TriQuint SEMICONDUCTOR

Application Note

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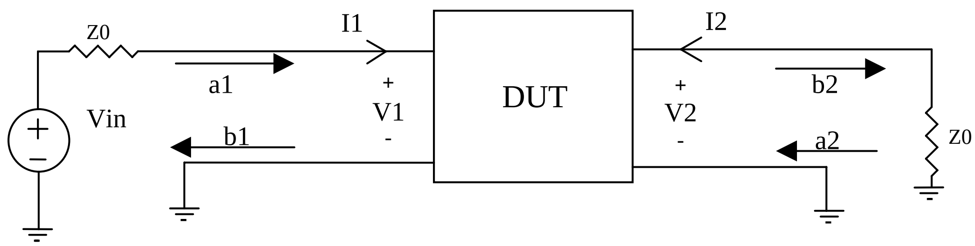
Extracting Scattering Parameters from SPICE

Scattering, or S parameters are easily generated by an AC analysis in SPICE. S parameters are defined with respect to incident (a1 and a2) and reflected (b1 and b2) voltage waves defined by the diagram below and by:

$$a1 = (V1 + Z0 * I1) / (2 * SQRT(Z0)) \qquad \text{(Eq. 1)}$$
$$b1 = (V1 - Z0 * I1) / (2 * SQRT(Z0)) \qquad \text{(Eq. 2)}$$
$$a2 = (V2 + Z0 * I2) / (2 * SQRT(Z0)) \qquad \text{(Eq. 3)}$$
$$b2 = (V2 - Z0 * I2) / (2 * SQRT(Z0)) \qquad \text{(Eq. 4)}$$



The scattering matrix relates the incident and reflected waves.

$$\text{(Eq. 5)} \quad \begin{vmatrix} b1 \\ b2 \end{vmatrix} = \begin{bmatrix} S11 & S12 \\ S21 & S22 \end{bmatrix} \begin{vmatrix} a1 \\ a2 \end{vmatrix}$$

S11 = b1/a1 for the case when a2 = 0. a2 is the wave reflected from the load back toward the output port and is zero when the load is the impedance used to define the S parameters. Hence,

$$\text{(Eq. 6)} \quad S11 = \frac{V1 - Z0*I1}{V1 + Z0*I1}$$

Noting that $Z1 = Z_{in} = V1/I1$, we have the familiar

$$\text{(Eq. 7)} \quad S11 = \frac{Z1 - Z0}{Z1 + Z0}$$

Let the Voltage Source be 2 Volts. This value simplifies calculations without compromise, for an AC run in SPICE is a small signal analysis. The voltage at the network input is then found from a voltage divider as:

$$V1 = (2 * Z1) / (Z1 + Z0) \qquad \text{(Eq. 8)}$$

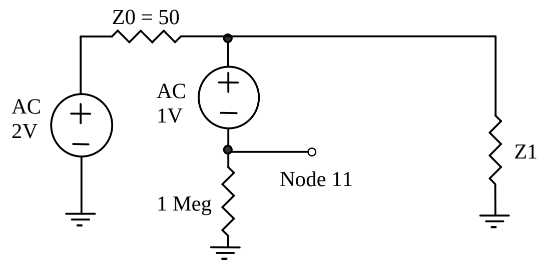
Solving for Z1 in terms of V1 results in:

$$Z1 = (V1 * Z0) / (2 – V1) \qquad \text{(Eq. 9)}$$

Substituting this in Eq. 7 yields the simple result:

$$S11 = V1 – 1 \qquad \text{(Eq. 10)}$$

This is more intuitive than many S parameter results. If a 2 volt source is used at the input, and if the input is perfectly matched, V1 will be 1 Volt. Subtracting 1 from that leaves S11 of zero, a "proper" result for a perfect impedance match. In SPICE, 1 is easily subtracted from V1 by attachment of a 1 Volt AC generator to the V1 node. The voltage at node 11 below is S11 in both magnitude and phase.



S21, the forward scattering parameter, is defined as S21=b2/a1. Using equations 1 and 4, this becomes

$$S21 = (V2 - I2*Z0) \;/\; (V1 + I1*Z0) \qquad \text{(Eq. 11)}$$

S21 is defined by Eq. 11 for the case when there is no reflection from the load, setting a2 = 0. This produces I2 = -V2/Z0, and Eq. 11 reduces to

$$S21 = (2*V2) / (V1 + Z0 + I1) \qquad \text{(Eq. 12)}$$

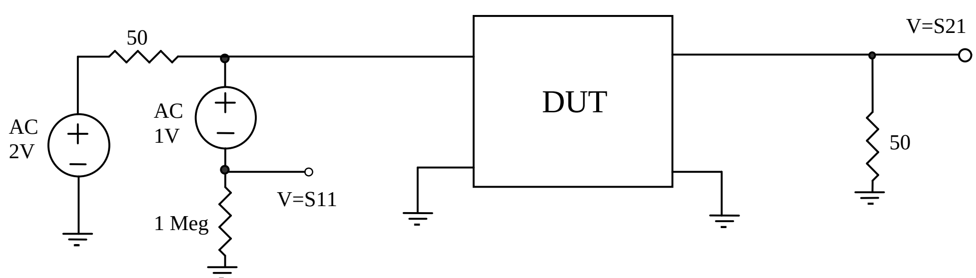
Consider our special case for a 2 volt driving generator. I1, the input current, is related to the input voltage, I1 = V1 / Z1. Substitution into Eq. 12 yields

$$S21 = (2*V2/V1) / (1 + Z0/Z1) \qquad \text{(Eq. 13)}$$

Equation 9 relates Z1 to V1; substitution simplifies to the almost trivial result that

$$S21 = V2 \qquad \text{(Eq. 14)}$$

Hence, S21 and S11 are calculated in SPICE from



S12 and S22 are calculated with similar ease with