Building Electro-Optical Systems:
Making It All Work

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Appendix B
Chapter Problems

There's always a simple solution to every human problem—neat, plausible, and wrong.
-- H. L. Mencken

Problems For Chapter 1

1.1 Phase Matching. A 1 mm long piece of nonlinear optical material, excited by a field at $e^{i\mathbf{k}\cdot\mathbf{x}-\omega t}$, produces a polarization at $e^{i2\mathbf{k}\cdot\mathbf{x}-2\omega t}$. The amplitude of the light radiated by this polarization in any direction is approximately proportional its 3-D Fourier transform. Neglecting dispersion and optical anisotropy, compute the front-to-back ratio (i.e. the ratio of the light intensity at 2ω along **k** to that along **-k**. b) If the beam is a circular Gaussian beam of small NA, find the complete intensity pattern of the beam. Is the selectivity better front-to-back or front-to-side? Why?

1.2 Photon budget for a TDI camera. Time delay and integration (TDI) is a method for improving the photon statistics of a CCD line scan camera, using a narrow 2D array (perhaps 4096x64 pixels).. If the image crosses the array at velocity $\boldsymbol{v}$, the CCD is clocked at a rate v/p, where p is the pixel pitch. a) How much image blur occurs? b) How much does N stages of TDI improve the photon budget, including the effects of blur? c) What is a reasonable tolerance for the magnitude and direction of v?

1.3 **ABCD Matrix for a Zoom Lens.** A zoom lens can be made from 3 lens elements, the rear two of which move. a) Construct the ABC matrix for the system. b) Find an expression relating the focal lengths and lens motions so that the focal length of the combination changes but the image of an object at infinity does not move.

1.4 **Improving Étendue by Immersion.** A LED die has a refractive index of about 3.5. a) If the light is generated uniformly in all directions, and the die is very wide and thin, what proportion of the light escapes through the top surface on the first try, including Fresnel losses? b) If the die is placed at the centre of a large hemisphere of radius r and index n, how much light escapes on the first bounce?

1.5 **Paraxial Wave Equation.** a) Prove that the Huygens integral and the TEM_{00} Gaussian beam are solutions of the paraxial wave equation. b) The true solution to the Helmholtz wave equation can be found by decomposing the field on the boundary, propagating each component, and recombining. The Huygens integral replaces the true propagator $e^{i\mathbf{k}\cdot\mathbf{x}}$ by e^{ikz} times $e^{ikz(u-1)}$, and then approximates u-1 by the first term in its binomial expansion. Show that for a general field, this requires that $z \gg (r^4/\lambda)^{1/3}$. c) Use a stationary phase analysis to show that for sufficiently slowly varying Θ, although the phase errors do become large at large r, the large-r contribution to the Huygens integral go strongly to zero as $z \to z'$, so that the Huygens integral remains valid.

1.6 **Rules Of Thumb For Beams.** How many fringes in the pupil does it take to move the waist of a Gaussian beam sideways by one $1/e^2$ diameter? How many fringes do you get in the pupil if you interfere two Gaussian beams, identical except that one beam is defocused by 1 Rayleigh range with respect to the other? If you're using a tunable laser with a fixed grating, how does the number of resolvable spots you can get from a given tuning range depend on the grating diameter and incidence angle? Hint: it's independent of angle, and should go as (tuning range x grating width)/(grating pitch in wavelengths).

1.7 **Confocal Imaging.** A laser confocal microscope is based on a heterodyne interferometer, with two identical laser beams (from the sample and reference surfaces) interfered at their pupils. a) Show that the in-focus amplitude point spread function of the device is the square of that of a simple imaging system. b) If the pupil function is a uniform circular disc of radius NA, show that the amplitude spatial frequency response of the system is

$$h(r) = \frac{2}{\pi}\left(\cos^{-1}(r) - r\sqrt{1-r^2}\right) \qquad \textbf{(B.1)}$$

where r=u/2NA.

1.8 **Confidence levels of dark and bright field.** Although the 1σ noise levels of dark field and coherent detection systems are the same, this is no longer the case if better than 1σ confidence is needed, or if we need to be sure that no signal exists. Discuss.

Problems For Chapter 2

2.1 **Pulling a He-Ne with a Cavity.** A He-Ne laser with a cavity length of 15 cm has an output coupler with a reflectivity of 99.5%. It is used with an optical spectrum analyzer with a finesse of 200 and one-way path length 5 cm, situated 20 cm away. How far does the cavity pull the laser? (b) How about if we switched to one with a finesse of 20,000, coupled with 1 metre of silica fibre?

2.2 **Michelson Interferometer with a Multimode Laser.** You need to build a Michelson interferometer from a multiple longitudinal mode He-Ne laser. The laser has 3 modes, one in the middle and one on each side, about 1/3 the power of the main one. Derive the fringe visibility in the Michelson as a function of path difference. What is its period?

2.3 Tungsten Bulb Lifetime. (a) Assuming that bulb failure occurs at 1% tungsten loss, plot normalized bulb lifetime vs. temperature. (b) Add a curve for total visible output (400-700 nm). (c) Assuming that the human eye response is a Gaussian centred at 550 nm with 100 nm FWHM and central value 683 lm/W, plot the luminous output *vs.* lifetime.

Problems for Chapter 3

3.1 **Fresnel losses in a 1-D concentrator.** (a) Plot the overall efficiency η vs. θ_i for an air to plastic (n=1.5) to uncoated silicon ($n\approx$3.5) system with the two planar interfaces parallel, assuming the light to be wideband so that multiple bounces add in intensity. (b) For a 1-D concentrator, use the unfolded view of Figure 3.9 and the result of (a) to investigate the photon efficiency as a function of position and angle, for a cone angle of 15°. (c) What limits the capture efficiency? Is there a better way?

3.2 **Arrhenius Relation and Dark Current.** The shunt resistance of a diode arises from the diffusion currents going back and forth across the junction. Using the diode equation*, where V_{bg} is the band gap of the semiconductor material, how much does the zero-bias dark resistance of a photodiode decrease with temperature?

3.3 **Response of a Split Detector.** (a) Compute the response of a bi-cell detector with a kerf of width d between cells to a Gaussian beam of $1/e^2$ radius w as a function of its position x. Assume the centre of the kerf is x=0. (b) Find the best straight line through the data for $x \in [-0.75w-d/2, 0.75w+d/2]$. At what values of w does the residual reach 1%, 5%, and 10%? (c) For a constant beam radius, what is the shot noise limited 1σ position resolution? (d) Compute the sensitivity function (partial derivative) of the imputed position of the beam vs. beam diameter. For a 500μm diameter beam centred at x=100μm and a 50μm kerf, how big a diameter uncertainty do you need to dominate the shot noise of a 100μA total photocurrent?

3.4 **Lateral Effect Cell.** A highly linear 1-D lateral effect cell of 2 mm diameter has a resistance R between its two anodes, and is illuminated by a small diameter beam producing photocurrent I. (a) What is the position uncertainty caused by the Johnson and shot noise contributions, and at what photocurrent do they become equal? Typical values of R range from 1 kΩ to 100 kΩ. If the total capacitance is uniformly distributed, use a 1-D RC transmission line model to estimate the phase shift between the two anode currents as a function of position and frequency.

3.5 **Error Propagation in Detector Calibration.** Consider a MOS imager with reasonable dark current, about 0.5 nA/cm^2, and low readout noise, about 5 e^- RMS (measured on different readings of the same pel, with the same clock speed and bandwidth used in the actual measurement). Its full-well capacity is 6×10^4 e^-, and its pel pitch is 10μm. Micro-lenses on its

* Diodes don't follow the diode equation closely; the exponent grows 1.5× to 3× more slowly than this in real devices (see Section 14.6).

surface improve its fill factor to 80% and η to 85%. Its output sensitivity is 15μV/e^-. Unfortunately, its fixed-pattern noise is 5 mV rms and its sensitivity in the instrument varies as much as ±25% depending on position, because of poor lens/pixel alignment, obliquity, and vignetting. We're determined to use it to make faint-object astronomical observations, and want to use the universal calibration scheme of Section 3.7.15, with a calibration error budget of 4e^- rms per pel. Assume initially that the error budget is equally apportioned between bias, dark, and flat field frames. (a) How many short and long dark field frames are needed to get good a good bias frame? (b) The exposure time can range from 1s to 3 hr; what integration times do the long dark frames need? Can the short-exposure dark frame be replaced with a bias frame? (c) The flat fields should be located at infinity, because otherwise the vignetting and obliquity will be different. The dawn sky at zenith is a convenient flat field source, but dawn doesn't last long, and becomes less uniform in brightness as day approaches (you can also lose data if it clouds over before you take your flats). Can you think of an approach to accumulating flat fields that will work well under these circumstances? Would your approach work as well with a frame transfer CCD?

3.6 **Noise of APDs.** If an APD has a noise exponent m=0.3, what is the optimum gain as a function of R_L and i_{diode}? How far is this from the shot noise? At what photocurrent does it become worthwhile using an APD?

3.7 **Photon Counting Photomultiplier vs APD.** A 6 mm diameter APD has a quantum efficiency of 80% at 700 nm, and a dark-count rate r_{d} of 1 kHz in Geiger mode. An extended-red PMT of 20 mm photocathode diameter has η_{pmt}=0.12 there, r_d=200 Hz, but ten times the area. If quenching the APD imposes a maximum count rate of 1 MHz and the PMT can go 30 MHz, at what flux density (W/m^2) does the APD start to give better data? How does this change if the total flux is the same?

3.8 **Shadow Mask Sensor.** (a) Design a 3-D shadow mask sensor from three 5 mm square solar cells and an equilateral triangle shadow mask. How do you combine the photocurrents to get x, y, and total intensity (often related to z)? (b) Compute its angular sensitivity and noise-equivalent angular error. What do you think will limit its accuracy in practical cases?

3.9 **Reaching the Shot Noise.** What optical flux density is required to reach the shot noise, as a function of D^* and λ?

Problems for Chapter 4

4.1 **Pellicle Properties.** (a)Compute the transmittance and reflectance of a nitrocellulose pellicle (n=1.5) as a function of thickness, wavelength, and angle of incidence. How thick should a pellicle be so as to have no major effect on the colour of the reflected light? How about the reflected light?
(b) The pellicle is 20 mm in diameter, and is vibrating by approximately ±1μm in the centre due to an acoustic disturbance. Assuming the surface distortion to be parabolic, what is the maximum power of the resulting surface in dioptres? Would this present serious problems in a measurement?

4.2 **Athermalizing an IR lens.** Infrared optical materials exhibit little dispersion but large

temperature coefficients, so athermalization is more of a concern than achromatism. An IR afocal beam expander has two positive elements made of silicon (n=3.5, $\partial n/\partial T$ = +1.5×10^{-4}), with focal lengths f_1 and f_2, separated by f_1+f_2.
(a) Assuming the lens barrel has zero CTE, find how thick the two elements should be in order that the magnification and focus of the resulting beam expander is temperature independent. (b) The two elements are held tightly against an aluminum spacer (CTE=23 ppm/K) by O-rings and threaded end caps. Now what should the thicknesses be?

4.3 **Etalon fringes from a 5-bounce beam.** Consider an optical Dewar in which by heroic efforts, all etalon fringes have been eliminated except for a single one, which has taken 6 bounces off BK-7 surfaces and now arrives exactly in line with the main beam from which it was derived. If its total path length is 300 cm, how large is the resulting fringe?

4.4 **Polarization Shift From Fresnel Formulae.** Compute the polarization shift from a solid cube corner reflector made of BK-7 as a function of azimuth and elevation. What are the maximum and minimum polarization shifts, and where do they occur?

4.5 **TIR Inside A Lens.** At what focal ratio does a plano-convex BK-7 lens exhibit TIR with on-axis plane wave illumination, if used backwards?

4.6 **Imaging Michelson Interferometer.** Show that an imaging Michelson interferometer with unequal arms can have no magnification error if the longer arm has a block of glass of length $\delta\ell/(n-1)$ in it. How well does this work over angle?

Problems for Chapter 5

5.1 **Wood's Horn (1-D Case).** Consider a very long, V-shaped piece of fused quartz, of opening angle α and negligible wall thickness, with light coming in at θ to its line of symmetry. (a) Its outside is covered with black spray paint, which is a good index match to the glass. What is the total reflectance of the horn as a function of α and θ? (b) If the inside is covered with a matte black finish which scatters light as $\exp(-10(\sin\theta-\sin\theta_r)^2)$, where θ_r is the angle of the specular reflection, but whose total reflectance is half that of the glass, approximately what happens then? You may find it helpful to use the fact that when two random variables are added, the probability density of the sum is the convolution of their PDs.

5.2 **Heat Absorbing Glass.** Consider a 250 W tungsten bulb of colour temperature 3400 K. If we put a very sharp cutoff 750 nm short pass filter in as a heat absorbing glass, what proportion of the total bulb output does it absorb? If the condenser is 60% efficient, how much power is absorbed? (b) If instead we develop a coating which accurately turns a 3400 K thermal spectrum into a more sunlike 5000 K one from 400-800 nm, how efficient can it be if we assume that its absorption is 10% at 400 nm?

5.3 **Recursive Transmission Calculation.** (a) Using (5.7) and Figure 5.4, find the recursion relation for $\hat{t}$. Which way do you have to apply it? Use your result to calculate and plot the reflection and transmission coefficients for the V-coating of Example 5.3, ignoring dispersion, and verify that $R+T=1$. (b) What is the phase retardation of the reflected and transmitted waves (i.e. the phase shift between *s* and *p*) at 30° incidence? If light comes in linearly polarized at 45° to the plane of incidence, with $\theta_i=30°$, find the lengths and directions of the major and minor axes of the resulting elliptical polarization (see Section 1.2.7).

5.4 **Enhanced Aluminum Mirrors.** A multilayer dielectric coating over an aluminum base layer can produce very good mirrors indeed. The simplest kind is the "enhanced aluminum" coating, which is one $\lambda/4$ layer each of SiO ($n_1=1.7$) over MgF_2 ($n_2=1.38$) over Al. (a) Calculate the reflectance of such a mirror at the wavelengths given in Table 4. (b) How much loss in the detected signal occurs per bounce at normal incidence? At 45° (*s* and *p*)? (c) What is the phase retardation between *s* and *p* as a function of incidence angle?

5.5 **Beer's Law and Diabatie.** Consider a grade of fused silica, which has a pronounced absorption band at 2.7μm. The peak is Gaussian in shape for very thin samples, with a $1/e^2$ width of about 100 nm. A 1 mm thick piece absorbs 50% in the centre of the band. (a) Plot its internal transmittance for thicknesses of 0.2, 1, 5, and 10 mm on the same axes. What is the dependence of the FWHM of the notch on thickness? (b) Plot the diabatie, marking the vertical scale where the internal transmission of a 10 mm thick piece is 100%, 99.9% 99.5%, 99%, 90%, 70%, 50%, 20%, 10%, 1%, 0.5%, 0.1%, and 0.01%. How much do you have to slide the vertical scale up and down to get the correct results for 2 mm thickness?

Problems for Chapter 6

6.1 **Polarization Compensator.** Show that any fully polarized state of a monochromatic wave can be transformed into any other, using a compensator made from two quarter wave plates, independently rotatable. This is used very often in fibre optics, where polarization instability is a constant problem.

6.2 **Étendue of a wave plate.** (a) Calculate the change in retardation of zero- and 20^{th}-order quarter waves plate made of quartz. as a function of incidence angle, for rotations about *x* and about *y*. If we can tolerate crosstalk of 1%, what is the maximum angle of incidence in each case? (b) Suppose we have a $\lambda/4$ plate cut for a 514.5 nm, which we want to use at 488 nm. How much do we have to tilt each wave plate, in which direction, and how is the angular acceptance altered?

6.3 **Wollaston and Nomarski Prisms.** (a) Find the fringe locus (i.e. the place where the two rays from a single source point cross or appear to cross) for a calcite Wollaston prism of wedge angle 4°. (b) repeat for a Nomarski prism, where the optic axis on the incoming beam makes an angle of 15° with the input face. (c) What is the phase shift as a function of angle for an on-axis point?

6.4 **Compensating plate for C-axis normal sapphire window.** We sometimes need to use sapphire windows in hostile environments, e.g. in detecting liquidborne particles in dilute HF ($n\approx1.34$), which rapidly etches SiO_2. Use a plate of MgF_2 to cancel the effects of a C-axis normal sapphire window. How well does it prevent polarization shifts as a function of incidence angle? What effect does it have on light focused through the window, as a function of NA?

6.5 **Polarization Insensitive Faraday Isolator.** If you had two identical 45° Faraday rotators and two identical walkoff plates, explain how to build a polarization insensitive Faraday isolator. Can you do it with only one Faraday isolator?

6.6 **Symmetry of the Dielectric Tensor.** Show that the dielectric tensor $\underline{\underline{\varepsilon}}$ must be real and symmetric for lossless dielectrics with no optical activity. What are the symmetry conditions on $\underline{\underline{\varepsilon}}$ in an isotropic, optically active material?

6.7 **Waveplates and Mirrors.** A retarder is coated with a perfectly conducting mirror on one side, and is illuminated at normal incidence with an elliptically polarized plane wave. What is the relationship between the polarizations coming in and going out? Does the effect cancel as in optical activity, double as in Faraday rotation, or neither?

Problems for Chapter 7

7.1 **Hologon Scanner.** Derive the diffraction angle of a transmission hologon scanner operated at minimum angular deviation (i.e. $\theta_d=\theta_i$ in the middle of the facet) a function of shaft angle. Is this better or worse than an polygon? If the shaft wobbles a bit, find the resulting change in the diffracted wave (amplitude and phase). If the tolerance for out-of-plane wobble is δ, how much shaft wobble can we stand in each case?

7.2 **Sloppy Monochromator Alignment.** Consider a Czerny-Turner monochromator illuminated by an ideal single-frequency laser beam, whose alignment is poorly controlled. For fixed slit positions θ_i and θ_d, and neglecting diffraction at the slit, compute the first order wavelength making it through the exit slit as a function of the angles of incidence on the slit. (b) If the height of the slit is large enough to produce a significant k_y at the grating, how much spectral shift does this produce? (c) What trade-off does this predict between resolving power and étendue for the polychromatic case?

7.3 **Lossless Grating Theory**. Compute the approximate diffracted field of the lossless grating of Figure 7.1, by propagating each order independently to the plane z=0, and then finding the angular spectrum. Pay particular attention to conservation of energy. Plot the results as a function of angle for λ=0.5μm, and a grating frequency of 600 grooves/mm. Where are the anomalies, and how large are they?

7.4 **Moving Grating** A radially ruled grating on a circular substrate is turned at an angular frequency ω What frequency shift is induced in the reflected light?

7.5 **Moire zone plates.** (a) Show that two identical zone plates of focal length f on top of each other cancel out to leading order. If one is stretched radially by a factor 1+ε, show that the resulting moire pattern is also a zone plate, and compute its focal length as a function of f and ε.

7.6 **Compound Grating.** Compute the dispersion of 2 gratings arranged as in Figure 7.8.

7.7 **Fibre Bundle Spectrometer.** If you were using an f/8 OMA grating spectrometer with a multimode fibre bundle consisting of three 100 μm core, 120 μm OD fibres jammed right up against its slit, as shown in Figure B.1, what change in the spectrum do you expect with fibre misalignment in position? In angle?

Problems for Chapter 8

8.1 **Disappearing Modes?** Since $d/dr|J_m(\kappa a)| <0$ for allowed modes, what happens to the LP_{01} mode when $J_0'(\kappa a)$ crosses 0?

8.2 **Coupled Modes With Finite Beat Length.** Find analytic expressions for the eigenvalues and eigenvectors valid to leading order in $c/\Delta k$ in both the long and short beat length limits. How rapidly does the coupling fall off with decreasing λ_b?

8.3 **Ray Optics Model of the Étendue.** Consider a fibre whose index varies parabolically from n_1 on axis to n_2 at $r=a$, and is then constant at n_2 for $r>a$. In the ray optics approximation, and ignoring evanescent wave effects, calculate the range of **k** vectors that will be totally reflected,

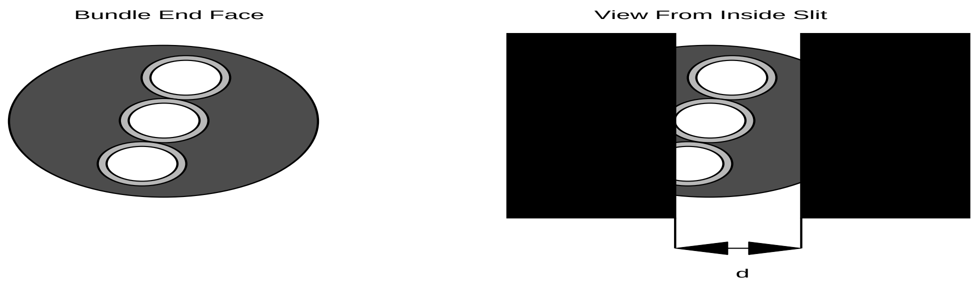

Figure B.1 Entrance Slit of A Fibre-coupled OMA Spectrometer

as a function of r. Integrate the solid angle subtended by this range of **k** over the end face to find the étendue of the fibre.

8.4 **Fibre Bundles.** (a) For a large bundle of 125 μm OD, 100 μm core, unjacketed fibres, find what proportion of the bundle area is in a fibre core? (b) Assuming that the illumination of the facet is spatially uniform inside the core and 0 elsewhere, and uniform in angle up to the NA limit, how far out from the end of the bundle do you have to go before the spatial nonuniformity of illumination is less than 1% in the visible? (c) Under the same assumptions, compute the MTF of a hexagonally packed, 6-fibre bundle placed at focus.

8.5 **OCDR Advantages.** (a) A LED has a Gaussian power spectrum with $1/e^2$ points of 800 and 900 nm, and couples 50 μW of power into a single mode fibre. The fibre feeds a 3 dB coupler, whose outputs form a fibre Michelson interferometer. The LO arm has a mirror at the end, and the signal arm is not. Assuming a shot noise limited detection system and a bandwidth of 0.5 Hz, what is the 1 σ uncertainty in an OCDR measurement of fibre length? (Remember to include coupler losses, or explain how to avoid them.) (b) Is that better than a superluminescent diode with a bandwidth of 1 THz and 1 mW of power in the fibre? How does this compare with an OTDR whose duty cycle is 0.01% and whose peak source power in the fibre is 10 mW? (c) How does the result of (b) change if the SLD has a ripple of 5% in its PSD at a delay of 20 ps?

Problems For Chapter 9

9.1 **Symmetrical Lenses.** If a compound lens is symmetrical front-to-back, and is used at 1:1 magnification, some of the aberrations cancel and some add. Split the lens down the middle, and consider the aberrations of the two halves. When the two contributions are combined at the end, which of the W coefficients add and which cancel? Describe the relevant distinction in a sentence.

9.2 **Longitudinal magnification.** Show that an optical system of magnification M has a longitudinal magnification M^2, and that therefore (paraxially speaking) a sphere of radius *a* gets imaged onto another sphere of the same radius.

9.3 **Snell's Law.** Use (9.10) to derive a generalized Snell's Law for a gradient index region between two regions of constant index. (b) Take the limit as the transition becomes infinitely sharp and recover the standard form of Snell's Law.

9.4 **SNR And Strehl Ratio.** The heterodyne confocal microscope of Example 9.3 works by interfering the beam reflected from the sample with a good quality uniform beam at the pupil, after both have been recollimated. (a) Show that if the beam from the sample is unaberrated, the CTF is the Chinese Hat function. (b) Assuming that the laser's excess noise is negligible at the measurement frequency, find an expression for the SNR as a function of the pupil function aberrations of the beam from the sample.

9.5 **V(z).** Consider a heterodyne confocal reflection microscope, whose detected output is proportional to the interference term between two pupil functions, $\tilde{A}$ and $\tilde{A}'$ from the two branches of a focused-beam Michelson interferometer. The two start out with plane wave spectra $\tilde{A}=\mathrm{circ}((u^2+v^2)/\mathrm{NA}^2)$, where NA is not necessarily small. (a) Show that the output does not change when both are defocused together. (b) Show that if one is in focus, and the other is defocused by a distance z, to leading order in NA, the output $V(z)$ is

$$V(z) = \mathrm{sinc}\left(\frac{z}{\lambda}(1-\cos\theta)\right)e^{ikz(1+\cos\theta)} \qquad \textbf{(B.2)}$$

where θ is the cone angle of the beam (a change of variable to cosθ may be of help). (c) Complete the exact calculation, and show that

$$V(z) = \frac{2}{\beta^2}(\alpha\sin\beta + \sin\alpha(\beta\cos\theta\cos\gamma - \sin\gamma) + i(-\alpha\cos\beta + \sin\alpha(\beta\cos\theta\sin\gamma + \cos\gamma))) \qquad \textbf{(B.3)}$$

where $\beta=kz$, $\alpha=\beta\sin^2(\theta/2)$, and $\gamma=\beta\cos^2(\theta/2)$. Use your favourite math program to plot the amplitude, phase, and z-derivative of the phase of this as a function of z for NA=0.1, 0.5, 0.8, and 0.95.

At what NA is the z-derivative of phase in (B.2) off by 1% near z=0?

9.6 **MTF With Aberrations.** Calculate numerically and plot the MTF and Strehl ratio of a small-aperture lens with (a) no aberrations; (b) ±0.1, 0.25, 0.5, and 1.0 waves p-p of spherical aberration; (c) the same as (b), but with defocus added to make the average phase of the pupil function 0.

9.7 **Underwater Camera.** Underwater camera housings use flat windows to keep the camera dry. Consider a plane air-water interface (n_2=1.33). What is the astigmatism as a function of aperture and field angle? What's the diffraction-limited field of view in the green as a function of NA on the object side?

9.8 **CTF With Fringes.** Consider a microscope cover glass of 100 μm thickness, introduced into the pupil of a 40X, 0.7 NA microscope lens illuminated with a uniform pupil function from a He-Ne laser. (a) Including the effects of multiple reflections in the glass, find the pupil function and PSF. (b) Repeat (a), assuming that the lens is cover glass corrected, i.e. has a compensating aberration designed in.

9.9 **Cleaning up a diode laser beam with an aperture.** A diode laser has an elliptical output beam, with about a 3:1 ratio of major to minor axis, and with moderate astigmatism; the long direction wants to focus closer-in than the short direction. If the laser has 1 wave of astigmatism, what is the Strehl ratio at the best focus point?
(b) If we put a circular aperture whose diameter is the minor $1/e^2$ diameter of the beam, how what is the p-p wavefront error and Strehl ratio? What about the photon efficiency? Is this a good method?

9.10 **Illumination Diagrams.** Sketch the illumination diagrams at the centre and edge of the field for a low power microscope of 0.10 NA, 10 mm field diameter, and 25 mm working distance, with (a) a 3 mm diameter, 100/125 µm step-index multimode fibre bundle illuminator 40 mm underneath a transparent sample; (b) the same bundle, mounted on one side of the lens, 15 mm from the axis; (c) critical illumination, with the bundle expanded to fill the field, and defocused enough to keep the nonuniformity; (d) Köhler illumination, with the field scaled to the 50% points of the (nearly Gaussian) angular spectrum.

Problems For Chapter 10

10.1 **Optimal sensitivity.** Given a shot noise limited measurement with laser power *P*, what absorption value gives the best SNR of the measurement?

10.2 Dim Field SNR. Consider a polarimeter specialized for small deviations, e.g. in electro-optic sampling. A linearly polarized input beam goes through a sample, which has some small retardation δ, through a crystal analyzer. (a) Find the optimal orientation and CNR of the analyzer as a function of optical power, if the laser power is *P*, the front end amplifier has a noise current spectral density i_N, and the detected fractional laser RIN is ρ per root hertz. If the source is a stabilized He-Ne with low frequency $\rho=10^{-7}$, P = 200 µW, and i_N=2 pA/√Hz, what are the optimal orientation and CNR? (b) How about if we used a Wollaston and differential detectors that gave 30 dB worth of dc rejection? (c) What is the best way to improve the measurement, and how good would you expect it to get?

10.3 **Zeroing Out The Fundamental.** (a) How small an extinction can the particle counter of Figure 10.8 theoretically detect if the laser is a 5 mW red He-Ne? (b) What factors limit how well the fundamental signal can be nulled in practice? (c) How large would you expect each one to be? How much worse is this than a dark field system with the same detection NA?

Problems for Chapter 12

12.1 **Quad Cell Aid.** What effects degrade the alignment accuracy of a quad cell with a Gaussian beam? Which do you think is the limiting factor when (a) the background is strong; (b) when it is weak; (c) in the presence of vignetting?

12.2 **Dust Doughnuts.** How far out of focus does a dust particle have to be before there is no umbra (deep shadow), as a function of its diameter and the NA of the beam it sits in? How far before it causes less than a 1% irradiance change?

12.3 **Aligning Imperfect Beams.** Consider aligning two Gaussian beams, one of which has been vignetting by falling slightly off one edge of a rectangular mirror; it has been chopped off at the $1/e^2$ point. Explain the difficulty this causes in beam alignment, and what you'd do about it (assuming you couldn't just fix it). Extra credit: just what does it mean for two beams whose aberrations are different to be aligned perfectly?

12.4 **Interacting Adjustments.** Consider aligning two counterpropagating beams using a pair of two-tilt mirror mounts in *one* of the beams, spaced a distance d apart, where the two beams have to be overlapped in position and angle at a distance s from M2. The mounts tilt by $\delta\theta$ per turn of the adjustment screws. In the small angle approximation (perhaps by using augmented ABCD matrices as in Section 1.3.10), compute the interaction between the controls for angle and position, as a function of d and s. What condition on d and/or s should be observed to make the adjustment easy?

12.5 **Lens Clamping.** Discuss the relative merits of the two lens mounting approaches in Figure 12.1, (a) in a lab environment; (b) in a high vibration environment; (c) under extreme temperature swings.

Problems for Chapter 13

13.1 **Frequency Plan.** (a) Design a single conversion frequency plan for a scanning heterodyne interferometer similar to the one in Example 9.3 but based on a two-axis Bragg cell. The scanning beam is diffracted twice on transmit, and the reference beam twice on receive; the signal frequency is $2(f_x+f_y)$, where f_x and f_y both range from 55-110 MHz. There will be some slight interference from pickup at f_x and f_y, but high order mixing products will not be a problem since the detector is very accurately quadratic in the optical fields. (b) Given signal sources at f_x and f_y, design a system to provide a clean LO signal for the mixer in (a).

13.2 **ADC Error Budget:** An astronomical CCD camera has a full-well capacity of 10^6 electrons, a dark current of 2 e^-/s/pel, and a readout noise of 5 electrons rms. The dynamic range of astronomical images is greater than 10^{12} (electrical). How many ADC bits are needed for the camera performance to be dominated by photon noise, as a function of exposure time? (b) If the sky brightness is equivalent to 20 e^-/s/pel, how does this change?

13.3 **Optical SSB mixer.** In Chapter 3, we saw that a coherent optical detector loses 3dB in sensitivity when the beat frequency is not 0, due to optical image noise. Design an optical SSB mixer using two waveplates, a polarizing beamsplitter, and two detectors (assume that the incoming signals are perfectly linearly polarized). How good does the polarization purity have to be to get a 2.5 dB SNR improvement? What will limit its performance?

13.4 **Detecting Weak Pulses.** You have a single photomultiplier tube, whose dark pulse rate is 200 Hz. You are trying to detect weak luminescence from a biological sample by chopping between a dark box and the sample, using a chopper wheel whose frequency can be adjusted from 1 Hz to 1 kHz with a 50% duty cycle. What is the weakest luminescence you can detect with 3σ confidence in an integration time *T*?

13.5 **Getting I and Q On The Sly.** Consider an unequal-armed interferometer using a current-tuned diode laser. Using (13.9), construct a block diagram and filter specifications for an *I*/*Q* interrogation system. show how the fundamental and second harmonics of the detected signal can be used as surrogates for *I* and *Q*. What exactly should the modulation index *m* be, and why? What is the SNR penalty, if any, of using this technique vs. using a PLL to keep the operating point of the interferometer constant? How much noisier is this than using the optical SSB mixer of Problem 13.2?

Problems For Chapter 14

14.1 **Impedances and Transmission Lines.** (a)Show that a pot of resistance *R*, loaded by two resistors of value *R'* from the ends to the wiper, is equivalent near null to a single pot of value 2*RR'*/(*R*+2*R'*) and *R*/(2*R'*)+1 turns full scale.
b) Show that the resistance of a parallel *RC* circuit goes as f^{-2} for large *f*, but that its conductance is constant.
(c) Verify (14.12). Show that a shorted line less than $\lambda/4$ long appears inductive, whereas an open-circuited line of the same length looks capacitive, and derive the equivalent values of *L* and *C*.
(d). Consider a very short piece of coax of length *dx*, terminated with a resistor Z_0. Assuming that the coax can be modelled as a differential series inductance *Ldx* and shunt capacitance *Cdx*, show that the requirement that the input impedance be frequency independent to leading order requires that $Z_0=(L/C)^{1/2}$.

14.2 **Noise Figure Of An Attenuator.** Consider a signal processing system with an *M* dB noise figure and $Z_{in}=50\Omega$. (a) Assuming a 50Ω source, how much does the noise figure deteriorate if a well matched *N* dB pad is put in front of it? (b) How does this change if Z_{in} is not 50Ω?

14.3 **Temperature Compensating A Tank Circuit.** A parallel-resonant LC circuit uses a 250 pF capacitor *C* and a 750 nH inductor *L*. It is found to have a temperature coefficient of ω_0 of -40 ppm/°C. We want to temperature compensate it without disturbing its room-temperature ω_0, so we intend to add two N750 capacitors, C_1 across *L* and C_2 in series with *C*. Find the values of the two capacitors required. About how much improvement do you expect to get this way? (b) Consider a single-layer air core solenoid. Assuming that the diameter and length both expand solely due to the CTE of copper, and that (14.3) is exact, what is the TC as a function of *a* and *b*?

14.4 **The Bazooka Balun.** Consider a long piece of 50 Ω coax, with a coaxial metal sleeve $\lambda/4$ long around the outside at one end. The sleeve is insulated from the braid along its length, but shorted to it on the inner end. Show that this device functions as a BALanced-to-UNbalanced converter, or *balun*. What impedance should the load be for a matched condition?
Does it matter what the sleeve diameter is? (b) Show that a common-mode choke made by winding the coax around a toroid core will work too. What must the inductive reactance of the choke be to keep the VSWR below 1.5?

14.5 **Diode Bridge Mixer Operation.** (a) Draw the output of the series diode clipper of Figure 14.12, as the input voltage goes from -12 V to +12 V. How sharp are the knees? (b) Consider a 50Ω diode bridge mixer, whose rf input consists of two -dsm sine waves at 1.0 and 1.15 MHz, in white Gaussian noise of -140 dBm/Hz. The preceding amplifier has a 2-pole Butterworth rolloff with $f_{-3\text{ dB}}$ = 5 MHz, and bandwidth limitations in the transformers are not important. Describe its operation in detail, half-cycle by half-cycle, when the LO waveform is: (i) a long +2 V pulse; (ii) a symmetrical square wave of 1.0 MHz, in phase with the 1 MHz rf signal; (iii) a 1.075 MHz, 1 V sine wave.

14.6 **Half-Flash ADC.** A half-flash 8-bit ADC uses a 4-bit flash converter (15 comparators) to generate the MSBs of the output word, then uses a 4-bit DAC to subtract off that part and an error amplifier to amplify the residual, which is then converted itself to produce the LSBs. Does the coarseness of the first conversion relax the settling time requirement for the T/H? Why? If so, by how much? If not, can you think of a way to modify the half-flash scheme so that it would?

14.7 **Absolute Value Circuit.** (a) Using the Ebers-Moll BJT model, and assuming that β>>1, show that the voltage at the emitter junction of a differential pair biased by a pure current i_{BIAS} and driven with $\pm V_{\text{in}}$ on the bases is

$$V_E = \frac{kT}{e}\ln\left(\frac{2I_s}{I_E}\cosh\frac{eV_{in}}{kT}\right) \qquad \textbf{(B.4)}$$

What is the character of this nonlinearity at small signals? If the base-emitter reverse breakdown voltage BV_{EBO} is 6 V, and the transistors have V_{BE}=0.65 V at 1 mA, what is the signal range over which the nonlinearity is less than 1% of the reading?

(b) Can you suggest a way to remove the nonlinearity accurately in analogue? (Hint: you can generate the inverse of a nonlinear function using a feedback loop, but watch out for common mode signals.) (c) How about using a successive approximation technique with a matched detector and a multiplying DAC?

14.8 **BJT Amplifiers.** Consider the BJT amplifier of Figure 14.13. Show by taking partial derivatives that: (a) the small-signal dc base impedance is $r_{\text{in}}=(\beta+1)(r_E+R_E)$; (b) the dc collector circuit impedance (i.e. the total small-signal impedance from C to ground) is $r_{\text{outC}}=V_{\text{Early}}/I_C \parallel R_L$; (c) the emitter circuit impedance is $r_{\text{outE}} = (R_B+r_B)/(\beta+1)+r_E \| R_E$, (d) the common-emitter gain $(\partial V_C/\partial V_B)$ is $A_{VCE}=R_{\text{outC}}/(R_E+r_E)$, and (e) the common-collector gain $(\partial V_E/\partial V_B)$ is $A_{VCC}=R_E/(r_E+R_E)$ (ignoring Early effect). What are the numerical values of these parameters if β=90, R_E=100, R_B=50, R_C= 2k, and I_C=500μA?

14.9 **Capture Effect.** Here we investigate the details of the capture effect in an ideal limiter, which faithfully preserves the zero crossings of the signal plus spur waveform. (a) Prove that the slope condition (14.25) holds in this case, irrespective of the phases of signal and spur. (b) What happens to the capture effect if the limiter has a deadband (total hysteresis) of V_{d}, as will occur with an IC comparator?

14.10 **Common Centroid Design for Linear And Quadratic Terms.** (a) Find all common-centroid 1:1 layouts for series connections with the 8-resistor array of Figure 14.1. Are they all mirror-symmetric? (b) Under the same conditions, how would you get two matched resistors in a

2:1 ratio, where both linear and quadratic gradients are cancelled? (Ignore the gradients within each resistor element.)

(c) Give a combinatoric rule for when a series connection is common-centroid.

Problems For Chapter 15

15.1 **Noise And Distortion Budget.** (a) If we had to do the laser bug zapper over again without the on-line calibration, what would have to be different? (b) Recalculate the signal and noise level table, choosing the attenuation and LNA gains with the assumption that the worst-case compression from all sources had to be <0.5% (don't forget that the δ of two cascaded stages adds). (c) Is it still possible with the inexpensive components, or if not, which ones would you have to gold-plate?

15.2 **Calibrating A Phase Digitizer.** A calibration fixture for a phase digitizer uses two identical frequency synthesizers, driven from a common clock. One of them produces a fixed 0 dBm output, the other is settable from 0 to -50 dBm. (a) This calibrator can be made essentially perfect, with one exception; explain why. (b) How good must the isolation between sections be so that the peak phase error is less than 0.1° at any relative phase with matched loads? (c) The phase digitizer uses a phase detector, which has an LO-RF isolation of 25 dB. If the LO power is +7 dBm, how good must the impedance match on the RF input be to guarantee <0.1° phase error even at -50 dBm input? How would you make sure that the phase digitizer is insensitive to mismatches? (d) The digitizer is a nulling design, using a phase shifter to keep the phase detector output at 0 V. What are the advantages of this design?

15.3 **Fast Envelope Detection.** (a) What are the rms and worst-case scallop loss of the envelope detector of Figure 15.9 as a function of the number of phases used? (b) How are these affected by random errors in the phase steps? How about systematic errors, where the phase steps are wrong by the same factor (e.g. due to a frequency shift)? (The Carré and Hariharan algorithms for phase shifting interferometry are relevant here.) (c) Design a fast envelope detector by paralleling the outputs of several emitter-coupled detectors. What are the strengths and weaknesses of this approach? Does component matching matter?

15.4 **Frequency Compensation.** (a) If an op amp has a GBW of 1 MHz, and has one additional pole at 1.5 MHz, what is the phase margin of a unity gain inverting amp with resistive feedback? (b) If we wanted to make an integrator with a gain of 10^5 s^{-1} with a 10 kΩ R_I, what should the feedback capacitor be? What is the phase margin then? (c) If we wanted to make a differentiator with the same time constant, what is its phase margin? Can you think of a way to improve this?

15.5 **Compensating a PLL.** You have a PLL frequency synthesizer consisting of an oscillator with frequency tuning $\Omega = K_{\mathrm{VCO}} \cdot V + 10^9$ rad/s where $K_{\mathrm{VCO}} = 10^8$ rad/s/v, followed by a ÷ 16 prescaler to get it to the comparison frequency ω_c and a phase detector whose output near null is $V_\phi = K_\phi \sin(\Delta\phi)$, where $K_\phi = 1$ V/rad and $\Delta\phi$ is the phase difference between the signal and reference. (a) Ignoring the $2\omega_c$ ripple from the phase detector, show that connecting the output of the phase detector back to the VCO will produce a feedback loop with a closed-loop unity gain bandwidth of 6.25×10^6 rad/s if $\omega_{\mathrm{ref}} = 6.25\times10^7$ rad/s. (b) What is the steady-state phase error as a function of f_{ref}? (c) How much phase modulation is produced by the $2\omega_c$ ripple, in both modulation index and sideband power? (d) Add an op amp integrator between the phase detector and the VCO to reduce the phase error, and reduce the bandwidth to 10^4 rad/s. What modifications do you have to make to improve its phase margin to 45°? (e) If the VCO's tuning is nonlinear, so that its gain varies from $5{\cdot}10^6$ to $1.5{\cdot}10^7$ rad/s/v, how does this affect the phase margin and loop bandwidth? How would you change the frequency compensation to fix it?

Problems For Chapter 16

16.1 **Pigtails.** Consider a 3 m piece of RG-58A/U coax (shield diameter 5 mm), with one end connected to a 50Ω generator tuned to 100 MHz. The other end has been stripped so as to leave a 3 cm pigtail of braid, soldered at its end to the ground plane of a prototype circuit. What is the inductive reactance of the pigtail? How will that influence the operation of the circuit?

16.2 **Strays.** Give a zero-order approximation for the self-capacitance of a sphere of radius r cm in diameter, centred at the origin, when the half-space $z<0$ is full of G-10. (b) Plot the parallel-plate capacitance of a disk of radius r, a distance h above a ground plane, with a G-10 dielectric, for h=0.1, 0.3, 0.5, 1.0, and 1.5 mm. About where would you expect the error in the parallel plate capacitor formula to reach 50%? Will the true capacitance be above or below the parallel plate value?

16.3 **Tuning a Pi Network.** Using your favourite math program or spreadsheet, calculate the return loss of a lowpass LC π-network as a function of L, C_1, and C_2, for the case of 50Ω source and load. (b) Starting with a Q of 2 (i.e. $L/R^2C = 4$, where C is the series combination of C_1 and C_2 and R is R_s+R_L), and an upper 3 dB frequency of 1 MHz, tune it numerically by varying elements individually (showing plots of Γ vs. f for each choice) to a maximally-flat configuration. How many iterations does it take to get Γ to within 0.1dB of the desired Butterworth response $|H(f)|^2=1/(1+f^4/f_0^4)$?

16.4 **Error Estimate For Coax Trick.** Calculate the error sensitivities of the coax standard capacitor and open-circuit resonance methods for measuring capacitors, in terms of $\partial C/\partial f_0$. Which is more accurate? Which would you choose for prototyping, and why?

16.5 **Modelling A Pad Stack.** Use a simple aluminum foil model and a DVM with a capacitance range to measure the capacitance of a square pad over a ground plane as a function of h/W, for air dielectric.

Problems For Chapter 17

17.1 **Window Normalization.** (a) Calculate the correct normalization for a 128-point Von Hann windowed DFT which is to be used as a spectrum analyzer. (b) Repeat for a Hamming window.

17.2 **Unwindowed DFT As Filter Bank.** Show that each sample of an unwindowed DFT is really equivalent to a sinc function filter.

17.3 **Windowed FIR Filters.** Design N point FIR lowpass filters of cutoff frequency f_c=0.22ν, for N = 5, 11, 12, and 25. Use both rectangular and von Hann windows (the points where the window is 0 don't count). Compute and plot their transfer functions and step responses on common scales. Discuss the effects of N and the choice of window on the passband accuracy, stopband rejection, f_c accuracy, and the transition band width.

17.4 **Numerical Jitter Experiment.** Perform a numerical investigation of the effects of sampling jitter on the computed spectrum of a pure sine wave input. Use your compiler's random number generator to compute samples of sin νt/4, where the samples have uniformly distributed p-p jitter of 1%, 5%, 10%, and 25% of the sampling period. Compute properly windowed DFTs, and plot the signal power and total noise power in the spectrum vs. p-p jitter. How do these vary with the jitter? How would you characterize the degradation behaviour in a sentence?

17.5 **OMA Spectrometer.** Consider an OMA (optical multichannel analyzer) spectrometer, which produces 512 output channels corresponding to the total optical power hitting each CCD element. Assuming that the CCD has rectangular pixels and a 100% fill factor (i.e. no spaces between), the CCD covers exactly 1 octave in frequency, the optical spectrum is equally dispersed in wavelength terms, and the 512 samples are oversampled by a minimum of 2×, resample it to 512 equally spaced frequencies. Make sure you preserve the local optical energy in the process. How many DFT samples does it take? How much of the red and blue ends do you lose if you require that aliasing is kept below 10^{-3} of full scale under all conditions?

Problems for Chapter 18

18.1 **Capacitance Multipliers.** Consider an instrument based on ac detection of a 100 kHz signal from an electronically chopped super-bright LED. Chopping the LED puts a 100 mV rms triangle wave on the power supply, and the total coupling capacitance from the supply into the front end is about 0.05 pF (the photodiode bias is filtered already). (a) If the total transimpedance is 10^{10} Ω, from a 100 kΩ transimpedance amp feedback resistor and a second stage gain of 10^5, what is the resulting spurious output voltage? If the measurement bandwidth is 25 Hz, how good a filter is needed on the supply to bring the spurious signal to 10 dB below the Johnson noise? (b) If the total front end supply current is 12 mA, design a capacitance multiplier that will do the job with a 10 dB safety margin.

18.2 **Constant Resistance T-Coil.** (a) Using the definition of mutual inductance, show that the negative-inductor circuit model for the coupled tapped coil is correct. (b) Give analytical expressions for the transimpedance, input impedance, and output impedance of the constant-resistance T-coil of Figure 18.19. What exactly is the constant-resistance property, and where does it apply? (c) Use your favourite math program to exhibit the bandwidth and settling time improvement claimed. Plot the improvement in settling time vs. Q for fixed R_L and C_d.

18.3 **Ultralow-light Detection.** (a) Assuming that the input bias current of a FET op amp is Poissonian, that the low frequency voltage noise is pure $1/f$, that temperature drift is indistinguishable from low frequency noise, that the feedback resistor has only Johnson noise, and that the shunt resistance of properly selected photodiodes is 1-50 GΩ at 20°C, what is the best low light performance obtainable in a photodiode infrared photometer using a Burr-Brown OPA-128 with a 10-30°C operating range? (b) What is the best dynamic range available, assuming excellent feedback resistor accuracy? (c) How much better does this get if we temperature stabilize the detector and amplifier to ± 0.1°? (d) And how does this compare with a room temperature PMT?

18.4 **Struggle for SNR.** Using the techniques of this chapter, design a front end that reaches the shot noise from dc to 50 MHz, and has a bandwidth of at least 100 MHz, using a photodiode with C_d=3 pF and I_d>50 μA. You don't need to provide a 50 Ω output, but do need to tolerate a second stage with 2 pF ∥ 20 kΩ input impedance and noise of 5 nV.√Hz and 0.2 pA/√Hz. (Hint: it needs 3 or fewer transistors.)

18.5 **Noise Canceller Improvements.** (a) Show that the negative feedback caused by the extrinsic emitter resistance $R_{E'}$ degrades the current splitting accuracy, but that it can be compensated by applying positive feedback to the bases of the differential pair. (b) How can you construct the necessary feedback voltage, given that you can't interfere with the summing junction of A_1? (Hint: use a current mirror, and exploit the fact that it's only ΔV_{BE} that matters.) Tricks like this are used in logarithmic amplifier modules. (c) What is the residual error due to higher order terms as a function of I_{sample} and I_{signal}?

Problems For Chapter 19

19.1 **Transfer Function for the Set Point.** The transfer function (20.15) connects heating power with temperature. What is the transfer function relevant to changes in the set point? Why?

19.2 **Diode Laser Temperature Controller.** Using (20.15), make a rough design of a temperature control loop for the common-centroid diode laser mount of the example. Assume that the TE coolers are 15x15mm, have the same per-area performance as the Ferrotec units in the other example, and cover nearly the whole back surface of the 8 mm thick 6061 aluminum cold plate. The temperature sensors are glass thermistors (thermal TC ≈ 1s) centred approximately 1.5 mm from the rear of the plate. Make sure the loop has a gain margin of at least 10 dB, and has infinite dc gain. How much better would a thin film thermocouple with a 10ms thermal TC be?

19.3 **Vacuum Insulation.** Find the "thermal resistance" of a 1 cm vacuum gap between two black bodies at nearly the same temperature. At what temperature does the vacuum conduct as well as 25°C air? As 25°C OFHC copper?

19.4 **Real Dewars.** A 1 litre Dewar has an interior surface area of 350 cm^2, and negligible excess losses through the lid and neck and through conduction by residual air. (a) What is the thermal time constant of a Dewar full of water, as a function of T_{hot}, T_{cold}, and ε? For the metal coatings on the two walls, ε is 0.02 to 0.05; in your experience with Thermos jugs, is this consistent with our assumption that heat transfer via the lid, neck, and residual air are negligible?

(b) Would it help to put additional shields inside the evacuated space?

19.5 **Two-Stage TEC.** Design a two-stage TEC, using the device of the example on the bottom and another of the same type but adjustable area on top. Let T_{sink}=25°C always. How low in temperature can you go? What is the cooling capacity at -50°C?

19.6 **InSb On A Cold Finger.** An InSb quadrant cell is mounted on a 4-stage TEC, which has almost no $\dot{Q}_{\max}$ at all, and is barely cold enough for the application. How would you connect the necessary 5 wires, and why?

19.7 **Thermal Mass and Heat Diffusion.** Show that the low-frequency limit of (20.15) gives the same frequency response we would expect from (20.16), and thus that the idea of thermal mass is consonant with our diffusion equation formulation.

19.8 **Lateral Temperature Drops.** Use a numerical or variational technique of your choice (e.g. Galerkin or collocation) to investigate lateral temperature drops in a 40 mm square alumina cold plate, 0.6mm thick, with a concentrated heat load in the top centre and a uniform heat flux across the bottom. Repeat with a 5mm thick piece of 1100-T0 aluminum. How much does the spreader plate help?