. 919         989         1. 425356           1. 846         890         1. 097345         2. 188         940         1. 225674         2. 943         990         1. 436532           1. 846         891         1. 099643         2. 207         941         1. 225674         2. 997         992         1. 444221           1. 865         892         1. 101750         2. 218         942         1. 225674         2. 992         993         1. 444221           1. 867         893         1. 106194         2. 226         943         1. 237591         3. 018         994         1. 461198           1. 867         894         1. 106194         2. 236         944         1. 237595         3. 071         996         1. 481324           1. 878         896         1. 110678         2. 246         946         1. 249066	38133	1.820	282	1. 0000 4E	9 161	936	1.211090	2,852	986	1.403269	0.100
1.830         B8Y         1.09000         2.180         938         1.216816         2.896         988         1.415722           1.835         888         1.092977         2.180         939         1.219712         2.919         989         1.422386           1.840         889         1.095445         2.188         940         1.225631         2.943         990         1.429367           1.846         890         1.097445         2.198         940         1.225674         2.943         991         1.436532           1.861         891         1.101750         2.218         941         1.225674         2.967         993         1.444221           1.865         893         1.101750         2.218         942         1.234561         2.992         994         1.451240           1.867         894         1.106194         2.226         944         1.234561         3.044         994         1.461198           1.878         896         1.1106194         2.226         946         1.240666         3.099         996         1.481324           1.878         896         1.110678         2.266         947         1.240666         3.099         997         1.49331	39953	1.825	000	1. 0000 t	170	937	1,213942	2.874	987	1.409877	0,040
1.835         889         1.092871         2.919         989         1.422350           1.840         889         1.096157         2.188         940         1.22631         2.943         980         1.422350           1.846         890         1.09543         2.198         940         1.225674         2.943         990         1.429257           1.861         891         1.09543         2.207         941         1.225674         2.967         991         1.436532           1.865         892         1.101750         2.218         942         1.225674         2.992         993         1.44221           1.867         893         1.106194         2.226         944         1.234561         3.018         994         1.46138           1.873         896         1.106194         2.238         944         1.234561         3.044         995         1.470755           1.873         896         1.110678         2.266         946         1.24066         3.099         996         1.481324           1.884         897         1.111293         2.268         947         1.24066         3.099         997         1.493317           1.890         889	91778		200	1,000001	2 180	938	1,216816	2,896	886	1,415722	0° 01.4
1,840         869         1,0907345         2,198         940         1,222631         2,943         990         1,42927           1,846         890         1,097345         2,198         941         1,22574         2,967         991         1,436532           1,861         891         1,00750         2,218         942         1,22554         2,992         992         1,444221           1,865         893         1,100760         2,226         943         1,231533         3,018         993         1,452407           1,867         894         1,106194         2,236         944         1,234561         3,044         994         1,461198           1,873         896         1,110678         2,246         945         1,24066         3,094         996         1,481324           1,878         896         1,110678         2,266         946         1,24066         3,099         997         1,493317           1,894         896         1,111503         2,278         946         1,24696         3,137         999         1,500796           1,895         899         1,117481         2,289         948         1,250049         3,187         999         1,507996	93608		0 0	1.006157	2.188	939	1,219712	2, 919	688	•	1770.0
1.846         890         1.09543         2.207         941         1.226574         2.967         991         1.436532           1.861         891         1.099543         2.207         942         1.226541         2.992         992         1.444221           1.865         893         1.10750         2.216         943         1.231533         3.018         993         1.452407           1.867         894         1.106194         2.286         944         1.234561         3.044         994         1.461198           1.873         896         1.1106194         2.246         946         1.24066         3.099         997         1.481324           1.874         896         1.111678         2.268         947         1.24696         3.099         997         1.493317           1.894         897         1.115203         2.278         948         1.26049         3.187         999         1.526071           1.895         1.117481         2.289         940         1.26049         3.187         999         1.570796	95443	1.840	800	1,00040	9 1 98	940	1, 222631	2,943	066	-4	200
1,861         691         1,000         2.218         942         1,228641         2,992         992         1,452407           1,865         892         1,101760         2.218         943         1,231533         3,018         993         1,452407           1,867         893         1,106194         2,226         944         1,234551         3,044         994         1,452407           1,873         895         1,106194         2,246         945         1,237595         3,071         996         1,470755           1,878         896         1,110678         2,257         946         1,24696         3,099         997         1,493317           1,884         897         1,112936         2,278         948         1,246892         3,157         998         1,500796           1,890         889         1,115203         2,289         948         1,26049         3,187         999         1,526071           1,895         1,117481         2,289         946         1,256336         3,218         1,000         1,570796	97283	1.846	9 6	1 000543	2.207	941	1,225574	2, 967	166		100.
1,856         993         1,452407           1,862         893         1,103968         2,226         943         1,23451         3,044         993         1,452407           1,867         894         1,106194         2,236         944         1,23451         3,044         994         1,461198           1,873         896         1,106194         2,246         946         1,23456         3,071         996         1,470755           1,878         896         1,110678         2,257         946         1,24666         3,099         997         1,493317           1,884         897         1,11298         2,278         948         1,26049         3,157         998         1,500762           1,890         889         1,115203         2,278         948         1,26049         3,187         999         1,526071           1,895         1,117481         2,289         940         1,26036         3,187         999         1,526071	99129	108.1	100	1 101750	2, 218	942	1,228541	2, 992	N 7	- 1	9
1,862         894         1,234551         3,044         994         1,461135           1,867         894         1,106194         2,238         945         1,237595         3,071         995         1,470755           1,873         895         1,110678         2,257         946         1,24966         3,099         996         1,481324           1,878         896         1,110678         2,257         946         1,248765         3,127         997         1,493317           1,89         1,1112935         2,268         948         1,246892         3,187         999         1,526071           1,89         1,1117481         2,289         949         1,250049         3,187         999         1,526071           1,895         1,1117481         2,289         940         1,25336         3,187         999         1,570796	08600	1,856	460	1.02968	2.226	943	1,231533	3,018	993	•	0.557
1.867         894         1.10010*         2.246         945         1.237595         3.071         995         1.47050           1.878         896         1.110678         2.257         946         1.243765         3.099         996         1.481324           1.878         896         1.110678         2.268         947         1.243765         3.127         999         1.493317           1.890         897         1.115936         2.268         948         1.246892         3.167         999         1.507642           1.890         889         1.117481         2.289         949         1.260049         3,187         999         1.526071           1.895         889         1.117481         2.289         940         1.25336         3,187         999         1.570796	02862	1.862	200	1 106104	988	944	1,234551	3.044	994		100.8
1.873     859     1.10078     2.257     946     1.240666     3.099     996     1.481234       1.878     896     1.110978     2.257     947     1.243765     3.127     997     1.493317       1.884     897     1.112935     2.268     948     1.246892     3.157     999     1.507642       1.890     899     1.117481     2.289     949     1.250049     3,187     999     1.526071       1.895     899     1.117481     2.289     950     1.250386     3,187     1000     1.570796	04698	1.867	488	1.100409	2.246	945	1, 237595	3, 071	966		11 000
1.878         896         1.11036         2.268         947         1.248765         3.127         997         1.493317           1.884         897         1.112203         2.268         948         1.246892         3.167         998         1.507542           1.890         898         1.117481         2.289         949         1.250049         3.187         999         1.526071           1.895         899         1.117481         2.289         949         1.250049         3.187         999         1.570796           1.895         899         1.117481         2.289         949         1.253286         3.218         1000         1.570796	06565		988	7.10040	0.00	946	1,240666	3,099	966	• • .	11.995
1.884 897 1.112503 2.278 948 1.246892 3.167 998 1.50742 1.890 898 1.115203 2.278 949 1.250049 3.187 999 1.526071 1.895 899 1.117481 2.289 949 1.250049 3.187 999 1.570796 1.895	08438		989	1.110006	988	947	1,243765	3, 127	997	_	14, 220
1.890 099 1.1220 2.289 949 1.250049 3,187 999 1.526071 1.895 899 1.117481 2.289 950 1.526071	10316		180	1 115909	2,278	948	1.246892	3, 157	866		15.029
1.895 689 1.111-151 2.000 1.25328 8,218 1000	12200		080	1,110401	988	949	1,250049	3, 187	666	•	44.120
	14090		S S S	1,11,401	000	950	1,253236	3, 218	7000	• •	

Table 1 - Sine - Angle Conversion Table (Sheet 5 of 5)

	<u> </u>			<u> </u>													_																								_										
Interp. Constant	000	000	880	800			. 900	.98.	. 988	. 987	. 987	i a	780		200	088.	. 987	986	. 986	986	. 989.	. 985	986.	. 989.	686	. 985	. 985	. 984	383	486.	. 964	200		984	982	. 983	. 983	. 983	. 982	. 982	. 982	982	. 981	. 981	. 982	. 980	. 981	. 981	. 980	086.	086
Sine	150497	151415	152404	153302	154380	1 55900	150000	000001	.157343	. 158331	159318	180305	161292		67.7201.	007601.	104252	165239	100220	167211	168197	169182	.170167	171153	172138	., 173123	.174108	175093	7,007.1	177002	170020	180013	180927	181980	182964	.183946	.184929	. 185912	.186895	.187877	. 188859	189841	.190823	191804	.192785	.193767	. 194747	.195728	.196709	197689	198669
Angle	52	155	153	154	, r.	7 1	2 5	707	158	159	160	185	162	100	201	* 6	100	166	,0T	168	ROT	2.5	17.1	7.17	27.1	174	07.7	0.7	7 7	2.4	100	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	188	188	200
Interp. Constant	995	982	982	. 884	982	000	004	+00.	<b>5</b> 88.	- 884	. 994	785	884	- 600	288.	100	288.	866.	288.	200	200	200	200	200	700	88.	788.	788	300	2001	1000	168	991	. 991	. 991	. 991	. 991	086	. 991	086	. 066	086	066.	088.	586	088.	.888	686	500	888	202
	100828	101823	.102818	103813	104807	105802	106796	001001	082201	.108784	. 109778	110772	111766	119780	112750	001011	. 114/41.	05/611	001011	117726	110719	190705	00,071	199800	000000	123062	195687	126850	197851	128643	129634	130626	131617	132608	133599	134590	135581	136572	137562	138553	139543	140533	141523	144010	143503	784487	7856FT	145471	147460	44044	
Angle	101	102	103	104	105	108	26	2 5	207	109	110	111	112	-10	114	1 4 F	211	011	777	0 1	190	1 1 1 1 1 1	191	192	3 6	1 64 1 9 5	2 6	197	200	120	130	131	132	133	134	135	136	137	•	•	•	ĺ	142	277	# # # # # # # # # # # # # # # # # # #	145	41	147.	148	•	
Interp. Constant	666	866	. 999	866	666	866	866	200	888	866	868	866	866	866	866	000	900	966	000	000	- 200	000	000	166	-	. 990	100	100	004	100	966	. 997	. 997	966	. 997	966 .	966	986	. 997	966.	. 995	986	986	900	CRO	0.00	CAR.	286	900	200	. 000
Sine	. 050978	. 051977	. 052975	.053974	. 054972	055971	. 056969	050000	20000	. 058866	.059964	. 060962	. 061960	062958	063956	084954	. CACAGO	068050	060000	060045	080049	070940	071070	072935	666640	074930	075927	076924	077921	078918	. 079915	. 080911	. 081908	. 082905	. 083901	, 084898	. 085894	. 086890	. 087886	. 088883	. 089879	. 090874	. 091870	000000	208580.	. 084607	. 083833	. 090848	000000	000843	. 000000
Angle	150	052	053	054	055	056	057	920		800	090	061	062	063	064	085	9 6	000	900	900	040	2.5	040	2.50	0.29	- CO	0.76	077	270	079	080	081	082	083	084	082	980	087	880	580 0	060	TRO	260	200	# # # P	660	0.00	280	000	100	3
Interp. Constant	1,000	1.000	1.000	1.000	1,000	1.000	1.000	000	200	7.000	1.000	1.000	1.000	1,000	666	000		86		30	666	000		1.000	000	1.000	000	666	000	1,000	666	1.000	666.	666 .	1.000	666	1.000	666.	686	686.	1.000	ARR.	666	666	666	888	666	AAA.	000	000	
Sine	. 001000	. 002000	. 003000	.004000	. 005000	. 006000	. 007000	00000		000000	. 010000	. 011000	. 012000	. 013000	. 014000	014999	01 5000	016990	000210	018000	019999	020998	02100	022998	800860	024997	025997	026997	027996	028996	. 029996	.030995	. 031995	. 032994	. 033993	. 034993	. 035992	. 036992	. 037991	. 038980	. 039989	. 04088	. 041988	04400	. 045300	045084	. 043804	. 040400	048980	049979	2122
Angle	100	005	003	8	005	900	004	800	88	8,6	010	011	012	013	014	51.5	910	2 5	018	200	020	021	200	023	024	025	026	027	028	029	030	031	032	033	034	035	036	22.0	200	989	040	041	2 6	3 5	# # E	946		# 0 0	049	020	

Table 2 - Angle - Sine Conversion Table (Sheet 1 of 5)

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Constant , 939	. 938	. 939	866	937	937	936	936	936	200	200	9 6	2000	934	- 934	. 934	. 933	000	200	288.	. 933	. 931	. 932	. 931	. 931	930	930	929	930	928	626	928	928	927	927	927	926	926	925			760	# F C C	***	# 00 C	646	36.	922	. 922	922	. 921	. 921	
343837	344776	345714	340000	341390 948598	240465 240465	250402	251938	959974	4 1 1 1 1 1 1	0.00000	. 3541.45	. 355080	. 356015	. 356949	357883	958817	- 10000	00/408	. 360683	. 361615	. 362548	363479	364411	365342	966273	367203	368133	369062	369992	370920	271849	444646	878705	974R32	275559	276486	07774	000000	000000	378703	380188	381113	. 382037	382961	. 383885	384808	. 385731	. 386653	387575	, 388497	.389418	
Angle	352	353	400	300	000	- 0	000	200	300	100	362	363	364	365	. e	9 6	200	368	369	370	371	27.2	2 6	010	# 200	920	0.00	24.0	9.0	000	200	100	700	200	# u	000	380	38.4	200	388	390	391	392	393	394	395	396	397	398	399	400	
ant '									~	~		-		2 5	= 9	2		920	020	040	200	040	848	948	948	947	947	947	947	946	946	945	946	945	944	944	944	944	943	943	943	942	942	941	942	941	940	940	040	040	020	202
Constant	•	•	•	•	•	•	•	•	•	•	33 . 951	14 952	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	٠	•	•	•	•	•	٠	•	•	٠	•	•	•	•			•	•	•	•	342898
	296475	298385	299339	300293	•	•	•	•	•	306011	•	•	•	٠	•	•	7 311717		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	٠	•	330657	•	, ,	•	•	•	•	•	•	•	•	•	•	•	350 . 342
Angle	301	208 208 208	304	305	306	307	308	309	310	311	610	1	313	314	315	316	817		210	818	320	321	322	323	324	325	32	327	328	329	330	331	332	333	334		-	337		-			-			· ·			·	~ ·	<b>∽</b>	
Interp. Constant	. 968	. 969	900	880	. 280	967	987	987	- 00	200	008.	996.	. 965	. 965	985	965		#QR •	. 965	. 963	. 964	963	984	962	983	282	282	896	1961	898	961	980	96.	. 680	0 40	90	200	608.	608.	a ca	20.6	800.	. 928	928	. 957	. 957	. 957	926	926	926	. 955	955
Sine	248373	249341	. 250310	. 251278 550545	. 202240	253413	. 204100	141007	*110CZ .	. 257081	. 258047	. 259013	259979	280944	000196	. 201.000	£1.9797	. 263839	. 264803	265768	286731	987805	000000	. 200000	20802.	. 67 COO 1	040104	. 27.4000	112017	004417	. 273334 074058	. 270330	1.131.1	7797.7	00.000	. 28UL90	.281197	. 282117	. 283076	. 284035	. 284994	, 285952.	. 286910	. 287868	. 288826	. 289783	290740	291697	292653	293609	294565	905590
Angle	251	252	253	254	255	256	257	258	229	260	261	262	263	284	F 10 0	202	586	267	268	989	200	2 5	1).2	272	273	274	272	276	27.7	278	279	280	281	282	283	284	282	286	287	288	289	290	291	292	293	294	295	296	200	808	900	667
Interp.	Constant	979	086	948	. 976	. 979	. 978	. 979	826	878	87.6	200	- 1	2.18.	. 977	. 977	776	928	240		976	. 975	926	. 975	. 975	. 975	. 975	. 975	. 974	. 974	. 974	. 973	. 973	. 973	. 973	. 973	. 972	973	126	972	640	140	170	1.00	020	0.60	0.60	2.5	•	•	696.	696
	1	199049	201608	202588	203567	204546	205525	206503	907482	208480	00400	. 409430	210416	211393	. 212370	213347	914824	100010	108612	. 216277	. 217254	. 218230	219205	220181	221156	222131	223106	224081	225056	. 226030	227004	227978	228951	229924	230897	231870	949843	992815	994499	00-400	. 400-00	230 (31	. 237703	. 238074	239040	240010	. 241586	242556	243526	244496	, 245466	. 246435
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Table 2 - Angle - Sine Conversion Table (Sheet 2 of 5)

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Interp. Constant	. 852	. 851	. 851	. 850	. 850	. 849	. 849	848	047	T C	1.88.	. 847	. 846	. 845	. 845	. 844	. 844	. 843	. 843	. 842	. 842	. 841	. 840	. 840	840	839	888	200		. 200	988	836	835	. 834	.834	. 834	. 833	. 832	. 832	. 831	. 831	. 830	. 829	.829	. 829	. 828	. 827	827	. 826	. 825	979
Sine	. 523539	. 524391	. 525342	. 526093	. 526943	. 527793	. 528642	529491	530330		081180	. 532033	. 532880	. 533726	. 534571	. 535416	. 536260	. 537104	. 537947	. 538790	. 539632	. 540474	. 541315	542155	542995	543835	544674	545512	546350	547187	548024	. 548860	. 549696	. 550531	, 551365	. 552199	. 553033	. 553866	. 554698	. 555530	. 556361	. 557192	. 558022	558851	. 559680	560509	561337	562164	562991	563817	250500
Angle	551	552	553	554	555	556	557	558	550	2 2 2	000	190	292	563	564	565	266	567	568	569	570	571	572	573	574	575	576	577	578	573	280	581	582	583	584	585	586	587	288	583	290	591	292	263	594	595	296	597	860	560	200
Interp. Constant	. 877	. 876	.876	. 875	0/2	. 875	. 874	.873	873	640	2.00	7,0	1,0.	.871	. 871	. 870	698	. 870	. 868	. 868	. 868	. 867	998.	998.	998.	. 865	. 865	864	. 863	. 863	. 863	. 862	. 861	. 862	.860	. 860	. 860	. 859	828	800	. 857	. 857	. 857	000	833	200	. 604 1.10	400	. 650	959	****
Sine	480303	. 481180	. 482056	462932	.463607	484682	. 485557	. 486431	. 487304	488177	480050	00000	22880# ·	. 490793	491004	. 492535	. 493405	. 494274	. 495144	. 496012	.496880	. 497748	. 498615	. 499481	. 500347	. 501213	. 502078	. 502943	. 503807	. 504670	. 505533	. 506396	. 507258	. 508119	. 508981	. 509841	510701	. 511561	. 512420	. 513278	. 514136	514893	515850	1070TC	5007 10	518418	50.32.0	520127	521834	522687	
Angle	501	202	200	4 2 3 3 3	000	900	201	208	209	. C. C.	511	100	910	210	916 6	515	216	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	0 45 145 145	020	030	200	900	20.0	040	146	242	240	* * * * * * * * * * * * * * * * * * * *	040		. 04.2	676	550	,,,,
Interp. Constant	208	880	660	000		0 0 0	7.68.	. 897	988.	896	895	200	-	200	***	. 883	. 894	268	288.	268.	288.	. 891	088	. 880	- 889	. 890	. 888	. 888	. 888	. 887	. 887	. 886	. 886	288.	200	200	*00*	2000	200	200	700	700	100.		100	820.	020	070	828	. 222	-
	485855	49766	499564	430469	440960	144950	807144.	442155	. 443052	. 443948	. 444844	445739	446634	447590	670127	. 448423	449316	450210	. 451102	. 451994	. 452886	. 453778	. 454669	. 455559	456449	. 457338	. 458228	. 459116	. 460004	.460892	. 461779	. 462666	. 463552	. 464438	400343	487009	487077	489880	469743	011001.	. #10040	000647	479971	474151	475,000	475011	476701	477669	. 478548	479428	
Angle	451 459	459	454	455	456	450	- 017	404 804	459	460	461	462	463	464	ARE	#00 #07	400	40.5	804	80#	2.5	17.4	4.(2	4.73	474	475	476	477	478	479	480	481	482	4 4 50 5	404 404	408	487	488	489	90	107	40 <i>b</i>	403	494	405	496	497	498	499	200	
Constant	920	0.00	920	918	816	810	250	0 10	.18.	. 917	. 917	. 916	918	25	916		* 18.0	* 7 6	*18.	210	910	216.	716.	276	.911	116.	. 910	. 910	. 910	606.	606	808	808	. 200	200	908	908	908	906	- 206	908	706	506	. 903	808	905	905	901	. 901	006	
Sine	. 391260	. 392180	393099	. 394019	394938	395856	396774	#	280786	. 398609	. 399526	. 400443	. 401359	. 402275	402190	404105	405010	40504	40000	407760	408679	40004.	403000	92#01# ·	.411410	412321	413232	. 414142	415052	. 415962	. 416871	417780	418688	420503	421410	422317	423223	424129	425035	425939	426844	427748	428652	429555	430458	. 431360	432262	. 433164	. 434065	. 434966	
Angle	402	403	404	405	406	407	408		) ) † ;	410	411	412	413	414	41.5	416	417	410	410	420	421	100	700	3 6	# C	07# 700	077	724	878	4 2 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	430	101	707	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	

Table 2 - Angle - Sine Conversion Table (Sheet 3 of 5)

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Constant	780	729	. 729	. 728	. 721	. 726	. 726	725	725	. PG2		. (23	7.72	722	. 721	. 720	. 720	. 719	. 718	. 718	. 717	. 716	. 715	. 715	.714	.714	. 712	712	717	710	709	. 708	. 708	707	707	. 705	60.	104	100	200		701	700	669	. 698	. 698	. 697	969	
Sine	062370	683831	684560	685289	686017	686744	687470	88196	60001	170000	689640	. 690370	. 691093	. 691815	. 692537	. 693258	. 693978	694698	695417	. 696135	696853	697570	698286	. 699001	. 699716	. 700430	. 701144	. 701856	702558	703000	704700	705409	. 706117	. 706825	. 707532	708239	446807.	7409048	. (10303	7111057	001111	713163	713864	714564	715263	715961	. 716659	.717356	
Angle	751	707	75.4	755	756	757	758	0 4 6	000	001	761	762	763	764	765	766	767	768	769	022	771	772	773	774	775	776	777	778	779	780	101	783	784	785	786	787	788	189	280	791	200	200	795	796	797	798	799	800	
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Interp. Constant	764	. 763	. (03	781	181	101	0 10	0 1	60.	. 758	. 757	. 757	. 756	755	755	457	754	- 646		. 136	101	10.5	5.5	748	749	747	747	. 746	. 746	. 744	745	742	742	742	740	.741	. 739	739	. 738	.788 85	. 736	. 736	130	200	73.4	28.6		187	1011
	644982	645746	60000	71.7).49	#000#0	040180	048000	650316	651075	651834	652592	653349	654106	654862	855617	. 00001 858879	0000	000/170	. 657880	. 658033	. 658385	, 600130	. 660887	100100.	663135	663884	664631	. 665378	. 666124	. 666870	. 667614	. 668359	201800.	670587	671329	672069	.672810	673549	674288	. 675026	675764	. 676500	677236	677972	01010	044870	200000	000000	ACOTOO.
Angle	701	702	703	704	602	92	707	708	409	710	1	6		714	# u	07.	01,	717	718	719	720	721	722	723	* 45 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4	- 12 92 93	7.27	728	729	730	731	732	788	407 404 404	2 - 4	787	738	736	740	741	742	743	744	745	740	747	748	(49	067
	$\vdash$											<u>.                                    </u>																		_	<del>-</del>				_			_									<u>-</u>	<del>.</del>	
Interp.	795	795	. 794	. 793	. 793	. 792	. 792	. 791	790	700		000	. 199	200	787	. 787	. 786	. 785	. 785	. 784	784	. 782	. 783	. 781	. 781	181	B C C	202.	778	777	777	. 776	. 775	772	***	4)		177	771	770	770	769	768	. 768	767	767	. 766	. 765	. 764
Sine	_	608777	607572	608366	609159	609952	610744	611536	612327	10100	613117	.613907	614696	. 615484	. 616272	. 617059	.617846	.618632	.6194:7	620202	620986	621770	. 622552	. 623335	. 624116	624897	. 625678	626457	62623	628793	629570	630347	. 631123	631898	632673	633447	103450	. 034894 A95766	000000	647408	838078	638848	. 639617	640385	. 641153	641920	642687	643453	. 644218
Angle	RE1	679	653	654	655	656	657	858	9 6	600	099	661	662	663	664	665	999	667	868	699	670	671	672	673	674	675	676	677	829	87.0	88	385 387	383	684	685	989	587	200	, co	980	180	808	694	695	969	697	869	669	200
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Interp.	Constant	. 824 426	#70°	828	600	. 660	200	1 60	. 821	818	. 820	. 819	. 818	.817	818	816	8 2			. 01.0	* 70.	* 6	. 645		. 811	. 811	. 810	. 810	808	808	208.	900	908	908	804	804	804	. 803	. 802	805	. 801	. 801	900	004	400	200	797	798	200
		565468	566292	20110	00100	701.000	208004 208004	570400	571227	572048	572867	573687	574506	575324	576141	676959	70000	01110	180910	. 57.8400	. 580221	. 581 655	. 581849	. 304004 509474	584286	585097	585908	. 586718	. 587528	. 588336	. 589145	589952	090,000	592372	593178	. 593982	. 594786	. 595590	. 596393	. 597195	597997	598798	599599	600388	001100	196T09.	90000	604300	. 004390 605186
	e le	•	•	•	•	•	909		608	609	610	611	612	613		415		910	617	618	619	620	621	770	624	625	626	627	628	629	630	631	222	200	, 6, 6,	636	637	638	639	640	641	642	643	644	640 C 250	940	- 64	250	649

Table 2 - Angle - Sine Conversion Table (Sheet 4 of 5)

Interp. Constant	580	280	220	24	2 6	- 6	0,6	926.	. 575	. 574	573	0 0	3 .	17.0.	17.6	270	. 569	. 568	. 567	. 567	. 566	565	564		. 683		188	100.	000		000		556	222	554	. 553	553	551	551	550	240	548	548	546	546	545	544		543	543	541	540
Sine	. 813997	814577	815157	815736	016910	£70070.	189010.	. 81.740.	. 818043	. 818618	819192	81078	20100	. 02000	. 820908	67.5179	822049	819228	. 823186	. 823753	.824320	.824886	825451	828015	826578	827140	827709	896969	898893	60000	820040	830407	831054	831610	. 832165	832719	. 833272	.833825	.834376	.834927	835477	.836026	836574	. 837122	. 837668	838214	838750	839303	839846	840389	840930	. 841471
Angle	951	952	953	954	955	920	0.00	).ca	928	959.	098	5	100	9 0	200	# u	865	900	296	968	696	046	971	972	973	974	975	920	977	840	646	086	981	982	983	984	982	986	987	988	989	066	991	992	993	984	988	966	997	966	666	1.000
Interp. Constant	, 621	.619	. 619	.618	817	617	212	2.5	010.	.614	.613	67	811	118	110	170	800.	800.	. 607	. 607	. 607	. 605	. 605	. 604	. 603	602	. 601	. 601	909	299	598	597	. 597	. 596	. 595	. 594	. 593	. 593	. 592	. 591	. 590	589	589	. 588	. 587	. 586	585	. 585	. 584	. 582	583	581
.Sine	.783948	. 784569	. 785188	785807	. 786425	787042	787650	10000	. 1882/3	. reese	. 789504	.790f17	790730	701347	70105	709560	700479	7) 100) .	187887.	794388	. 794995	. 795602	. 796207	. 796812	. 797416	. 798019	. 798621	. 799222	799823	.800423	.801022	.801620	. 802217	.802814	.803410	.804005	804599	. 805192	. 805785	806377	. 806968	807558	808147	808736	809324	809911	810497	811082	811667	812251	812833	813416
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Interp. Constant	. 659	. 658	. 657	. 657	. 656	. 655	. 654	854	F 60 0	200.	700.	. 651	. 650	. 650	648	849	647	272	978	0.00	. 640	. 644	. 644	. 643	. 642	. 642	. 640	. 640	. 639	. 638	. 638	. 637	. 636	. 635	. 634	. 634	633	. 632	. 631	. 631	. 630	. 629	. 628	. 627	. 627	. 626	. 625	. 625	. 623	. 623	. 622	. 621
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