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The few SPICE models of ultra fast P-i-N photodiode

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This paper considers SPICE modeling of the P-i-N photodiode response. Consideration is carried out accounting changes of bias voltage and of carriers velocities. We analyzed SPICE photodiode response in the case of three different input light excitations: Dirac's, Heaviside's and sinusoidal excitation. Results show good agreement with analytically derived results and prove correctness and efficiency of these SPICE models.

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1. Introduction

With the aim of establishing photodiode response arithmetically, a method was developed according to which photodiodes are represented by equivalent electric circuits [1], which can subsequently be calculated by means of *SPICE* program [2].

The existing literature which deals with the subject of electric model of photodiode seems to have given insufficient attention to the problem of the influence of photodiode voltage on the transport of carriers [3], [4]. In order to obtain more accurate information on how the change of photodiode voltage influences the transport of carriers, in depletion region of P-i-N photodiode, we included dealing with function $\Phi(t,\tau(t))$ where $\Phi(t)$ itself is time function of generating signal, and $\tau(t)$ is the time delay dependant on photodiode voltage. In order to include such effect in the *SPICE* simulator, which allows work with linear time transmission functions, it was necessary to create different equivalent electric models of photodiodes. In their basis, these different models are based on the completely same electrical scheme, where different forms of time functions are used for independent power and voltage sources (in subcircuits). Therefore, to determine integral effect of voltage influence on the time delay $\tau(t)$, we used a series of subcircuits in which value of $\tau(t)$ parameter changes, and summing the values from series of subcircuits we come to the total realized effect of photodiode voltage on the transport of carriers. This paper presents the results of *SPICE* analysis for Dirac's, Heaviside's and sinusoidal excitation, as the most interesting forms of time input excitations. First results were used for comparison with analytically derived results. Therefore, in our calculations we took the constant value for carriers' mobility, and after that the more realistic expression for the velocity of the electrons given by form (4) from [5] was introduced in the program.

2. Equivalent model

We considered P-i-N photodiode standard electric circuit as in [6]. We made several assumptions: 1) the width of the *P* region is much smaller than the width of the *i* region; 2) the diffusion current in the *P* region is neglected but in the *N* region it is taken into account; 3) there is no space charge effect; 4) the dark current is neglected; and 5) nonlinearity is the result of the voltage drop on the load resistance. All these assumptions are valid if we deal with weak incident light signal as in [4] and [6].

In creation of photodiodes equivalent electric circuit we start from the equation of the electric circuit:

$$U(t) = V_{CC} - RI(t). \tag{1}$$

The above expression describes the change of the photodiode voltage due to the flow of the photocurrent. In the above equation V_{CC} is bias voltage, R is load resistance and $I(t)$ the total current flow.

For Dirac's light input excitation:

$$G_{op}(x,t) = \alpha I \delta(t - t_0)\exp(-\alpha x), \tag{2}$$

we can obtain the mean value for carrier concentrations in *i* region and diffusion current on the border of *i-N*:

$$\langle n\rangle = \frac{I}{d}\left(1-\exp(-\alpha d)\exp\left(\alpha \upsilon_n (t-t_0)\right)\right), \tag{3a}$$

$$\langle p\rangle = \frac{I}{d}\left(\exp\left(-\alpha \upsilon_p (t-t_0)\right)-\exp(-\alpha d)\right), \tag{3b}$$

$$I_{pdif} = qSD_p\alpha^2 I\exp(-\alpha d)\exp\left(-\gamma(t-t_0)\right), \tag{3c}$$

where: $\gamma = \left(1 - L_p^2\alpha^2\right)/\tau_p$ .Here $G_{op}(x,t)$ represents the optical generation function, I is the intensity of incident excitation, α is the absorption coefficient, d is the width of *i* region, τ_p is the hole life time in *N* region, L_p is the hole diffusion length in *N* region, end $\upsilon_{n,p}$ are velocities of electrons and holes in *i* region. In above equations t_0 represents the time moment when impulse appears.

The photon flux of any arbitrary excitation can be presented as:

$$\Phi(t) = \int_0^{\infty} \Phi(t_0)\delta(t - t_0)dt_0\,, \qquad (4)$$

whereas the generation rate is given as :

$$G_{op}(x,t) = \alpha\Phi(t)\exp(-\alpha x)\,. \qquad (5)$$

We can find the necessary concentrations for arbitrary excitation using already derived results for Dirac's excitation [6] and using variable $\tau = t\text{-}t_0$.

$$\langle n\rangle = \frac{1}{d}\int_0^{\frac{d}{\upsilon_n}} \Phi(t-\tau)\left(1-\exp(-\alpha d)\exp(\alpha\upsilon_n\tau)\right)d\tau\,, \qquad (6a)$$

$$\langle p\rangle = \frac{1}{d}\int_0^{\frac{d}{\upsilon_p}} \Phi(t-\tau)\left(\exp(-\alpha\upsilon_p\tau)-\exp(-\alpha d)\right)d\tau\,, \qquad (6b)$$

$$I_{pdif} = qSD_p\alpha^2\int_0^{t}\Phi(t-\tau)\exp(-\alpha d)\exp(-\gamma\tau)d\tau\,. \qquad (6c)$$

It should be noted that we don't work with constant parameter τ but with $\tau(t)$ which depend on $U(t)$. Also, velocities are not taken as constants but they are calculated from the expression:

$$\upsilon_{n,p} = \mu_{n,p}\,\frac{U(t)-V_d}{d}\quad , \qquad (7)$$

where V_d is the punchtrough voltage for photodiode and $\mu_{n,p}$ are mobility of electrons and holes respectively.

Total current flow through the photodiode is given as:

$$I(t) = Sq\left(\langle n\rangle\upsilon_n + \langle p\rangle\upsilon_p\right) + I_{pdif} + R\,\frac{\varepsilon S}{d}\frac{dU(t)}{dt}\,. \qquad (8)$$

The last member in the above equation represents the displacement current.

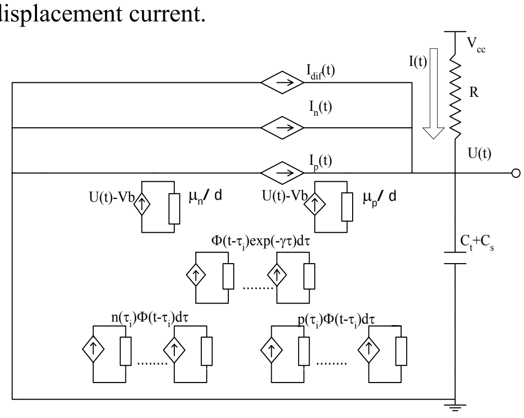

Fig. 1. The equivalent electric circuit for different light excitations of the P-i-N photodiode. The independent generators in the subcircuits depend on the time function of flux of the input light excitation Φ.

Now we should determine mean values for concentrations given in equations (6a), (6b) and (6c), which are reduced to summing per series of m subcircuits:

$$\langle n\rangle = \frac{1}{d}\sum_0^{d/\upsilon_n}\Phi(t-\tau_i)\left(1-\exp(-\alpha d)\exp(\alpha\upsilon_n\tau_i)\right)\Delta\tau \qquad (9a)$$

$$\langle p\rangle = \frac{1}{d}\sum_0^{d/\upsilon_p}\Phi(t-\tau_i)\left(\exp(-\alpha\upsilon_p\tau_i)-\exp(-\alpha d)\right)\Delta\tau\,, \qquad (9b)$$

$$I_{pdif} = qSD_p\alpha^2\sum_0^{t}\Phi(t-\tau_i)\exp(-\alpha d)\exp(-\gamma\tau_i)\Delta\tau \qquad (9c)$$

Here we introduce variable parameter τ_i which varies at each moment and obtains value:

$$\tau_i = \frac{i}{m}\frac{d}{\upsilon_{n.p}(t)}\,. \qquad (10)$$

Here i represents the serial number of a subcircuit and m represents the total number of subcircuits. The member $\Delta\tau$ depends on the total number of subcircuits m. It is obvious that for the different input excitation $\Phi(t)$ we use different function $\Phi(t\text{-}\tau(t))$ in subcircuits. From equations (1) and (8) we create the main electric circuit presented in fig 1. It contains two drift currents $I_n(t)$ and $I_p(t)$, and one diffusion current $I_{dif}(t)$ presented as current sources, the independent generator V_{CC} , load resistance R and summary capacitance of C_t = $\varepsilon S/d$ and parasitic capacitance C_s. The subcircuits from the first series in the equivalent electric circuit solve members from the sum in (9a). The remaining two series of the subcircuits solve the sums in (9b) and (9c). The two remaining subcircuits calculate the velocity of electrons and holes given by equation (7) for the first results, and later we use form (4) from [5] for the velocity of electrons.

3. Results and discussion

First results are obtained for Dirac's input excitation, without the subcircuits for integration, for parasitic capacitance C_s = 1pF, and they are shown in fig. 2. In these calculations we used next parameters of the *GaAs* photodiode: $\alpha = 10^4\text{cm}^{-1}$, $d = 5\mu\text{m}$, $\mu_n = 7500\ \frac{cm^2}{Vs}$, $\mu_p = 420\ \frac{cm^2}{Vs}$, $R = 50\Omega$, $V_d = 0.6\text{V}$, and $V_{CC} = 5\text{V}$. Dielectric constant is $\varepsilon_r = 12$, photodiode area $S = 700\mu\text{m}^2$, and the wavelength of input excitation is $\lambda = 0.8\mu\text{m}$.

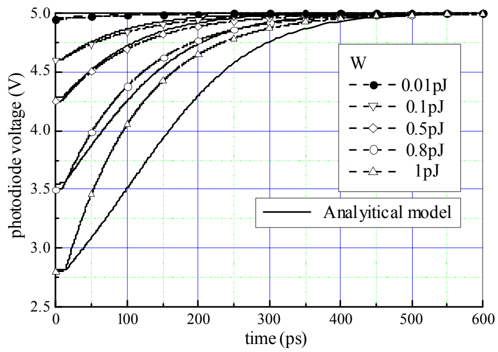

Fig. 2. The photodiode response on Dirac's excitation for the case of constant mobility of electrons. Comparison of SPICE results and analytically derived results [6]. Parasitic capacitance is C_s = 1pF.

The *SPICE* results for Dirac's light excitation show good agreement with the analytically derived results for smaller incident energies [6]. Then we repeated calculations for the same parameters except that we used form (4) from [5] for velocity of electrons. The results are presented in Fig. 3. It should be pointed once again that velocities change their values due to change of photodiode voltage – change of electric field.

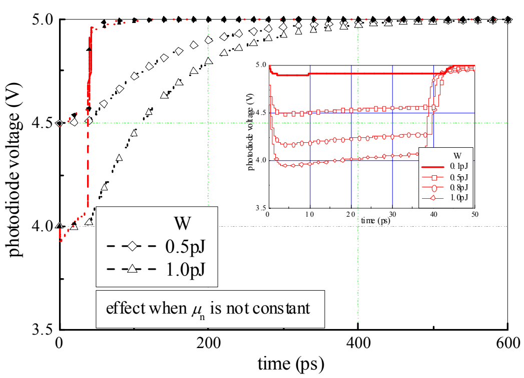

Fig. 3. The photodiode response on Dirac's excitation obtained with form (4) from [5]. Parasitic capacitance are C_s = 1pF and C_s = 0pF. Small picture presents the same results shown in another dimension.

There is more than obvious difference in the obtained results which is the consequence of the fact that now we use more realistic value for the velocity of electrons. The same figure shows results obtained for parasitic capacitance C_s = 0, when the time response of the photodiode has minimum value.

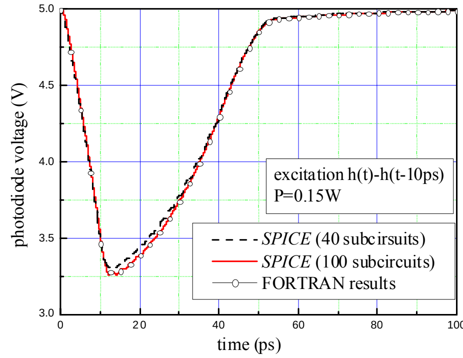

Fig. 4. The photodiode response on Heaviside's excitation. Comparison of SPICE results with 40, SPICE with 100 subcircuits and FORTRAN results.

For the case of Heaviside's time function of light excitation the arbitrary function Φ becomes Heaviside's function. We include it into the subcircuits and we obtain the *SPICE* simulation results. Fig. 4 presents photodiode responses for Heaviside's excitation obtained by *FORTRAN* simulation and *SPICE* simulations. We used 40 subcircuits and 100 subcircuits for simulation of integration in *SPICE*, whereas in *FORTRAN* we used step $d\tau$ = 0.01ps. Differences between *SPICE* and *FORTRAN* disappear when the number of subcircuits is increased. Due to the results, we suppose that it is sufficient to integrate with 100 subcircuits and in that case we can say that *SPICE* results are accurate.

The importance of taking into account the effect of changeable velocity is shown in Fig. 5.

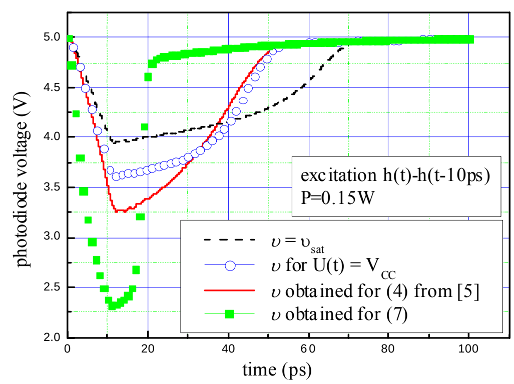

Fig. 5. The photodiode response on Heaviside's excitation. Dashed curve represents the results obtained from the velocity of electrons taken as constant equal to saturation velocity. Circular curve is obtained when we take form (4) from [5] but counting U(t) = V_{CC}. Full curve is obtained for (4) from [5] where we take into account change of U(t). The last curve is obtained for constant mobility of electrons.

The significant differences in time response and the minimum values for the photodiode voltage appear as a consequence of different values for velocity of electrons. Due to our consideration the most realistic value for the photodiode response is obtained when we take form (4) from [5] and we take into account the effect of change of velocity due to change of photodiode voltage – presented by the full curve.

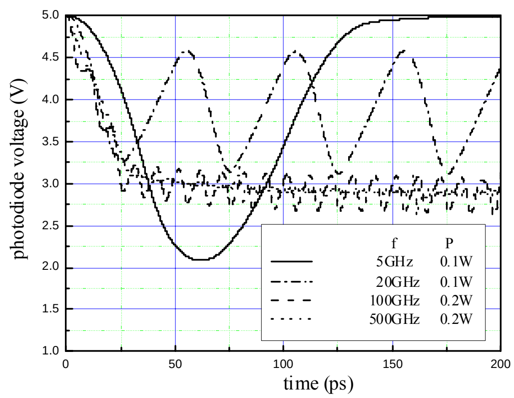

Fig. 6. The photodiode response obtained by SPICE on sinusoidal excitation for different frequencies of input signal.

Finally, fig. 6 shows response of the photodiode on the sinusoidal light excitation obtained by *SPICE*. In that case arbitrary function becomes:

$$\Phi(t) = \Phi_0 \sin(\varpi t) \text{ for } \sin(\varpi t) > 0,$$

$$\Phi(t) = 0 \text{ for } \sin(\varpi t) \le 0. \qquad (12)$$

Because photodiode has minimum time response of τ_r = 40ps (we consider time response as a time when photodiode voltage rises up to the value of 0.95 V_{CC}), for the intensity of input energy W = 1pJ, responses for input signal frequencies higher then 25GHz will not be apparent. So the photodiode responses for the input frequency of about f = 5GHz, which is much lower than photodiode cut-off frequency, gives expected behavior. For the frequencies about f = 20GHz output signal still follows the sinusoidal curve but doesn't reach the bias voltage. For higher frequencies, about f = 100GHz, diode response oscillates around an indefinite middle value. For extreme frequencies like f = 500GHz this particular photodiode is saturated.

4. Conclusions

Suggested equivalent electric circuit comprises all significant physical processes and can be simulated by *SPICE* program. In a simplified manner, the effect of the change of carrier's velocity is included in the circuit. As a result, generators in equivalent electric circuits depend on the type of input excitation. If we want to decrease error of integration process we must increase the number of subcircuits and, similar to *FORTRAN*, increasing the number of subcircuits decreases velocity of the program. Because of that, it is necessary to find the compromise between accuracy and speed. Our opinion is that *SPICE* can be efficiently used in calculating the photodiode response as well as it is used for laser diodes.

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