Shot Noise Limited Optical Measurements at Baseband with Noisy Lasers

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ABSTRACT

This paper describes a simple, all-electronic noise cancellation scheme which allows wideband, shot noise limited optical measurements at baseband, with noisy lasers, in many kinds of optical systems. With this system, it is usually possible to achieve the performance of a complex heterodyne system with a much simpler homodyne approach. Although it is similar to earlier differential and ratiometric techniques, its noise cancellation performance is much better, and it is highly effective at modulation frequencies up to tens of megahertz. The basic idea is to subtract photocurrents directly, under feedback control, to cancel excess noise (i.e. noise above the shot noise level) and spurious modulation of the beam. A sample is split off from the beam at the laser and detected with a photodiode similar to the main detector at the system output. Most optical systems and detectors have very wide temporal bandwidths and excellent linearity; thus at all frequencies of interest, the sample photocurrent has exactly the same instantaneous fractional excess noise fluctuations as the laser beam itself, with no differential gain or phase. If a fraction of the sample photocurrent is subtracted from the main detector output, with feedback controlling the division ratio to keep the DC component of the result at zero, the excess noise cancels identically. The actual noise cancellation bandwidth is very wide, and does not depend on the feedback bandwidth, only on that of the differential bipolar transistor current divider. Two noise-cancelled outputs are available, one a high-pass filtered version of the signal imparted by the optical system, the other, a low-pass filtered voltage related to the log ratio of the intensities of the two optical beams. The noise floors of the outputs depend only on the shot noise of the signal beam. Measurements using a prototype of the device are presented which demonstrate this performance.

2. INTRODUCTION

Excess noise, spurious modulation, and power drift in lasers are common problems in optical measurements. In gas lasers, the noise levels can easily reach 50 dB above shot noise, even quite far from DC. Since the noise and spurious signals are usually worst at low modulation frequencies, most high precision optical measurement systems apply some sort of modulation to the beam, thus making their output signals periodic in time at a frequency as far from the low-frequency noise as necessary. One typical example is a heterodyne interferometer, in which an acousto-optic cell is used primarily to escape baseband noise; the gain associated with heterodyne systems is also available with a homodyne system, as long as baseband noise does not swamp the measurement. Another one is chopping the beam, followed by lock-in detection to recover the desired signal while rejecting the excess noise; this is effective at eliminating the out-of-band noise, but does nothing for that in-band (in this paper, "excess noise" refers both to true noise above the shot noise level, and to spurious modulation of the beam intensity, due for example to mode beating or power supply ripple). The idea is to make the laser noise enter the measurement as a gain term rather than an additive term.

Drift and low-frequency noise are dealt with by allowing the laser to stabilize itself through long running (often ineffective), or by taking a sample of the output beam, and applying negative feedback to the laser operating current or to an external optical attenuator to keep the sample photocurrent constant. Some of these techniques work fairly well, but except with diode lasers, they tend to be complicated or expensive and can at best bring the signal-to-noise ratio of the resulting beam up to the signal-to-shot noise ratio of the sample beam. Since the sample beam is usually appreciably weaker than the main one, and thus contains relatively more shot noise, this may result in poor performance. In addition, since the noise suppression depends on the operation of feedback, the effective bandwidth of these systems is rather small. For example, if 40 dB noise suppression is needed to reach the noise floor, then the closed-loop gain error must be less than 1%; assuming an open-loop unity gain bandwidth of 0.5 MHz and a 1-pole rolloff, this occurs at 5 kHz.

3. ALL-ELECTRONIC NOISE SUPPRESSION

All-electronic schemes have been known for some time as well[1], but have not been widely used. These schemes differ from those discussed above in that no attempt is made to stabilize the laser beam itself, only the photocurrent at the output of the experiment. In them, the beam is sampled at the laser to obtain a photocurrent with the same fractional excess noise as that from the signal beam (representing the output of the optical system); the two are then combined in some fashion to obtain an output current which, ideally, is completely free of excess noise.

Such methods rely on two important properties of most optical systems, namely extremely wide temporal bandwidth and the use of highly linear photodetectors, such as photodiodes operated in the photoconductive mode. The wide bandwidth guarantees that if path delays are small, the optical system will not apply any differential gain or phase to the modulation of the beam—the instantaneous fractional excess amplitude noise of the sample beam will be identical with that of the signal beam—so that if the cancellation is done properly, the noise suppression should be essentially perfect. The photodiode linearity allows excellent cancellation performance with unmatched diodes, even if they are running at appreciably different current densities. With such fortunate circumstances, all-electronic noise cancellation should be extremely effective.

In the past, two all-electronic methods have been common, namely subtraction[2-4] and division. In a subtractive noise rejection scheme, the sample photocurrent is subtracted from the signal current. If the optical system is adjusted perfectly so that the two photocurrents are exactly equal, the noise and DC cancel, leaving only the signal. Subtractors can have wide bandwidths, since the photocurrents can be subtracted directly without prior conversion to voltages, and since the noise cancellation does not depend on feedback. Often, adding a properly adjusted subtractor can improve the excess noise of a system by 20 dB or so. The improvement is seldom more than that, because the adjustment required is finicky, and because the intensity of the signal beam usually varies somewhat during a measurement, so that the currents cannot be exactly equal at all times. In addition, since the shot noise currents of the signal and sample photocurrents are uncorrelated, both contribute to the noise of the output signal, and thus limit the system noise floor to 3 dB above the shot noise of the signal beam alone.

Dividers avoid the requirement for precise adjustment by dividing the signal current by the sample, hence cancelling the fractional (rather than absolute) noise deviations. They have great attractions in theory, since dividing out the instantaneous intensity provides compensation for drift as well as for excess noise. Dividers in principle need not suffer from the 3 dB additional noise problem of subtractors, since the sample beam can be made stronger than the signal beam (and so relatively quieter).

Unfortunately, dividers tend to be slow, so that the suppression bandwidth is limited, and are very noisy. They are inconvenient to use during the setup of the optical system, as one must supply a sample (denominator) voltage if they are to function at all. One of the best dividers available is the Burr-Brown MPY-634, which with a full scale (10 V) denominator, has a noise spectral density of about 1 $\mu V/\sqrt{Hz}$ with a zero numerator, and 2 $\mu V/\sqrt{Hz}$ with a 10 volt numerator, about 60 dB worse than the best operational amplifiers. Since its maximum input level is 10 V, its maximum signal to noise ratio (SNR) is 134 dB in 1 Hz bandwidth. With appropriately chosen current-to-voltage converter (transresistance amplifier) gain, and if the signal level is reduced by 3 dB to avoid clipping, this is equivalent to the signal to shot noise ratio of a photocurrent of 8 μA. With a red helium-neon laser and a silicon PIN diode (0.3 A/W), that implies that a 27 μW laser beam can be quieted to 3 dB above the shot noise, but a 3 mW beam, well within the linear operating regime of most photodiodes, can be quieted only to 20 dB above the shot noise, a poor performance. Dividers based on multipliers have poor high frequency performance; swept-sine measurements (0.2 V p-p plus 7 VDC on both numerator and denominator) on the unit mentioned above reveal 42 dB small signal suppression at 100 Hz, deteriorating steadily above about 5 kHz, to 22 dB at 100 kHz and virtually zero at 1 MHz.

Although both subtractors and dividers remove the additive excess noise, one difference between them deserves emphasis. Since any linear optical measurement is ultimately an extinction measurement (the signal level is proportional to the optical excitation), level fluctuations in the excitation beam intermodulate with the desired signal. Subtractors eliminate the additive excess noise, but can do nothing to suppress this noise intermodulation; dividers in principle eliminate both. For this reason, open-loop subtractors are most suitable for measurements of small changes in intensity, with lasers which are already reasonably quiet.

For some purposes, then, dividers and open-loop subtractors can be very useful, but in general their use does not allow truly shot-noise limited measurements with noisy lasers.

4. FEEDBACK NOISE CANCELLER

While noise, nonlinearity and slowness fundamentally limit dividers, the open loop subtractor can be improved substantially in many areas by the use of negative feedback. The DC balance problem is solved by arranging the optical system so that the sample beam is somewhat stronger than the signal beam, and dividing the sample photocurrent with a bipolar transistor pair as shown in Figure 1. The ratio of the collector currents of the two transistors Q1 and Q2 is related to the difference $\Delta V_{BE} = V_{BE2} - V_{BE1}$. in their base-emitter voltages (in the Ebers-Moll model) by

$$\frac{I_{C2}}{I_{C1}} = \exp(\frac{q\Delta V_{BE}}{kT}) , \qquad (1)$$

where q is the electron charge, T is absolute temperature, and k is Boltzmann's constant; thus by controlling this voltage, the current cancellation can be adjusted electronically. For constant, equal collector to emitter voltages V_{CE}, and using devices with good log conformance, this ratio is constant over several decades of collector current.

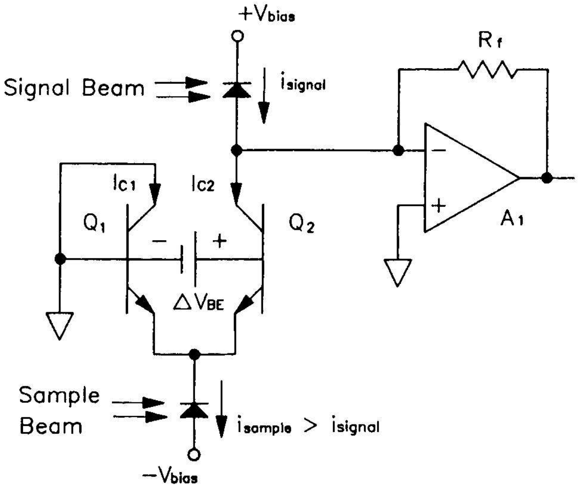

Fig. 1: Simplified schematic diagram showing the use of a bipolar transistor differential pair as a variable current divider. ΔV_{be} controls the current division ratio, and the ratio is independent of the magnitude of the sample photocurrent.

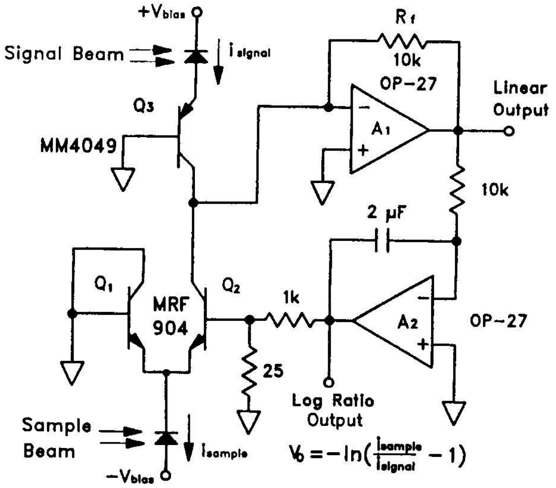

Fig. 2: Schematic diagram of a fully functional version of the feedback noise canceller. Transistors Q1 and Q2 form a current divider, controlled by servo amplifier A2 so that the DC level at the output of transresistance amplifier A1 is zero. This ensures that the signal and divided sample photocurrents are equal, and thus that the noise cancels identically.

Figure 2 shows a prototype noise canceller using negative feedback to keep the division ratio adjusted exactly. Besides the differential pair Q1-Q2, there is a transresistance amplifier A1 which converts the photocurrent to a voltage, and an integrating servo amplifier A2 which forces the output of A1 (and hence the current into A1's summing junction) to be zero. Cascode transistor Q3 prevents the capacitance of the signal photodiode from loading the summing junction. A1's output is a high pass filtered version of the signal photocurrent, minus the excess noise; its corner frequency is the 3 dB bandwidth of the feedback loop. Cancelling the DC photocurrent allows the use of large feedback resistors without saturating amplifier A1, and hence gives shot-noise limited performance at lower optical power levels than an ordinary transresistance amplifier. For pedagogical reasons, the circuit of Figure 2 does not exploit this advantage (see the section on performance). If the feedback resistor is made larger than 40kΩ, A1 should be replaced with a FET type (e.g. LF356A).

The exponential dependence of collector current on ΔV_{BE} is crucial, since it guarantees that the transconductance of the transistors is proportional to their collector currents, so any fluctuation in the sample photocurrent gets divided in exactly the same ratio as the DC, irrespective of feedback loop bandwidth, and hence that the cancellation bandwidth is independent of feedback loop bandwidth.

Since the instantaneous excess noise fluctuations of the photocurrent are exactly proportional to its DC level, as discussed above, application of negative feedback to one of the transistor bases to keep the net DC photocurrent at zero results in essentially perfect noise cancellation out to very high frequencies, regardless of the bandwidth of the feedback loop. As an added benefit, the feedback voltage offers an alternative output; since ΔV_{BE} is related to the ratio of the sample current to the signal current (within the feedback bandwidth) by the Ebers-Moll equation (1), then

$$\Delta V_{BE} = \frac{kT}{q}\ln\left(\frac{I_{sample}}{I_{signal}} - 1\right) . \tag{2}$$

If the feedback signal is used as the output, the performance is much like that of a divider, in that intermodulation between noise and signal is suppressed (V_{BE} depends only on the ratio of the two photocurrents). One important difference is that the new system does not get noisier as its loop bandwidth is approached, since the DC cancellation guarantees the cancellation of additive noise at all frequencies of interest; only the suppression of noise intermodulation declines. There is nothing in the system which forces the feedback loop to be slow. If only the logarithmic output is desired, A1 can be eliminated, and the photocurrents delivered directly to the summing junction of A2.

The transistors can be very fast; matched transistors are available with current gain bandwidth products f_T of several gigahertz, so that the noise cancellation can be excellent even at high modulation frequencies. If the utmost temperature stability of the logarithmic output is not required, the transistors need not be matched; unmatched transistors with 5 GHz f_t were used in the prototype.

This system is intrinsically very quiet; on the A1 output, outside the loop bandwidth, its total equivalent input noise current is

$$i_{nT} = \sqrt{\frac{e_{nA}^2}{R_f^2} + i_{nA}^2 + i_{nSignal}^2 + \beta i_{nSample}^2 + i_{nR}^2} \quad , \tag{3}$$

where e_{nA} is the RMS noise voltage of the amplifier, R_f is the feedback resistance, β is the current division ratio, and i_{nA}, i_{nR}, $i_{nSample}$, $i_{nSignal}$, and are respectively the RMS current noise of the amplifier and feedback resistor, and the shot noise of the sample and signal photocurrents. Normally the noise will be dominated by the resistor noise for small currents and by shot noise for larger ones. Inside the loop bandwidth, at the A2 output, the noise is a similar combination of shot noise and the input noise voltage of the divider transistors.

5. PERFORMANCE

Figures 3 and 4 show the performance of the prototype noise suppressor of Figure 2. Q1 and Q2 were MRF 904s, and Q3 was an MM4049, all having f_Ts of 4 GHz or more. The feedback bandwidth was about 10 Hz. The laser used here was a multiple longitudinal mode, 2 mW helium-neon unit which exhibited severe intermittent mode beating. The mode beating could be started and stopped by cooling the laser housing by hand. The signal and sample photocurrents were 170 μA and 280 μA, respectively; output was taken from A1, and the feedback resistor R_f was 4.76 kΩ. The expected shot noise current spectral density i_{ns} of a photocurrent is given by

$$i_{ns} = \sqrt{2qi} \ , \tag{4}$$

in this case 7.4 pA/$\sqrt{\text{Hz}}$ from signal current shot noise alone.

Figure 3 shows the performance of the feedback noise canceller. In each plot, the the output noise with the canceller running is compared with the noise with the sample beam blocked, so that the circuit operated as an ordinary transresistance amplifier. Figure 3(a) is an averaged plot of the input-referred noise spectral density during a quiescent period (no mode beating). The flatband noise spectral density after cancellation is 11.2 pA/$\sqrt{\text{Hz}}$, in excellent agreement with the expected value of 7.4 $\sqrt{2}$ pA/$\sqrt{\text{Hz}}$ $\simeq$ 10.5 pA/$\sqrt{\text{Hz}}$. Figure 3(b) is a plot of 100 frequency sweeps taken with an FFT analyzer while mode beats passed through zero frequency, with the MAX HOLD enabled, to accumulate a crude envelope of the spurious peaks. It shows real-world noise suppression of 50 dB without adjustments.

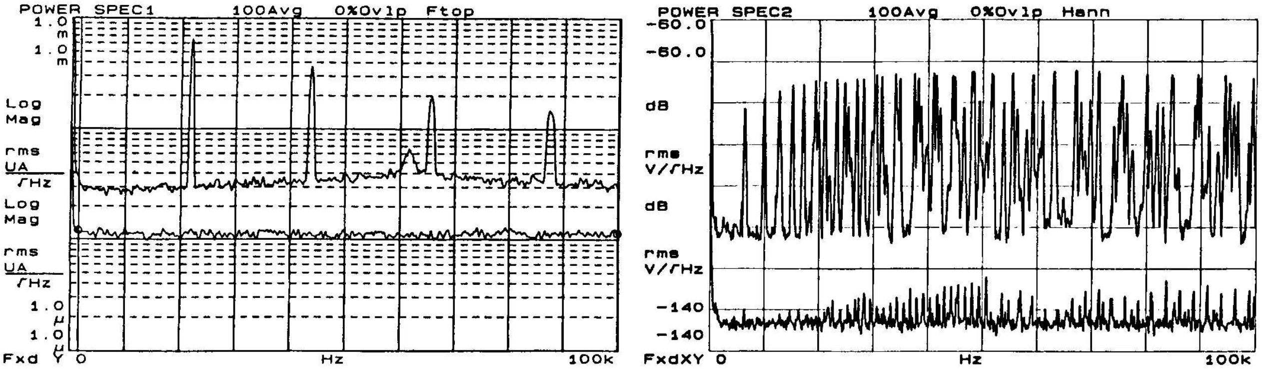

Fig. 3: Performance of the feedback noise canceller in actual use with a 2 mW He-Ne laser exhibiting severe intermittent mode beating and considerable 22 kHz power supply ripple. In each plot, the two curves show data taken with cancellation operating and with the sample beam blocked, so that the device was operating as an ordinary transresistance amplifier.

(a) noise during a quiescent period. Noise after cancellation was 11.2 pA/$\sqrt{\text{Hz}}$, 0.6 dB above the theoretical shot noise value.

(b) mode beat passing through zero frequency; the maximum values at each frequency from 100 FFT analyzer sweeps are shown. Noise suppression is better than 50 dB.

Figure 4 is a swept-sine plot of the ultimate cancellation performance of the prototype. The beam was amplitude modulated by 30% before splitting, using a sine wave oscillator driving a Pockels cell, and an analyzer prism to convert the polarization shift to AM. The data is from a spectrum analyzer; one sweep was taken with the canceller operating, and a second with the sample beam blocked. The plot shown is the ratio of the two.

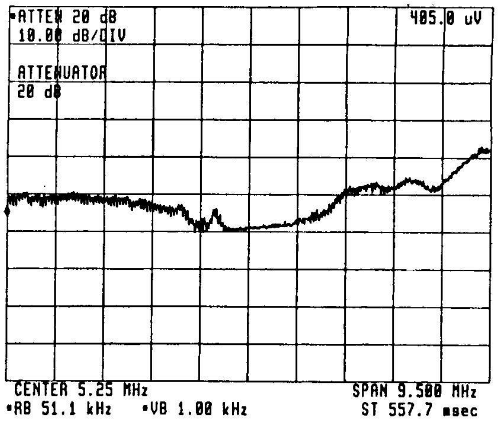

Fig. 4. Swept sine measurement of the ultimate cancellation performance of the prototype with large (30%) intensity modulation. The curve shown is the ratio of sweeps with the sample beam blocked and unblocked. Cancellation is better than 40 dB up to 10 MHz, although the feedback loop bandwidth is only 10 Hz.

Although the bandwidth of the amplifier was only 3 MHz, and that of the feedback 10 Hz, the ultimate cancellation was better than 50 dB up to 7 MHz, and 40 dB up to 10 MHz. As the frequency increases, the voltage swing at the summing junction of A1 increases, causing saturation of Q2 and Q3 on the signal peaks and hence poorer cancellation. Another version using current feedback amplifiers in a slightly different configuration is under development, with a design goal of 100 MHz bandwidth (-3 dB).

It should be noted that the performance level reached in this circuit is within 3 dB of that of an ideal heterodyne system, and is superior to that of an ideal chopped system, since choppers exhibit 3 dB additional loss due to a 50% duty cycle. A variation of this circuit which eliminates the 3 dB penalty is under development, and promises to allow this method to actually achieve the performance of the theoretical ideal heterodyne system.

6: APPLICATIONS AND DISCUSSION

Although the noise suppressor is an electronic device, its primary application is in optical experiments. Most laser-based optical systems suffer from excess laser noise and drift, and heroic measures are often necessary to reduce these effects. Some examples of current interest to my colleagues and me, in which the feedback canceller is being used or is planned for use, are two-beam IR absorption spectroscopy using a diode laser[5], detection of phase features in dielectric films and sub-0.1 micron particles on surfaces by scanning optical microscopy, coherent lidar, detection of submicron aerosol particles, detection of residual stress in thin films by stress-induced birefringence, and thermoreflectance thermometry. Others include laser scanning microscopy of biological specimens, and various forms of interferometry, especially those involving fringe tracking. Many of these measurements involve the need to measure a small shift on a large background.

Since the canceller reduces the noise by such a large factor, there are some subtle pitfalls in its use. Primary among these are the differences in laser noise in different parts of the beam cross-section and in different polarization states. Some diode lasers exhibit position-dependent noise, apparently due to the low finesse of their resonators, which results in poor rejection of higher order Gaussian modes. With any laser, it is usually best to add a polarizer at the laser output to insure that the sample and signal beam arise from the same polarization component; the spontaneous emission from the laser usually contains large amounts of power supply noise, which will appear on the signal output if this precaution is not taken.

In measurements which are limited by excess laser noise, there is no point in increasing laser power, since large lasers are often noisier than small ones. With the feedback canceller, this is no longer an issue, so that the signal to shot noise ratio can be improved by raising the laser power.

This device makes possible very great simplifications to many optical systems by obviating a lot of the measures commonly taken to avoid excess laser noise and spurious modulation; it is simple, inexpensive, and highly effective, and seems destined for wide application.

7. REFERENCES

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